PHYSICS 203B (2009)
Course Description

Instructor

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Lectures

M, W 9:30—10:50   Mayer Hall Addition 2623

Homework

Will be assigned during lecture, and solutions will be posted on the web at
http://physics.ucsd.edu/students/courses/spring2009/physics203b/

Close cousins to the homework problems will appear on the mid-terms and final.

Grade

Mid-Term	Mid-Term	Final
25%	25%	50%

Lecture notes will be posted on web at
http://physics.ucsd.edu/students/courses/spring2009/physics203b/

Texts

1. *Classical Theory of Fluids* by Landau and Lifshitz
2. *Classical Electrodynamics* by Jackson

Outline

1. Boundary value problems in electrostatics and magnetostatics
2. Electromagnetic waves and geometrical optics
3. Drude model for $\varepsilon(\omega)$
4. Magnetic diffusion and skin depth
5. Wave guides
6. Radiation

Units (Appendix in Jackson)

Gaussian system

Start with (cm, sec, gm, dyne, erg, etc) from mechanics.

Basic equations for E+M

$$\underset{\sim}{f} = \rho \underset{\sim}{E} + \frac{\underset{\sim}{J}}{c} \times \underset{\sim}{B}$$

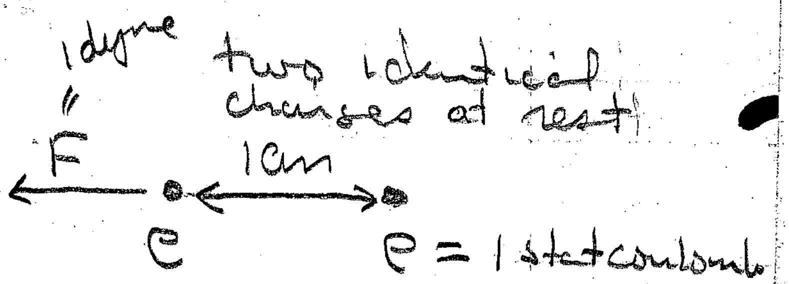
ρ + J are total charge + total current

$$\underset{\sim}{\nabla} \cdot \underset{\sim}{E} = 4\pi\rho \qquad \underset{\sim}{\nabla} \cdot \underset{\sim}{B} = 0$$

$$\underset{\sim}{\nabla} \times \underset{\sim}{B} = \frac{4\pi}{c}\underset{\sim}{J} + \frac{1}{c}\frac{\partial E}{\partial t}, \quad \underset{\sim}{\nabla} \times \underset{\sim}{E} = -\frac{1}{c}\frac{\partial B}{\partial t}$$

Charge

two identical charges at rest



$$\left.\begin{aligned} \underset{\sim}{F} &= e\underset{\sim}{E} \\ \nabla \cdot E &= 4\pi\rho \\ \nabla \times \underset{\sim}{E} &= 0 \end{aligned}\right\} \Longrightarrow \quad F = \frac{e^2}{r^2}$$

$$1 \text{ dyne} = \frac{(\text{statcoulomb})^2}{\text{cm}^2}$$

$\therefore$ statcoulomb $= \sqrt{\text{cm}^2 \text{dyne}}$

current

$$I = \frac{dQ}{dt}$$

$$\text{statampère} = \frac{\text{statcoulomb}}{\text{sec}}$$

$\underline{E}$, electric field

$$\underline{E} = \underline{F}/e$$

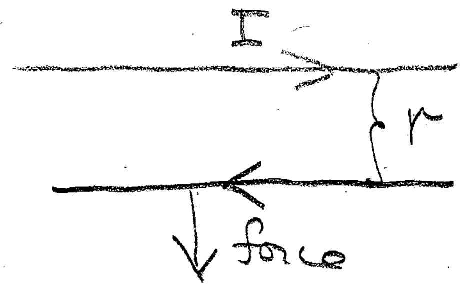
$$\frac{\text{dyne}}{\text{statcoulomb}} = \left(\frac{\text{erg}}{\text{statcoulomb}}\right)\frac{1}{\text{cm}} = \frac{\text{statvolt}}{\text{cm}}$$

electric potential statvolt

capacitance $\quad C = Q/V$

$$\frac{\text{statcoulomb}}{\text{statvolt}} = \frac{(\text{statcoulomb})^2}{\text{dyne cm}} = \text{cm}$$

Force between two current carrying wires



$$\left.\begin{aligned} \underline{f} &= \frac{\underline{J}}{c} \times \underline{B} \\ \underline{\nabla} \times \underline{B} &= \frac{4\pi}{c} \underline{J} \\ \underline{\nabla} \cdot \underline{B} &= 0 \end{aligned}\right\} \Rightarrow \quad \frac{\text{force}}{\text{length}} = \frac{1}{c^2}\left(\frac{2I^2}{r}\right)$$

previously defined

This experiment determines the constant $1/c^2$, and then Maxwell's equations predict speed of light, or measured speed of light and Maxwell predict force/length.

