HW 3 QM III

★ **Anomalous Hall effect:** Soon after the discovery of Hall effect in 19th century, Hall discovered the Hall voltage in ferromagnets without external magnetic field. The mechanism of this anomalous Hall effect has been in debating for many years. For a recent review, see arxiv:0904.4154. Today anomalous Hall effect remains a huge active field in condensed matter physics. One mechanism for the anomalous Hall effect is called the intrinsic mechnism due to the Berry phase effect.

Let us consider a spin-orbit coupling Hamiltonian of the Rasha type in the 2D free space

$$H = \frac{-\hbar^2\nabla^2}{2m} + \lambda\,(-i\hbar\vec{\nabla}\times\vec{\sigma})\cdot\hat{e}_z - \Delta\ \sigma_z$$

where $\vec{\sigma}$s are Pauli matrices, and Δ is an effective Zeeman field that originates from the ferromagnetic ordering; λ is the spin-orbit coupling constant.

① In the momentum representation, show that

$$H(k) = \frac{\hbar^2 k^2}{2m} + \vec{n}(k)\cdot\vec{\sigma}, \quad \text{where } \vec{n} = (-\lambda k_y, \lambda k_x, -\Delta)$$

$\vec{n}$ here is not normalized to 1. Solve the spectra of $H(k)$,

$$= \frac{\hbar^2 k^2}{2m} \pm \sqrt{n^2 + \Delta^2}$$ ②

which have two branches $\boxed{\mathcal{E}_\pm(k)}$ Plot the spectra $\mathcal{E}_\pm(k)$ as function of (k_x, k_y).

for both $\Delta = 0$ and $\Delta \neq 0$.

② Calcutate the eigenvectors $\psi_\pm(\vec{k})$, at $\Delta \neq 0$ Berry connetions for $\psi_+(k)$

$\vec{A}_+(k_x, k_y) = \langle \psi_+(k) | -i \vec{\nabla} | \psi_+(k) \rangle$, and Berry curvature

$$\Omega^+ = \partial_{k_x} A_{+,k_y} - \partial_{k_y} A_{+,k_x}.$$

③ Repeat the same calculation for the branch of $\psi_-(k)$,

Show that $\Omega^- = -\Omega^+$. Check that

$$-\int dk_x dk_y \, \Omega^+ = \int dk_x dk_y \, \Omega^- = \pi.$$

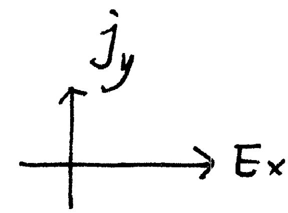
④ Plot Ω^- in the 2D momentum space at $\Delta \neq 0$

Where does Ω^- get the maximal value? Then decrease the value of Δ, how does the distribution of Ω^- change? Finally, set $\Delta = 0$, what happens?

5. The anomalous velocity arising from Berry curvature

$$\vec{v}_+(\vec{k}) = \frac{\partial \varepsilon_+}{\partial \vec{k}} - \frac{e}{\hbar}\vec{E} \times \Omega^+_z \hat{z},$$

$$\vec{v}_-(\vec{k}) = \frac{\partial \varepsilon_-}{\partial \vec{k}} - \frac{e}{\hbar}\vec{E} \times \Omega^-_z \hat{z}.$$



Show that this gives rise to the transverse current, i.e. the Hall current.

The corresponding Hall conductance defined as $\sigma_{xy} = \frac{j_y}{E_x}$.

Show: $$\sigma^{2D}_{xy} = \frac{e^2}{\hbar}\int \frac{dk_x dk_y}{(2\pi)^2} f(\varepsilon_+(k))\,\Omega^+ + f(\varepsilon_-(k))\,\Omega^-$$

$$= \frac{e^2}{\hbar^2}\int \frac{dk_x dk_y}{(2\pi)^2} \left[f(\varepsilon_-(k)) - f(\varepsilon_+(k))\right]\Omega^-$$

where $f(\varepsilon)$ is the Fermi distribution function.