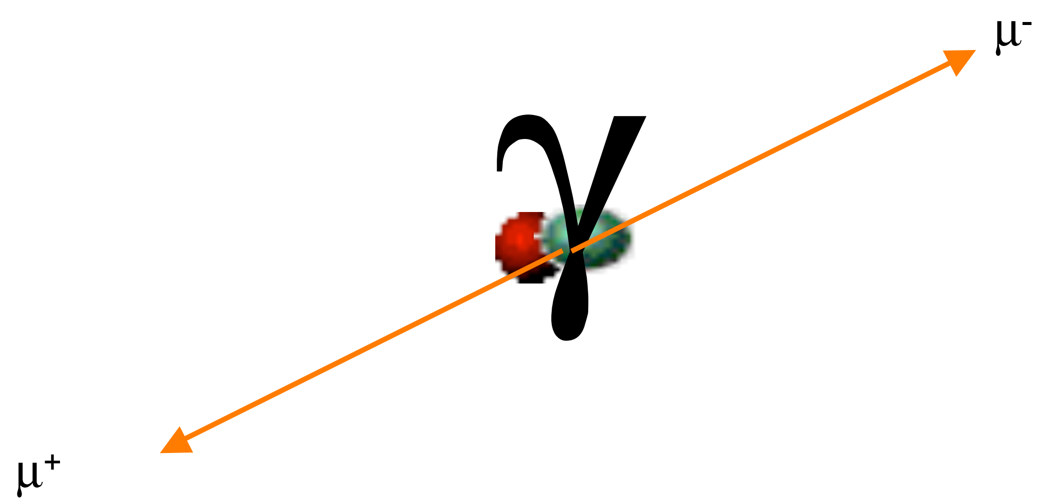
Physics 2D Lecture Slides
Lecture 9

April 14, 2009

Mass Can “Morph” into Energy & Vice Versa

- Unlike in Newtonian mechanics
- In relativistic physics : Mass and Energy are the same thing
- New word/concept : MassEnergy , just like SpaceTime
- It is the mass-energy that is always conserved in every reaction : Before & After a reaction has happened
- Like squeezing a balloon : Squeeze here, it grows elsewhere
  - **If you “squeeze” mass, it becomes (kinetic) energy & vice verca !**
    - **CONVERSION FACTOR = C^2**
    - **This exchange rate never changes !**

Creation and Annihilation of Particles

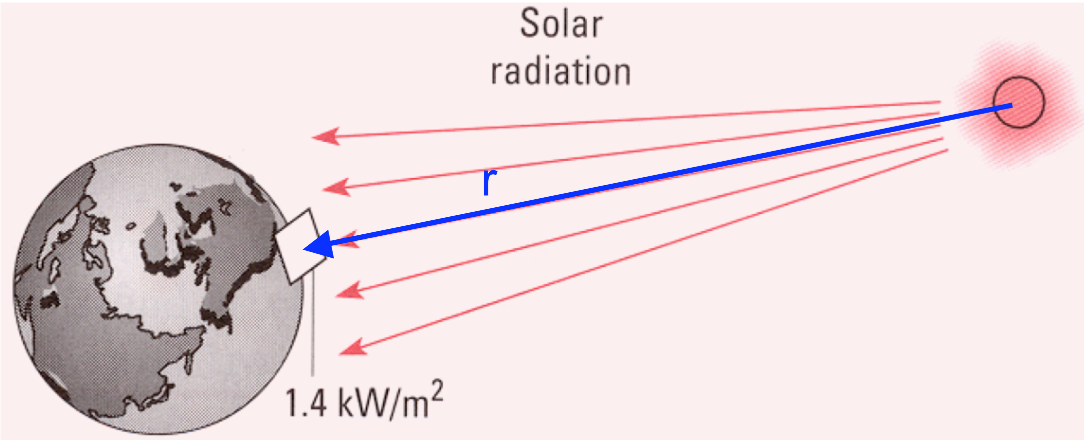


Sequence of events in a matter-antimatter collision:

$$e^{+} + e^{-} \rightarrow \gamma \rightarrow \mu^{+} + \mu^{-}$$

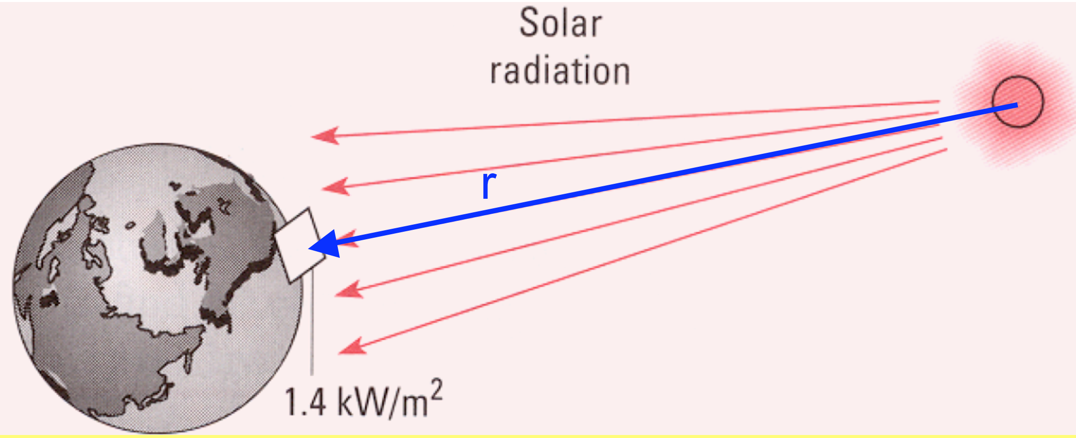
$E=mc^2 \Rightarrow$ Sunshine Won't Be Forever !