B Magnetic induction $F = Q \frac{v}{c} \times B$

1 Gauss is B-field that produces force of $\frac{v}{c}$ dynes on 1 statcoulomb moving $\perp$ to field.
[Gauss = dyne / statcoul.]

induction $L = \frac{emf}{\left(\frac{dI}{dt}\right)}$

$$\frac{\text{statvolt}}{\text{statamp/sec}} = \frac{\text{sec}^2}{\text{cm}}$$

Magnetized + Polarized material

$\underset{\sim}{P}$, Polarization

$$\frac{\text{dipole moment}}{\text{cm}^3} = \frac{\text{statcoulomb}}{\text{cm}^2}$$

$\underline{D}$, Electric Displacement

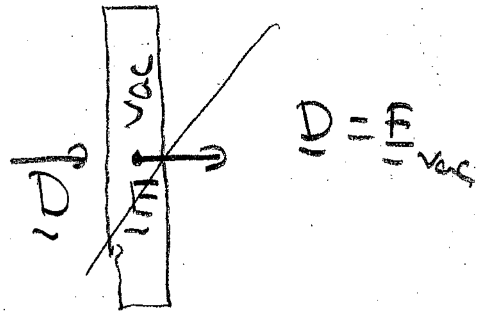
$$\underline{D} = \underline{E} + 4\pi \underline{P}$$

$$\underline{\nabla} \cdot \underline{D} = 4\pi \rho_f$$

$\underline{E} = \underline{D}$ in vac

in medium use

D_n continuous



$$\frac{\text{statvolt}}{\text{cm}} = \frac{\text{statcoul}}{\text{cm}^2}$$

$\underline{M}$, Magnetization

$$\frac{\text{dipole moment}}{\text{cm}^3} = \frac{\text{statcoulomb}}{\text{cm}^2}$$

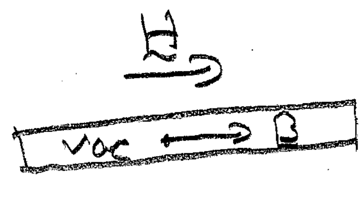
$\underline{H}$, Magnetic field

$$\underline{B} = \underline{H} + 4\pi \underline{M}$$

$$\underline{\nabla} \times \underline{H} = \frac{4\pi}{c} \underline{J}_f + \frac{1}{c} \frac{\partial \underline{D}}{\partial t}$$

$\underline{H} = \underline{B}$ in vac.

in medium use $\underline{H}_t$ continuous



$$\underline{H} = \underline{B}_{vac}$$

Oersted = Gauss

<u>Rationalized MKS units</u>

(m, sec, Nt, Joule, etc.) from mech

Basic equations for E+M

$\underset{\sim}{f} = \rho \underline{E} + \underline{J} \times \underline{B}$ ρ, J are total charge + current

$$\underline{\nabla} \cdot \underline{E} = \rho / \epsilon_0 \quad , \quad \underline{\nabla} \cdot \underline{B} = 0$$

$$\nabla \times \underline{B} = \mu_0 \underline{J} + \mu_0 \epsilon_0 \frac{\partial \underline{E}}{\partial t}$$

$$\nabla \times \underline{E} = -\frac{\partial \underline{B}}{\partial t}$$

Note that

$$\mu_0 \equiv 4\pi \times 10^{-7} \frac{Nt}{(\text{Ampere})^2}$$

defined still to be defined

$$c^2 = \frac{1}{\mu_0 \epsilon_0}$$ is experimentally determined constant

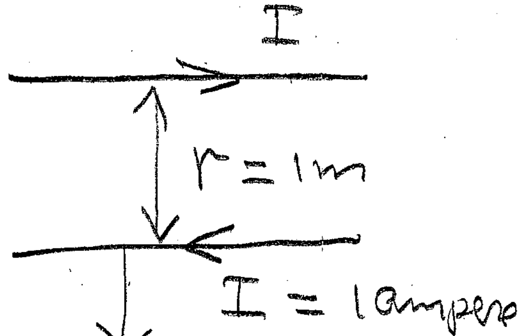
Coulomb's law

$$\underline{E}(\underline{r}) = \frac{1}{4\pi\epsilon_0} \iiint d^3\underline{r}' \frac{\rho(\underline{r}')(\underline{r}-\underline{r}')}{|\underline{r}-\underline{r}'|^3}$$

Biot + Savart

$$\underline{B}(\underline{r}) = \frac{\mu_0}{4\pi} \iiint d^3\underline{r}' \frac{\underline{J}(\underline{r}') \times (\underline{r}-\underline{r}')}{|\underline{r}-\underline{r}'|^3}$$

