Formula sheet

Constants and Factors

Speed of light: $c = 299,792,458$ m/s exactly (about 3×10^8 meters/sec)
Newton's constant $G = 6.67 \times 10^{-11}$ m^3/s^2 kg
Coulomb constant $k = 1/(4\pi\epsilon_0) = 9 \times 10^9$ N m^2/C^2, $\epsilon_0 = 8.85 \times 10^{-12}$ C^2/N m^2
Charge on electron $q_e = -1.6 \times 10^{-19}$ C
Mass of proton and neutron about 1.67 $\times 10^{-27}$ kg
Mass of electron: $m_e = 9.11 \times 10^{-31}$kg
Permeability constant: $\mu_0 = 4\pi \times 10^{-7}$N/A^2
1 dyne = 10^{-5} Newtons

Formulas

Coulomb law: $\vec{F}_{12} = \frac{kq_1q_2}{r^2}\hat{r}$
Electric field: $\vec{E} = \vec{F}/q = \frac{kq}{r^2}\hat{r}$; $\vec{E} = \int d\vec{E} = \int dqk\hat{r}/r^2$
Newton's acceleration law: $\vec{F} = m\vec{a}$
Newton's law of Gravity: $\vec{F} = -\frac{GMm}{r^2}\hat{r}$
Electric dipole moment: $\vec{p} = q\vec{d}$, from negative to positive.
Field far from electric dipole: $\vec{E} = -\hat{i}kp/y^3$ (perpendicular); $\vec{E} = 2\hat{i}kp/x^3$ (on dipole axis);
Electric field along axis of circular ring of charge Q, with radius a: $E = kxQ/(x^2 + a^2)^{3/2}$
Field far from line charge: $E = 2k\lambda/r$ radial direction; $\lambda = Q/L$
Torque on dipole: $\vec{\tau} = \vec{p} \times \vec{E}$; Potential energy of dipole: $\vec{\tau} = -\vec{p} \cdot \vec{E}$

Gauss's law: flux through surface, $\phi = \oint \vec{E} \cdot d\vec{A} = q_{enclosed}/\epsilon_0$, $4\pi\epsilon_0 = 1/k$
Flat sheet with uniform charge density: $E = \sigma/(2\epsilon_0)$ perpendicular to surface
At surface of conductor : $E = \sigma/\epsilon_0$ perpendicular to surface

Electric potential difference: $\Delta V_{AB} = -\int_A^B \vec{E} \cdot d\vec{l}$
Potential outside spherical charge distribution (from infinity): $V(r) = kq/r$
Potential of charge distribution: $V = \int dV = k\int dq/r$
Electric field from potential: $E_l = -dV/dl$

Energy density in electric field: $u_E = \frac{1}{2}\epsilon_0 E^2$
Total energy in electric field $U = \int u_E d\text{Vol} = \frac{1}{2}\epsilon_0 \int E^2 d\text{Vol}$
Capacitance: $C = Q/V$
General energy stored in capacitor: $U = \frac{1}{2}Q^2/C = \frac{1}{2}CV^2$
Parallel plate capacitor: Capacitance: $C = \epsilon_0 A/d$; stored energy: $U = Q^2 d/(2\epsilon_0 A)$
Cylindrical capacitor inner radius a, outer b: : $C = 2\pi\epsilon_0 L/\ln(b/a)$; $U = kL\lambda^2 \ln(b/a)$; $\lambda = Q/L$
Spherical capacitor: inner radius a, outer b: $C = 4\pi\epsilon_0 ab/(b-a)$; $U = \frac{1}{2}kQ^2(1/a - 1/b)$
Capacitors in parallel: $C = C_1 + C_2 + \cdots$; in series: $1/C = 1/C_1 + 1/C_2 + \cdots$
Dielectrics: $C \rightarrow \kappa C_0$; (constant Q: $E \rightarrow E_0/\kappa$; $U \rightarrow U_0/\kappa$)

Current: $I = dQ/dt$, Power $P = VdQ/dt = VI$; (I in Amps; t in sec; Q in coul)
Ohm's laws: $V = IR$; $E = \rho J$; (R in Ohms, ρ in Ohm meters, J is current density (Amps/m^2)
For an object of length l and area A: $R = l\rho/A$
Resistors in series: $R_{tot} = R_1 + R_2 + R_3 + \cdots$; in parallel: $1/R_{tot} = 1/R_1 + 1/R_2 + 1/R_3 + \cdots$
Kirchhoff's laws: voltage differences around closed loop = 0. Sum of currents at any node =0.
RC circuit charging: $Q(t) = CV_0(1 - e^{-t/RC})$; $V(t) = V_0(1 - e^{-t/RC})$; $I(t) = (V_0/R)e^{-t/RC}$
RC circuit discharging: $Q(t) = V_0 e^{-t/RC}$; $I(t) = (V_0/R)e^{-t/RC}$; $Q(t) = CV(t)$

Magnetic Force: $\vec{F} = q\vec{v} \times \vec{B} = qvB\sin\theta$; Total electromagnetic force: $\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$
Circular motion of charged particle in uniform magnetic field: $r = mv/(qB)$
Cyclotron frequency: $f = 1/T = qB/(2\pi m)$
Magnetic Force on a current: $d\vec{F} = Id\vec{l} \times \vec{B}$
Magnet dipole: $\mu = NIA$; torque: $\vec{\tau} = \vec{\mu} \times \vec{B}$; potential energy $U_{mag} = -\vec{\mu} \cdot \vec{B}$.
Biot-Savart Law: $d\vec{B} = (\mu_0/4\pi)Id\vec{l} \times \hat{r}/r^2$; $\vec{B} = \int d\vec{B}$

B-field from circle of radius a on loop axis: $B = \mu_0 I a^2/(2(x^2 + a^2)^{3/2})$; $(a \ll x) B \to \mu_0 \mu/(2\pi x^3)$
B-field for straight wire: $B = \mu_0 I/(2\pi r)$
Force between two parallel wires: $F_2 = I_2 l B_1 = \mu_0 I_1 I_2 l/(2\pi d)$
Ampere's law for steady current: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encircled}$

Magnetic Flux through surface A: $\phi_B = \int \vec{B} \cdot d\vec{A}$
Faraday's law: EMF around closed loop $= \oint \vec{E} \cdot d\vec{l} = -d\phi_B/dt$
Lenz's law: The direction of the induced EMF and current is such that the B-field produced will OPPOSE the change in B-field

Mutual inductance: $M = \phi_2/I_1$
Self inductance: $L = \phi/I$
Voltage across inductance: mutual: $V_2 = -M dI_1/dt$; Self inductance: $V_L = -L dI/dt$
LR circuit starting up with battery with voltage V_0, resistance R, and inductance L: $V_L = -V_0 e^{-Rt/L}$; $I = (V_0/R)(1 - e^{-Rt/L})$
LR circuit turning off with resistance R, and inductance L, and intial current I_0: $I = I_0 e^{-Rt/L}$
Magnetic energy in an inductor: $U_B = \frac{1}{2} L I^2$
Magnetic energy density: $u_B = \frac{1}{2} B^2/\mu_0$