Q: Solar Energy reaches earth at rate of 1.4kW per square meter of surface perpendicular to the direction of the sun. by how much does the mass of sun decrease per second owing to energy loss? The mean radius of the Earth's orbit is 1.5×10^{11}m.



- Surface area of a sphere of radius r is $A = 4\pi r^2$
- Total Power radiated by Sun = power received by a sphere whose radius is equal to earth's orbit radius

E= mc² ⇒ Sunshine Won't Be Forever !



Total Power radiated by Sun = power received by a sphere with radius equal to earth-sun orbit radius(r in figure)

$$P^{sun}_{lost} = \frac{P^{Earth}_{incident}}{A} A_{earth-sun} = \frac{P^{Earth}_{incident}}{A} 4\pi r^2_{earth-sun} = (1.4 \times 10^3 W / m^2)(4\pi)(1.5 \times 10^{11})^2$$

$$P^{sun}_{lost} = 4.0 \times 10^{26} W$$

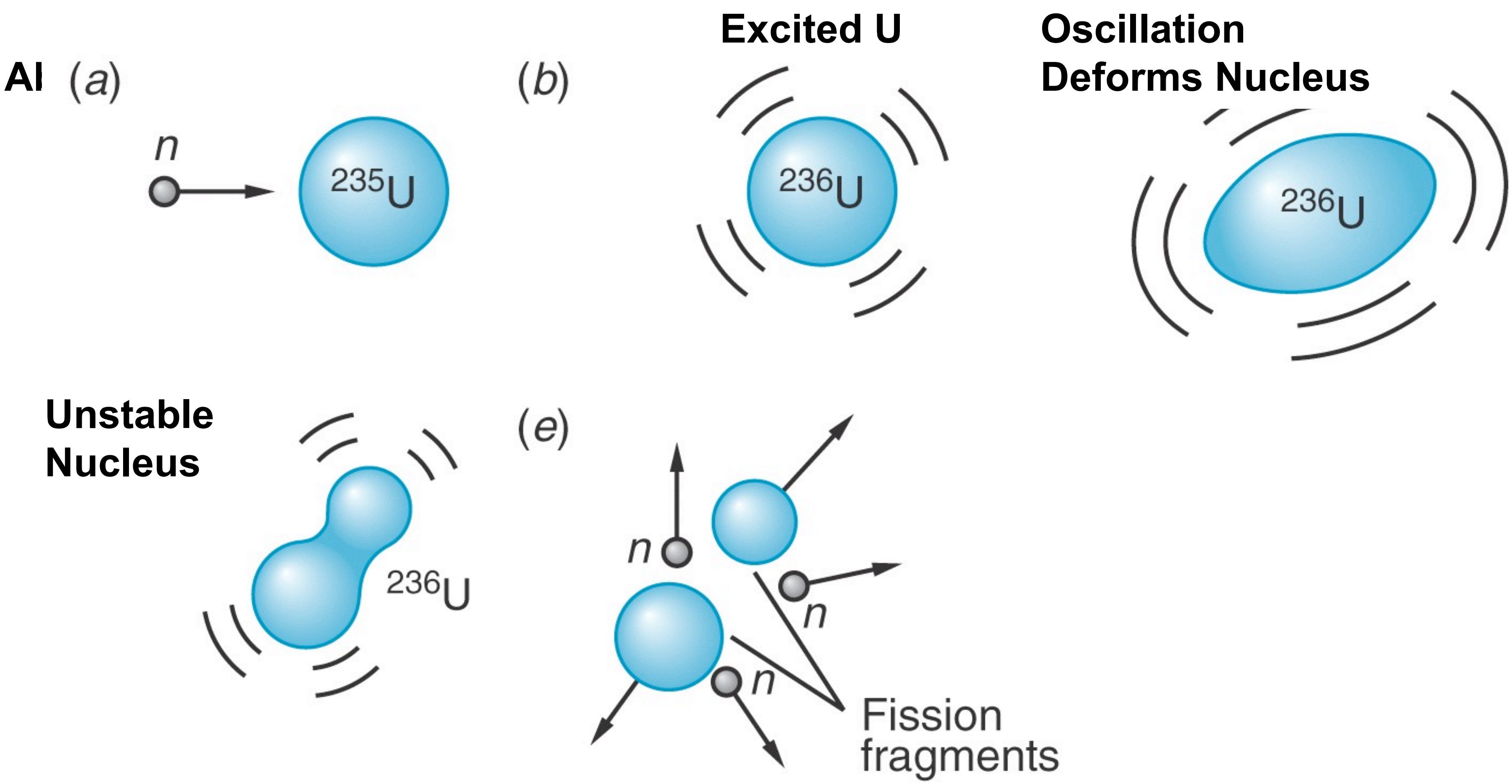
So Sun loses E = 4.0×10^{26} J of rest energy per second

Its mass decreases by $m = \frac{E}{c^2} = \frac{4.0 \times 10^{26}\,\mathrm{J}}{(3.0 \times 10^8)^2} = 4.4 \times 10^9\,kg$ per sec!!