<u>Current</u> defined through magnetic forces



$$\left.\begin{array}{l} \nabla \times \underset{\sim}{B} = \mu_0 \underset{\sim}{J} \\ \nabla \cdot \underset{\sim}{B} = 0 \\ \underset{\sim}{F} = \underset{\sim}{J} \times \underset{\sim}{B} \end{array}\right\} \Rightarrow \frac{\text{Force}}{\text{length}} = \frac{\mu_0}{2\pi} \frac{I^2}{r}$$

$$\frac{\text{force}}{\text{length}} = 2 \times 10^{-7} \frac{\text{Nt}}{\text{m}}$$

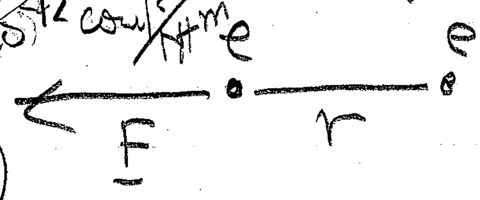
1 ampere in parallel wires separated by 1m produces 2×10^{-7} Nt/m force

<u>Charge</u>

Coulomb ≡ Ampere sec

<u>ϵ_0 can be determined experimentally</u> by measuring force between two charges (or by measuring speed of light)

$\epsilon_0 \simeq 8.854 \times 10^{-12}$ coul^2/Nt m^2



$$\left.\begin{array}{l} \nabla \cdot \underset{\sim}{E} = \rho/\epsilon_0 \\ \nabla \times \underset{\sim}{E} = 0 \\ \underset{\sim}{f} = \rho \underset{\sim}{E} \end{array}\right\} \Rightarrow F = \frac{1}{4\pi\epsilon_0}\left(\frac{e^2}{r^2}\right)$$

already defined

$\underset{\sim}{E}$, electric field $\quad \underset{\sim}{F} = e\underset{=}{E}$

$$\frac{Nt}{coul.} = \frac{Volt}{m}$$

ϕ, electric potential

$$Volt = \frac{Joule}{coul.}$$

C, capacitance

$$farad = \frac{Coul}{Volt}$$ ← (comment on fact that coul is not expressed in terms of units for force & length, so capacitance doesn't reduce to length)

$\underset{\sim}{B}$, magnetic induction $\quad \underset{\sim}{F} = e\underset{\sim}{v} \times \underset{=}{B}$

$$tesla = \frac{Nt}{coul.\ m/sec}$$

magnetic flux $\quad \Phi = \oint \underset{=}{B} \cdot d\underset{=}{S}$

$$weber = tesla\ m^2$$

Inductance

$$\mathcal{E}mf = L\frac{dI}{dt}$$

$$\text{henry} = \frac{\text{volt sec}}{\text{amp}}$$

Homework

(1) Convert the following to Gaussian units
volt, coulomb, henry, tesla, farad

Polarized and magnetized media

$$\rho_b = -\underset{\sim}{\nabla}\cdot\underset{\sim}{P} \quad , \quad \underset{\sim}{J}_b = \frac{\partial \underset{\sim}{P}}{\partial t} + \underset{\sim}{\nabla}\times\underset{\sim}{M}$$

$$\nabla\cdot\underbrace{(\epsilon_0\underset{\sim}{E}+\underset{\sim}{P})}_{\underset{\sim}{D} \text{ units coul/m}^2} = \rho_f$$

$$\nabla\times\underbrace{\left(\frac{\underset{\sim}{B}}{\mu_0}-\underset{\sim}{M}\right)}_{\underset{\sim}{H} \leftarrow \text{units amp/m}} = \underset{\sim}{J}_f + \frac{\partial}{\partial t}\underbrace{(\epsilon_0\underset{\sim}{E}+\underset{\sim}{P})}_{\underset{\sim}{D}}$$

$\underset{\sim}{\nabla} \cdot \underset{\sim}{B} = 0 \qquad \nabla \times \underset{\sim}{E} = -\frac{\partial \underset{\sim}{B}}{\partial t}$

all constants gone

<u>MKS units + atomic physics</u>

batteries tend to have potentials of order Volt.

for few meters of thin copper wire

$R \simeq$ ohm

$\uparrow \frac{\text{amp}}{\text{Volt}}$

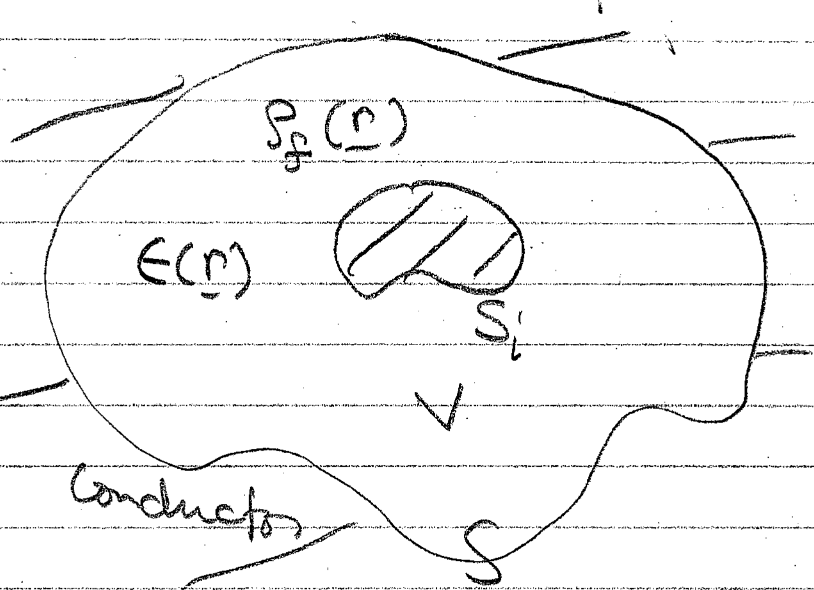
$\therefore I = \frac{V}{R} \sim$ amp

Often symmetry not adequate for direct use of integral representation of Maxwell's equations. Also only told where free charge is, not all of charge. Free charge on surface of conductor may not be specified. Potential is specified.

Boundary value problems

J. Chapters 1,2,3
L+L Vol 8, chapters 1,2
Brau Chapters 3

Uniqueness theorem (motivate)



$$\nabla \cdot \underline{D} = 4\pi \rho_f$$

$$\underline{D} = \epsilon(\underline{r})\, \underline{E} \quad , \quad \underline{E} = -\nabla \varphi$$

$$\nabla \cdot \epsilon(\underline{r}) \nabla \varphi = -4\pi \rho_f(\underline{r})$$

Consider two sol'ns $\varphi_1(\underline{r})$ & $\varphi_2(\underline{r})$

$$\nabla \cdot \epsilon(\underline{r}) \nabla \underbrace{[\varphi_2 - \varphi_1]}_{\Phi} = 0$$

$$\nabla \cdot \Phi \epsilon \nabla \Phi = \epsilon (\nabla \Phi)^2 + \Phi \underbrace{\nabla \cdot [\epsilon \nabla \Phi]}_{= 0}$$

$$\oiint_S d\underline{S} \cdot \Phi \epsilon \nabla \Phi = \iiint_V d^3\underline{r}\, \epsilon (\nabla \Phi)^2$$

note that $\epsilon(\underline{r}) \geq 1$ for thermal equil (L+L Vol 8 page 63)

1. Dirichlet b.c.

$0 = \nabla \Phi = \nabla \varphi_2 - \nabla \varphi_1$ in V if

$0 = \Phi = \varphi_2 - \varphi_1$ on S

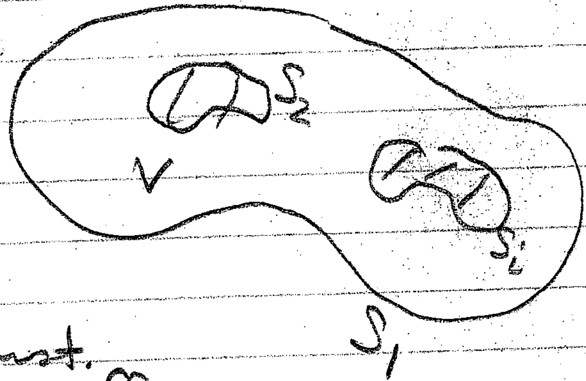
$\therefore \varphi_1 = \varphi_2$ in V

2. Neumann b.c.

$$0 = \nabla \Phi = \nabla \varphi_2 - \nabla \varphi_1 \quad \text{in } V$$

if $0 = \hat{n} \cdot \nabla \Phi = \hat{n} \cdot \nabla \varphi_2 - \hat{n} \cdot \nabla \varphi_1$ on S

modified Neumann b.c.
(appropriate for conductors with total charge specified)



specify $\varphi = \text{const.}$
on S_i and $4\pi Q_i = -\oint\!\!\oint_{S_i} \epsilon \nabla \varphi \cdot d\mathbf{s}$