If the Sun's Mass = $2.0 \times 10^{30} kg$ So how long with the Sun last ?

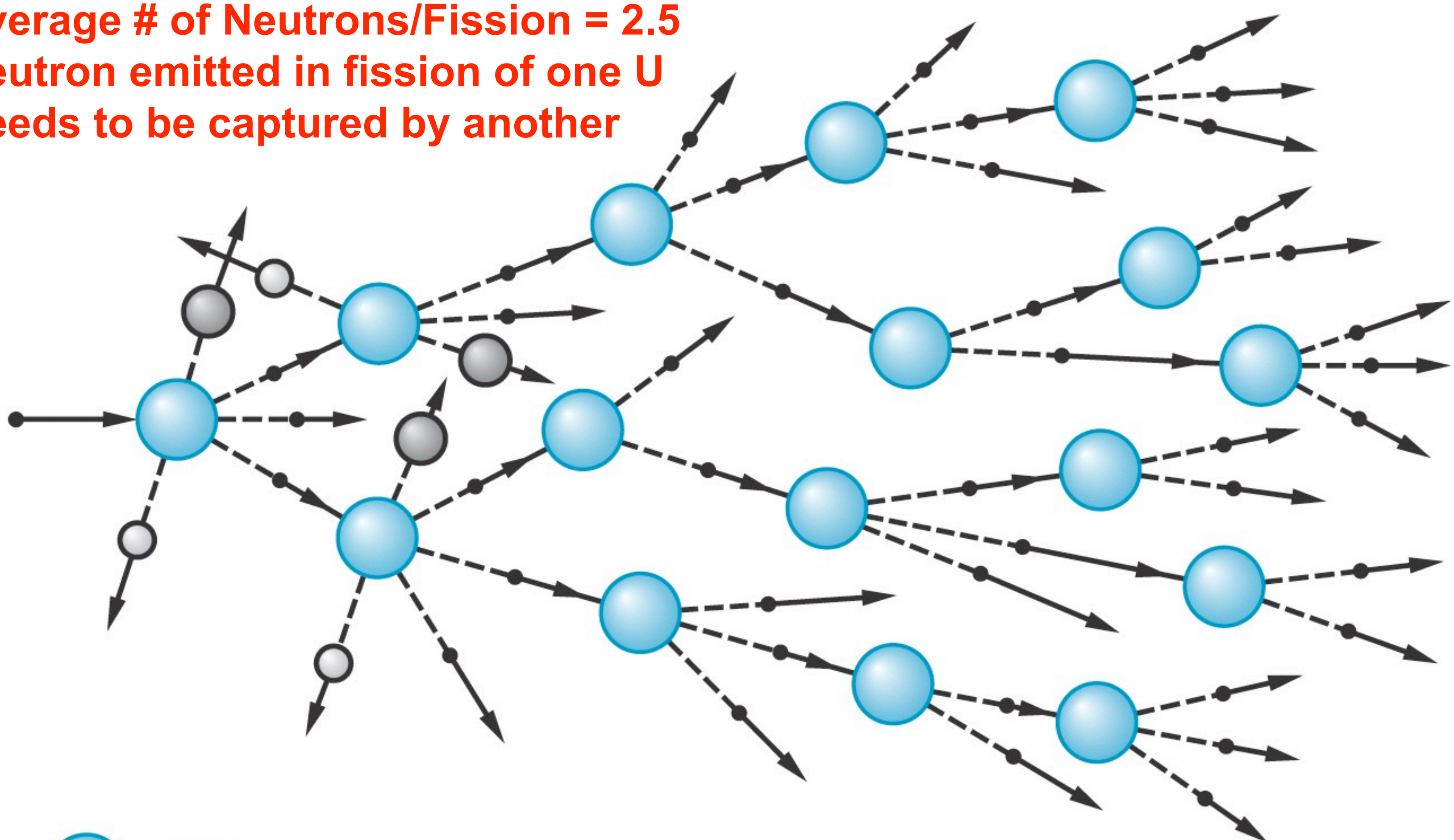
One day the sun will be gone and the solar system will not be a hospitable place for life

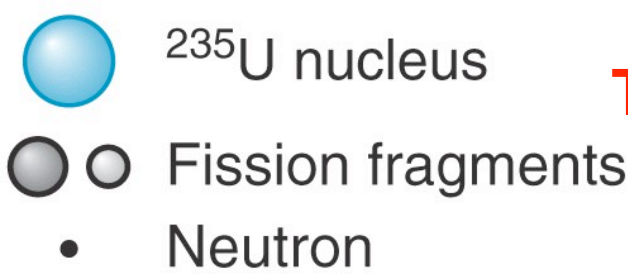
Nuclear Fission Schematic : Tickling a Nucleus



Sustaining Chain Reaction: 1st three Fissions

Average # of Neutrons/Fission = 2.5
Neutron emitted in fission of one U
Needs to be captured by another

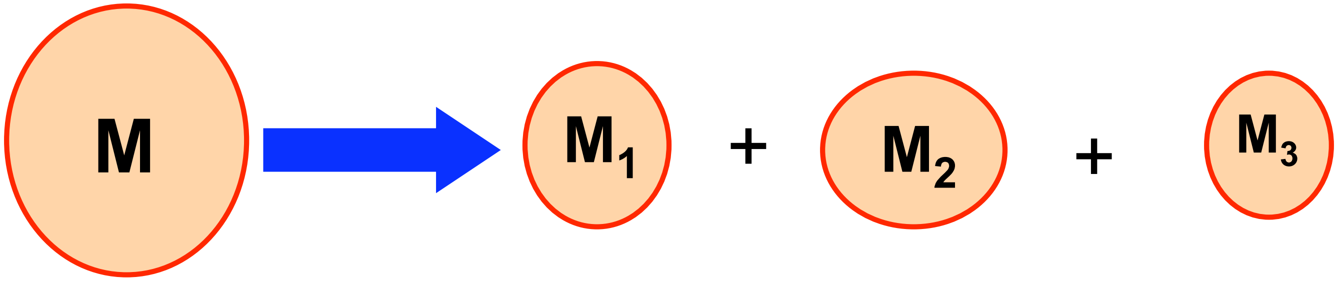




To control reaction => define factor K

Supercritical K >> 1 in a Nuclear Bomb
Critical K = 1 in a Nuclear Reactor

Conservation of Mass-Energy: Nuclear Fission



Nuclear Fission

$$Mc^2 = \frac{M_1c^2}{\underbrace{\sqrt{1-\frac{u_1^2}{c^2}}}_{<1}} + \frac{M_2c^2}{\underbrace{\sqrt{1-\frac{u_2^2}{c^2}}}_{<1}} + \frac{M_3c^2}{\underbrace{\sqrt{1-\frac{u_3^2}{c^2}}}_{<1}} \Rightarrow M > M_1 + M_2 + M_3$$

Loss of mass shows up as kinetic energy of final state particles

Disintegration energy per fission $Q=(M-(M_1+M_2+M_3))c^2 = \Delta Mc^2$

$^{236}_{92}\mathrm{U} \rightarrow {}^{143}_{55}\mathrm{Cs} + {}^{90}_{92}\mathrm{Rb} + 3\,{}^{1}_{0}\mathrm{n}$ (1 AMU= $1.6605402 \times 10^{-27} kg = 931.49$ MeV)

$\Delta m = 0.177537\mathrm{u} = 2.9471 \times 10^{-28} kg = 165.4$ MeV= energy release/fission =peanuts

What makes it explosive is 1 mole of Uranium = 6.023×10^{23} Nuclei !!

Energy Released by 1 Kg of Fissionable Uranium

$1 \text{ Mole of Uranium} = 236 \text{ gm}, \quad \text{Avagadro''s \#} = 6.023 \times 10^{23} \text{ Nuclei}$

$$\text{So in 1 kg } N = \frac{6.023 \times 10^{23}}{236g / mole} \times 1000g = 2.55 \times 10^{24} \text{ nuclei}$$

$1 \text{ Nuclear fission} = 165.4 \text{ MeV} \therefore \; 10^3 \; g = 2.55 \times 10^{24} \times 165.4 \text{ MeV}$