$$0 = \nabla \Phi = \nabla \varphi_2 - \nabla \varphi_1 \quad \text{in } V$$

if $\Phi = \text{const}$ on S_i and

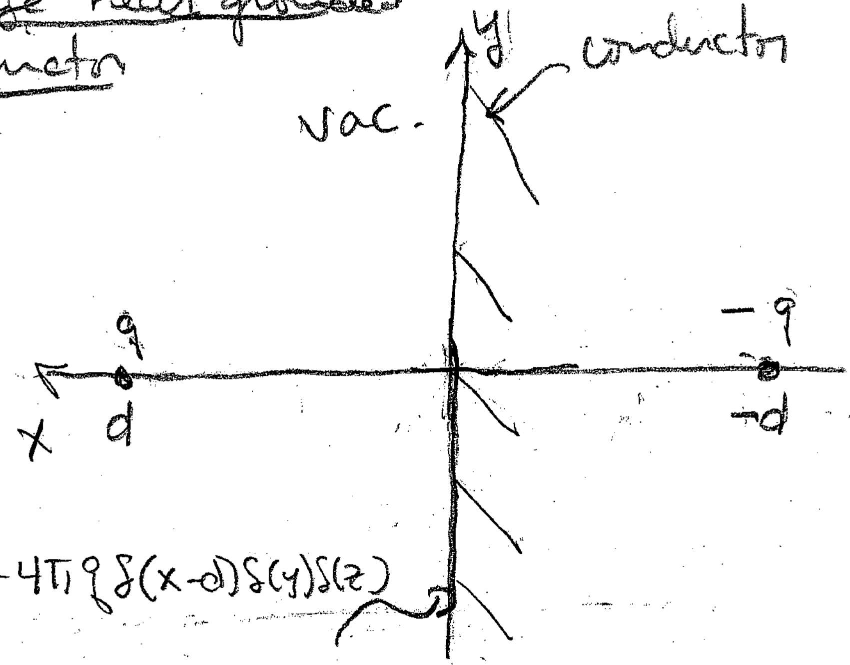
$$0 = \oint\!\!\oint_{S_i} \epsilon \nabla \Phi \cdot d\mathbf{s} = \oint\!\!\oint_{S_i} \epsilon \nabla \varphi_2 \cdot d\mathbf{s} - \oint\!\!\oint_{S_i} \epsilon \nabla \varphi_1 \cdot d\mathbf{s}$$

for all i

Any method that yields soln consistent with b.c. is ok.

<u>method of images</u> (J. Chapter 2)

<u>point charge near grounded plane conductor</u>



$$\nabla^2\varphi = -4\pi q\,\delta(x-d)\,\delta(y)\,\delta(z)$$

$\varphi = 0$

$\varphi(\underline{r}) \to 0$ as $|\underline{r}| \to \infty$

$$\varphi(x,y,z) = \frac{q}{[(x-d)^2+y^2+z^2]^{1/2}} - \frac{q}{[(x+d)^2+y^2+z^2]^{1/2}}$$

$\varphi = 0$ for $x=0$ and $\varphi \to 0$ as $r \to \infty$

discuss surface charge + lines of force

~~read J. sec 4.4 for point charge in front of dielectric interface~~

comment on uniqueness theorem for surface at $|\underline{r}| = \infty$.
$\oint(\varphi_1-\varphi_2)\nabla(\varphi_1-\varphi_2)\cdot d\underline{S} \to 0$ as $|\underline{R}| \to \infty$

Work required to remove particle to ∞

let particle be located at x (i.e., d=x.)

$$F_x(x) = -\frac{q^2}{(2x)^2}$$

$$\text{Work} = \int_x^\infty [-F_x(x')]\,dx' = q^2\int_x^\infty \frac{dx'}{(2x')^2}$$

$$= \frac{q^2}{4x} = \frac{1}{2}\left(\frac{q^2}{2x}\right)$$

Factor of 1/2 because no work done on image charge.

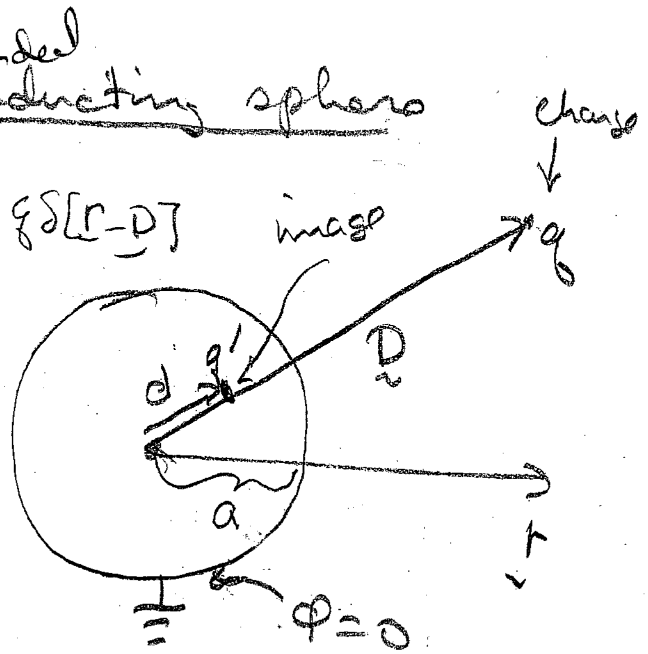
~~alternately, field energy fills 1/2 of space~~

point charge near grounded conducting sphere

$$\nabla^2\varphi = -4\pi q\,\delta[\underset{\sim}{r}-\underset{\sim}{D}]$$

let $q' = -q\frac{a}{D}$

$$\underset{\sim}{d} = \underset{\sim}{D}\left(\frac{a}{D}\right)^2$$



$\varphi(\underset{\sim}{r}) \to 0$ as $r \to \infty$

outside sphere

$$\varphi(\underset{\sim}{r}) = \frac{q}{|\underset{\sim}{r}-\underset{\sim}{D}|} - \frac{q\,a/D}{|\underset{\sim}{r}-\underset{\sim}{D}\frac{a^2}{D^2}|}$$

$$\frac{q\frac{a}{D}}{|\underset{\sim}{r}-\underset{\sim}{D}\frac{a^2}{D^2}|} = \frac{q}{|D\frac{\underset{\sim}{r}}{a}-\frac{\underset{\sim}{D}}{D}a|} \underset{\uparrow}{=} \frac{q}{|\underset{\sim}{D}-\underset{\sim}{r}|}$$

for $|\underset{\sim}{r}| = a$

$\therefore\ \varphi(a) = 0$ also $\varphi(r) \to 0$ as $r \to \infty$

discuss surface charge on sphere and lines of force

let vac $\to$ diel ϵ

Suppose that the condition $\Phi(a) = 0$ is replaced by the condition that net charge on sphere is Q.

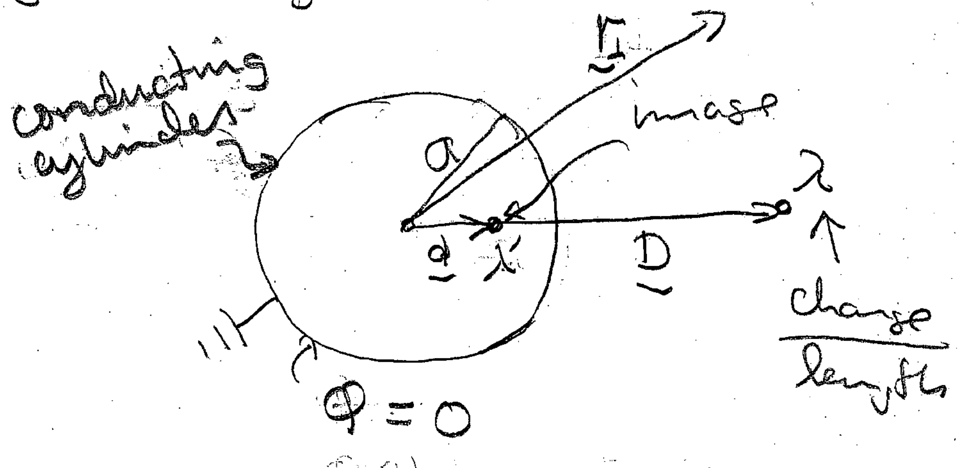
$$\Phi(\underset{\sim}{r}) = \frac{q}{|\underset{\sim}{r} - \underset{\sim}{D}|} - \frac{q\frac{a}{D}}{|\underset{\sim}{r} - \underset{\sim}{D}\frac{a^2}{D^2}|} + \frac{Q + q\frac{a}{D}}{|\underset{\sim}{r}|}$$

Discuss

$$-\oint\!\!\oint \underset{\sim}{\nabla}\Phi \cdot d\underset{\sim}{S} = 4\pi\left[Q + \cancel{q\frac{a}{D}} - \cancel{q\frac{a}{D}}\right]$$

~~electron charge~~ near gold atom

<u>line charge near grounded conducting cylinder</u>



let $\lambda' = -\lambda$

$$\underset{\sim}{d} = \frac{a^2}{D^2}\underset{\sim}{D}$$

outside cylinder

$$\varphi(\underset{\sim}{r}) = -2\lambda \ln|\underset{\sim}{r} - \underset{\sim}{D}| + 2\lambda \ln\left|\left(\underset{\sim}{r} - \frac{a^2}{D^2}\underset{\sim}{D}\right)\frac{D}{a}\right|$$

$$= -2\lambda \ln \frac{|\underset{\sim}{r} - \underset{\sim}{D}|}{\left|\frac{\underset{\sim}{r}}{a}D - a\frac{\underset{\sim}{D}}{D}\right|}$$

$\therefore$ $\varphi(\underset{\sim}{r}) = 0$ for $|\underset{\sim}{r}| = a$

$$\lim_{|\underset{\sim}{r}| \to \infty} \varphi(\underset{\sim}{r}) = -2\lambda \ln\left(\frac{a}{D}\right)$$

not localized charge dist.