$\text{Note } 1 \text{ MeV} = 4.45 \times 10^{-20} \; kWh$

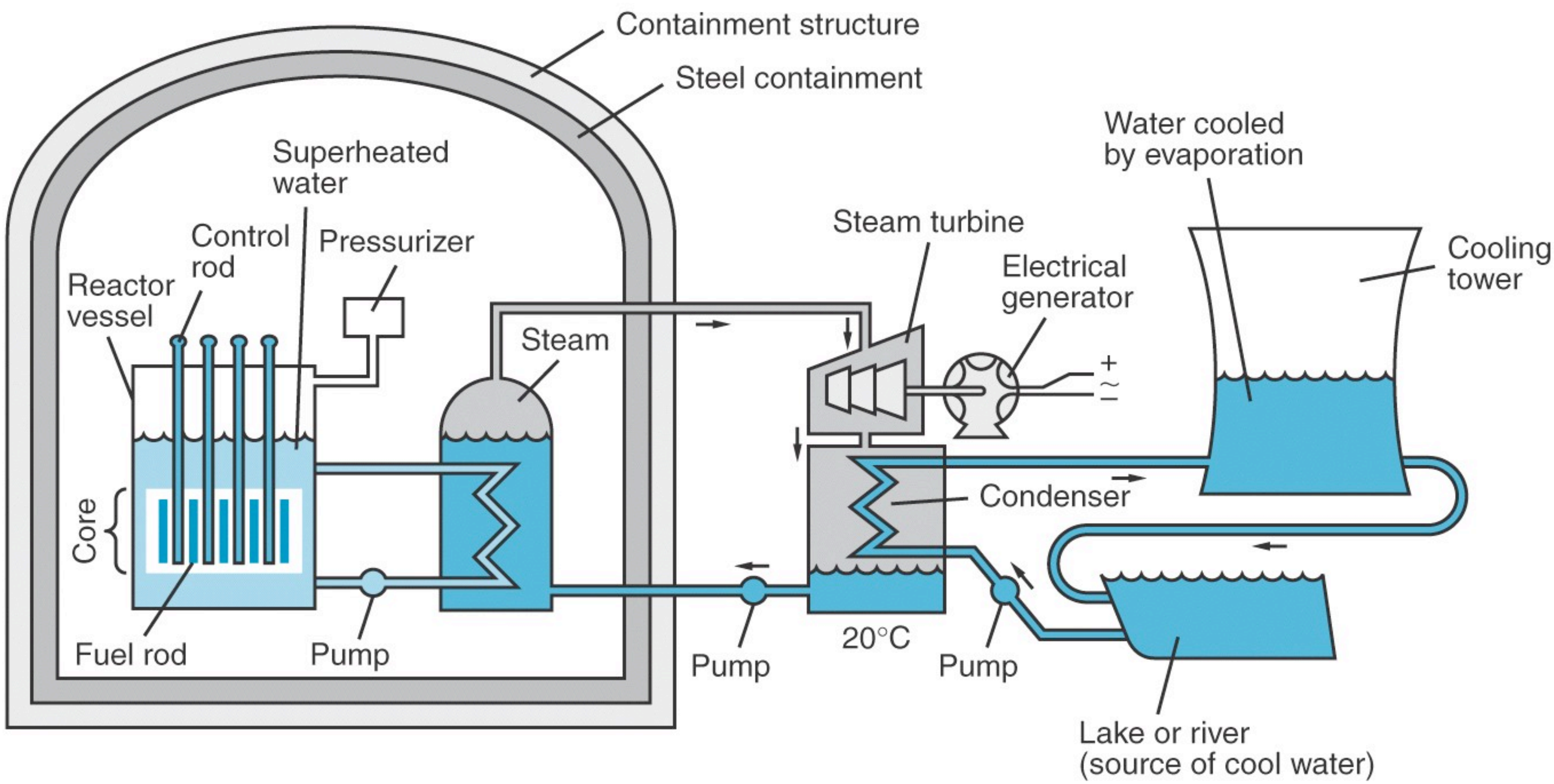
If the power plant has conversion efficiency = 40%

$\text{Energy Transformed} = 748 \times 10^6 \; kWh$

$\Rightarrow$ 1 $100W$ lamp can be lit for 8500 years !

Schematic of a Pressurized-Water Reactor

Water in contact with reactor core serves as a moderator and heat transfer Medium. Heat produced in fission drives turbine



Nuclear Fusion : What Powers the Sun

Opposite of Fission

Mass of a Nucleus $<$ mass of its component protons+Neutrons

Nuclei are stable, bound by an attractive "Strong Force"

Think of Nuclei as molecules and proton/neutron as atoms making it

Binding Energy: Work/Energy required to pull a bound system (M) apart leaving its components (m) free of the attractive force and at rest:

$$Mc^2 + BE = \sum_{i=1}^{n} m_i c^2$$

$${}^{2}_{1}H \quad + \quad {}^{2}_{1}H \quad = \quad {}^{4}_{2}He \quad + \quad 23.9\ \text{MeV} \quad =$$

Deuterium Deuterium = Helium + Released Energy

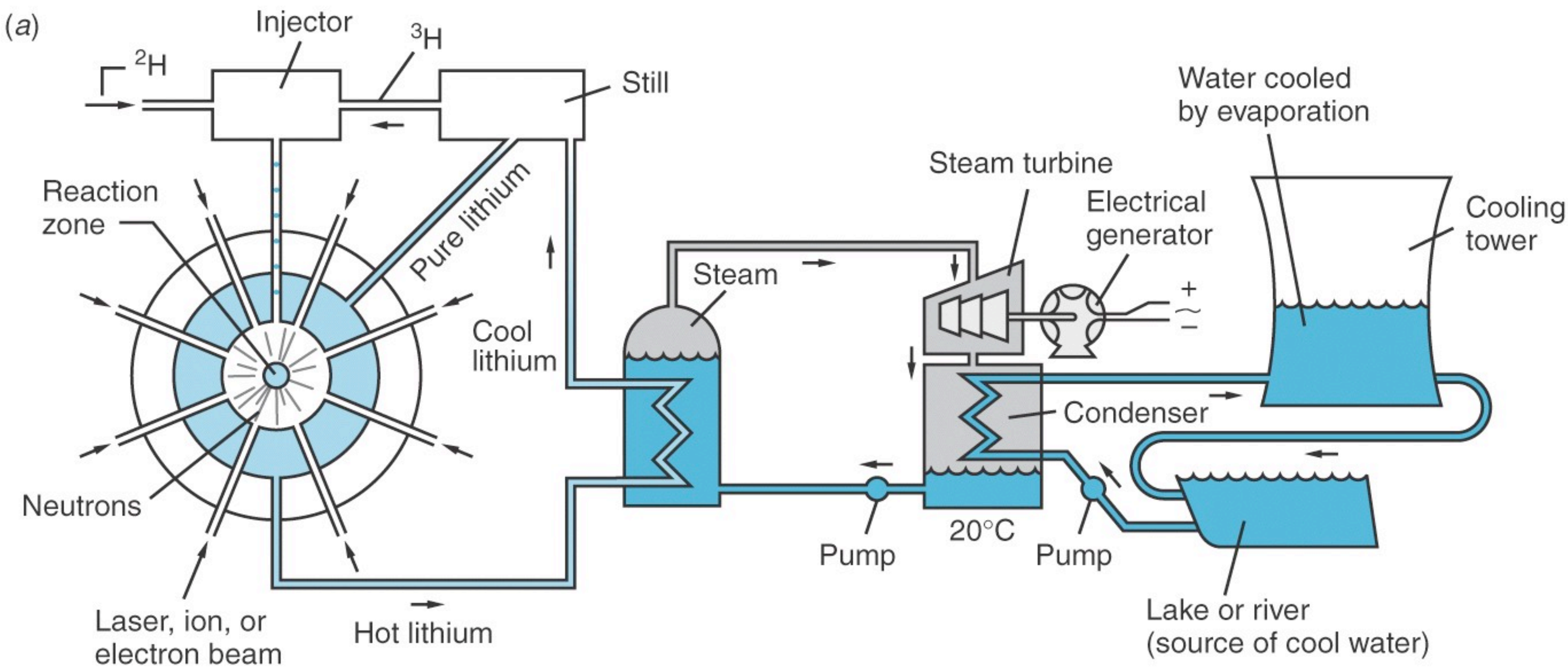
Think of energy released in Fusion as Dissociation energy of Chemistry