The condition $\varphi = 0$ at ∞ is replaced by φ remains finite at ∞. For example, this eliminates term

$$\lambda'' \ln \frac{r}{a}.$$

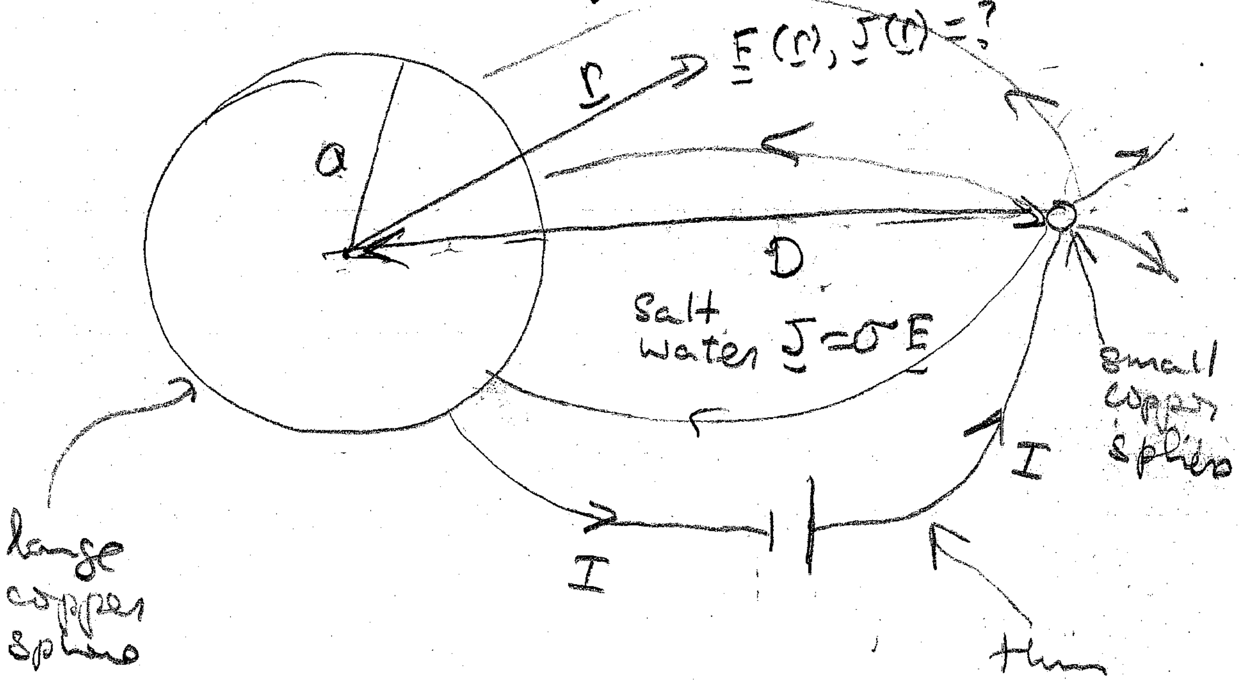
Discuss surface charge and lines of force.

let vac $\rightarrow$ diel ϵ

Homework

(2) J. 2.2
(3) J. 2.3
(4) J. 2.9 } read sec. 2.5 (sphere
(5) J. 2.10 } in uniform E-field)

<u>Example</u> steady state current



$\sigma_{copper} \gg \sigma_{\substack{salt \\ water}}$

find current dist. neglect field of wire

$$\underset{\sim}{E} = -\underset{\sim}{\nabla}\varphi \quad , \quad \underset{\sim}{J} = \sigma \underset{\sim}{E}$$

$$0 = \frac{\partial \rho}{\partial t} = -\underset{\sim}{\nabla}\cdot\underset{\sim}{J} = +\sigma\nabla^2\varphi$$

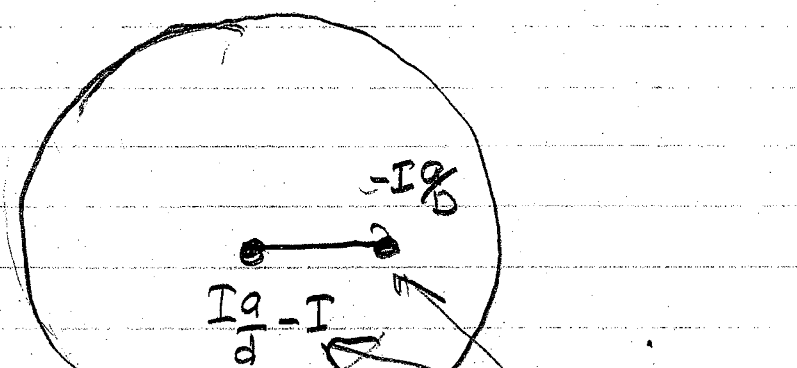
$$\underset{\substack{\text{small}\\\text{sphere}}}{\oiint} \underset{\sim}{J}\cdot d\underset{\sim}{S} = I \quad , \quad \underset{\substack{\text{large}\\\text{sphere}}}{\oiint} \underset{\sim}{J}\cdot d\underset{\sim}{S} = -I$$