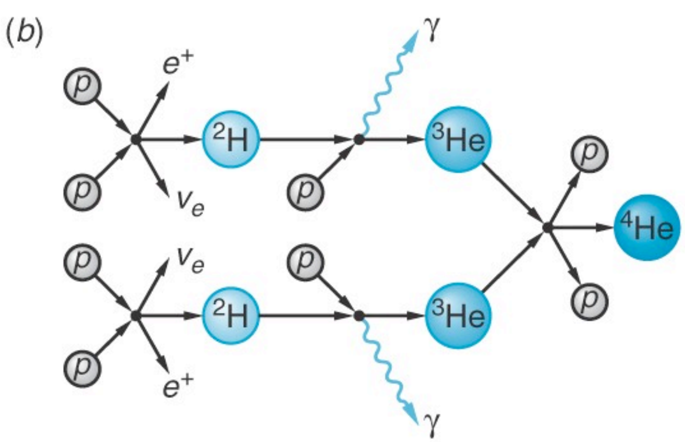
Sun's Power Output = 4×10^{26} Watts $\Rightarrow 10^{38}$ Fusion/Second !!!!

Nuclear Fusion: Wishing For The Star

- Fusion is eminently desirable because
  - More Energy/Nucleon
    - (3.52 MeV in fusion Vs 1 MeV in fission)
    - $^{2}H + {}^{3}H \rightarrow {}^{4}He + n + 17.6$ MeV
  - Relatively abundant fuel supply, No danger like nuclear reactor going supercritical
- Unfortunately technology not commercially available
  - What's inside nuclei => protons and Neutrons
  - Need Large KE to overcome Coulomb repulsion between nuclei
    - About 1 MeV needed to bring nuclei close enough together for Strong Nuclear Attraction → fusion
    - Need to
      - heat particle to high temp such that thermal energy $E = kT \approx 10$keV → tunneling thru coulomb barrier
      - Implies heating to $T \approx 10^{8}$ K (like in stars)
      - Confine Plasma (± ions) long enough for fusion
        - » In stars, enormous gravitational field confines plasma

Inertial Fusion Reactor : Schematic





Pellet of frozen-solid Deuterium & tritium bombarded from all sides with intense pulsed laser beam with energy ≈10^6 Joules lasting 10^{-8} S

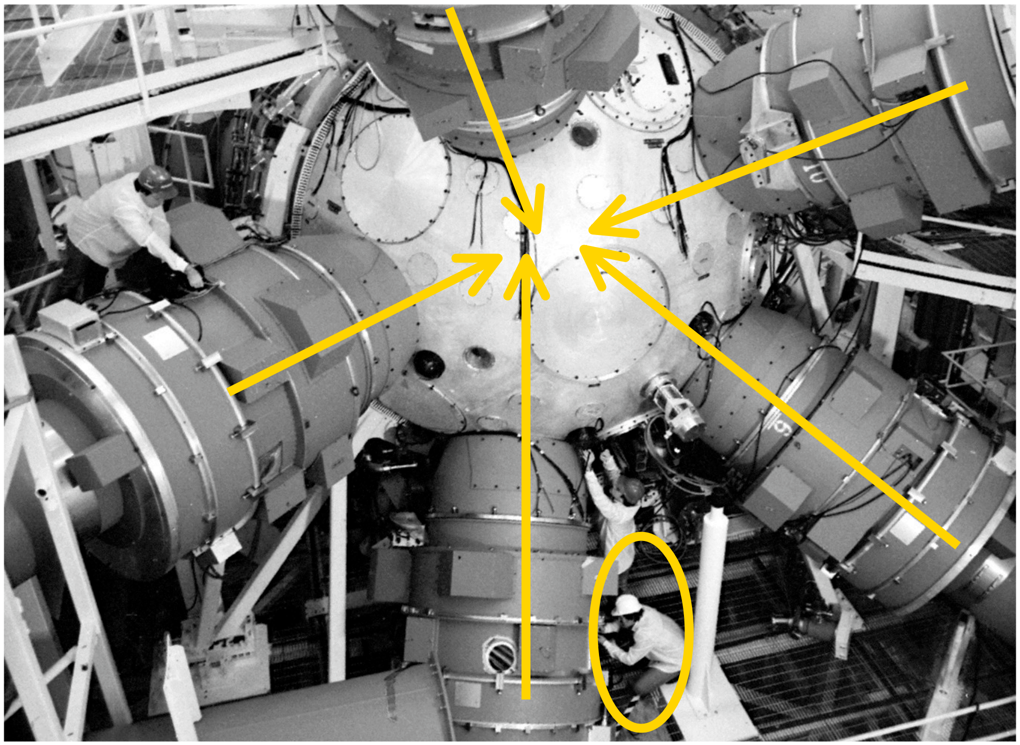
Momentum imparted by laser beam compresses pellet by 1/10000 of normal density and heats it to temp T ≈ 10^8 K for 10^{-10} S

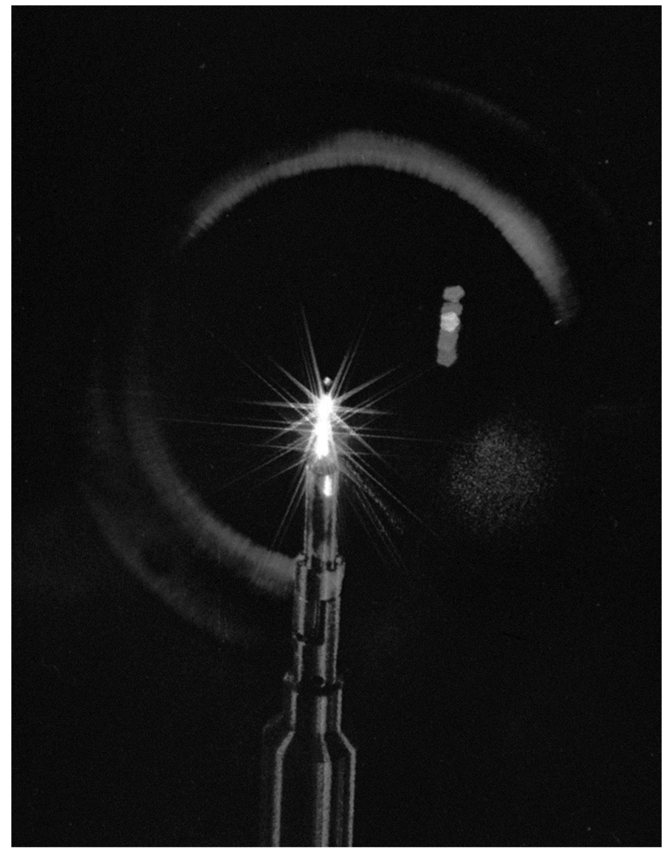
Burst of fusion energy transported away by liquid Li

World's Most Powerful Laser : NOVA @ LLNL

Size of football field, 3 stories tall

Generates 1.0 x 10^{14} watts (100 terawatts)