φ = const. on large sphere
solve by image method

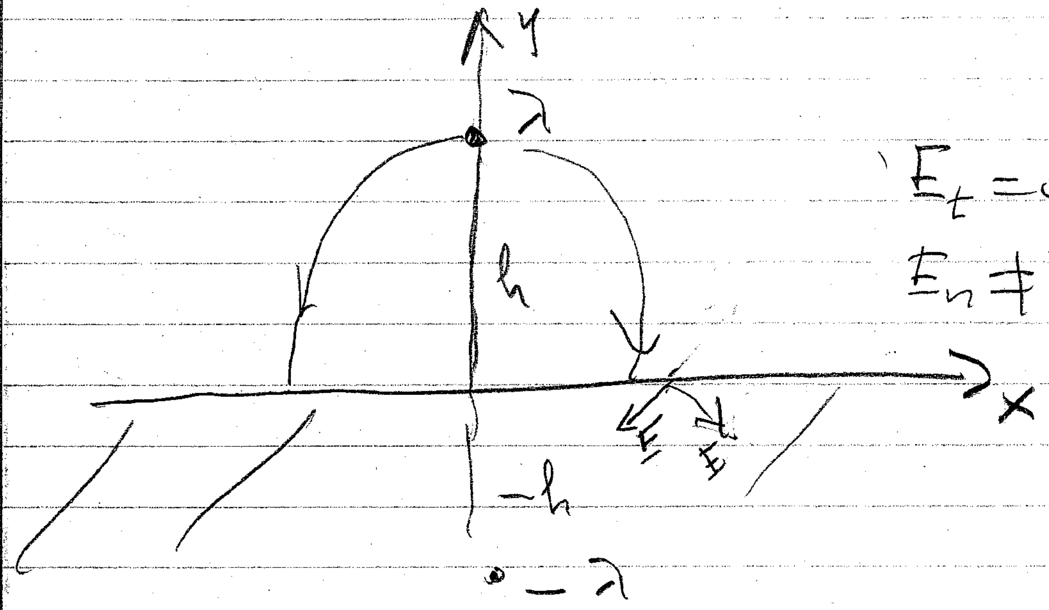
$$\varphi(\underset{\sim}{r}) = \frac{I}{4\pi\sigma|\underset{\sim}{r}-\underset{\sim}{D}|} - \frac{I\frac{a}{D}}{4\pi\sigma\left|\underset{\sim}{r}-\underset{\sim}{D}\frac{a^2}{D^2}\right|}$$

$$+ \frac{I\frac{a}{D} - I}{4\pi\sigma|\underset{\sim}{r}|}$$

due to images in sphere



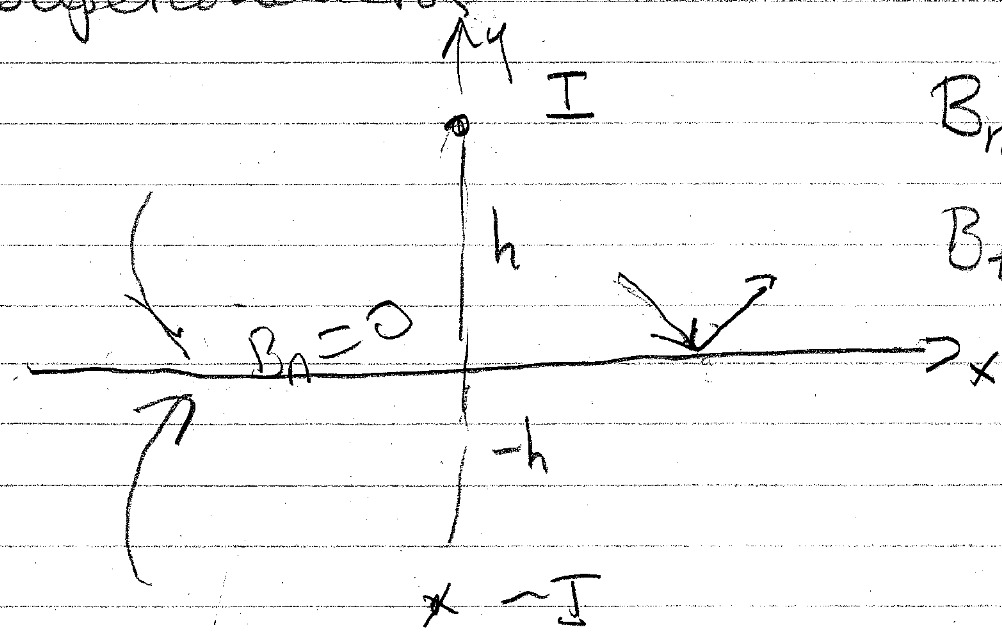
Charged wire above conductor



$E_t = 0$

$E_n \neq 0$

Current carrying wire above superconductor



$B_n = 0$

$B_t \neq 0$

Use vector potential for current carrying wire

$$\nabla^2 A_z = -\frac{4\pi}{c} J_z = -\frac{4\pi I}{c}\delta(x)\,\delta(y-h)$$

at surface require

$$0 = B_y = \hat{y}\cdot\nabla\times\hat{z}A_z(x,y) = \frac{\partial A_z}{\partial x}$$

A_z = const on surface, so

$A_z(x,y)$ is like $\Phi(x,y)$