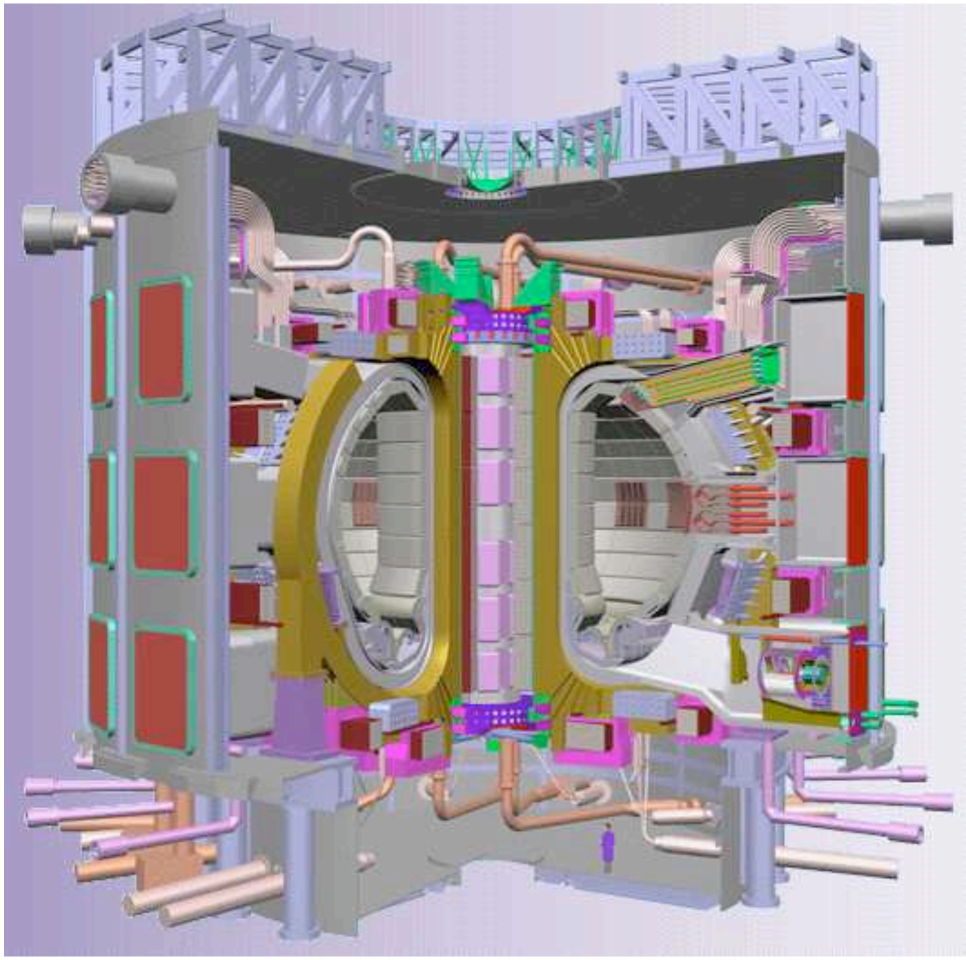




10 laser beams converge onto H pellet (0.5mm diam)

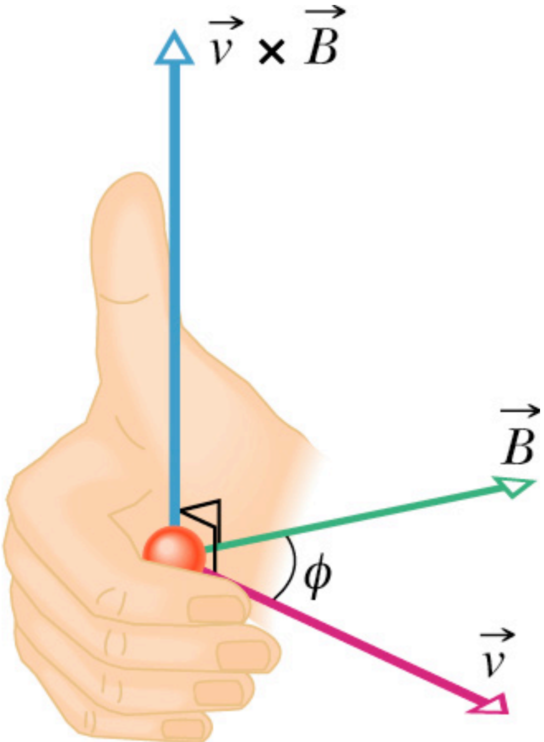
Fusion reaction is visible as a starlight lasting 10^{-10} S
Releasing 10^{13} neutrons

ITER: The Next Big Step in Nuclear Fusion



Visit www.iter.org for Details of this mega Science & Engineering Project
This may be future of cheap, clean Nuclear Energy for Earthlings

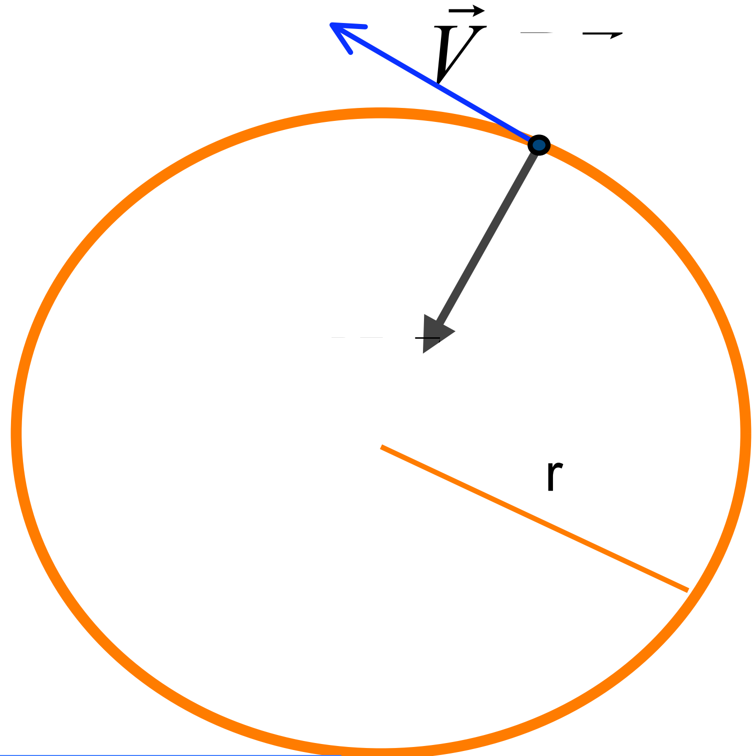
Magnetic Confinement & Circular Particle Accelerator



Classically

$$F = m\frac{v^2}{r}$$

$$qvB = m\frac{v^2}{r}$$

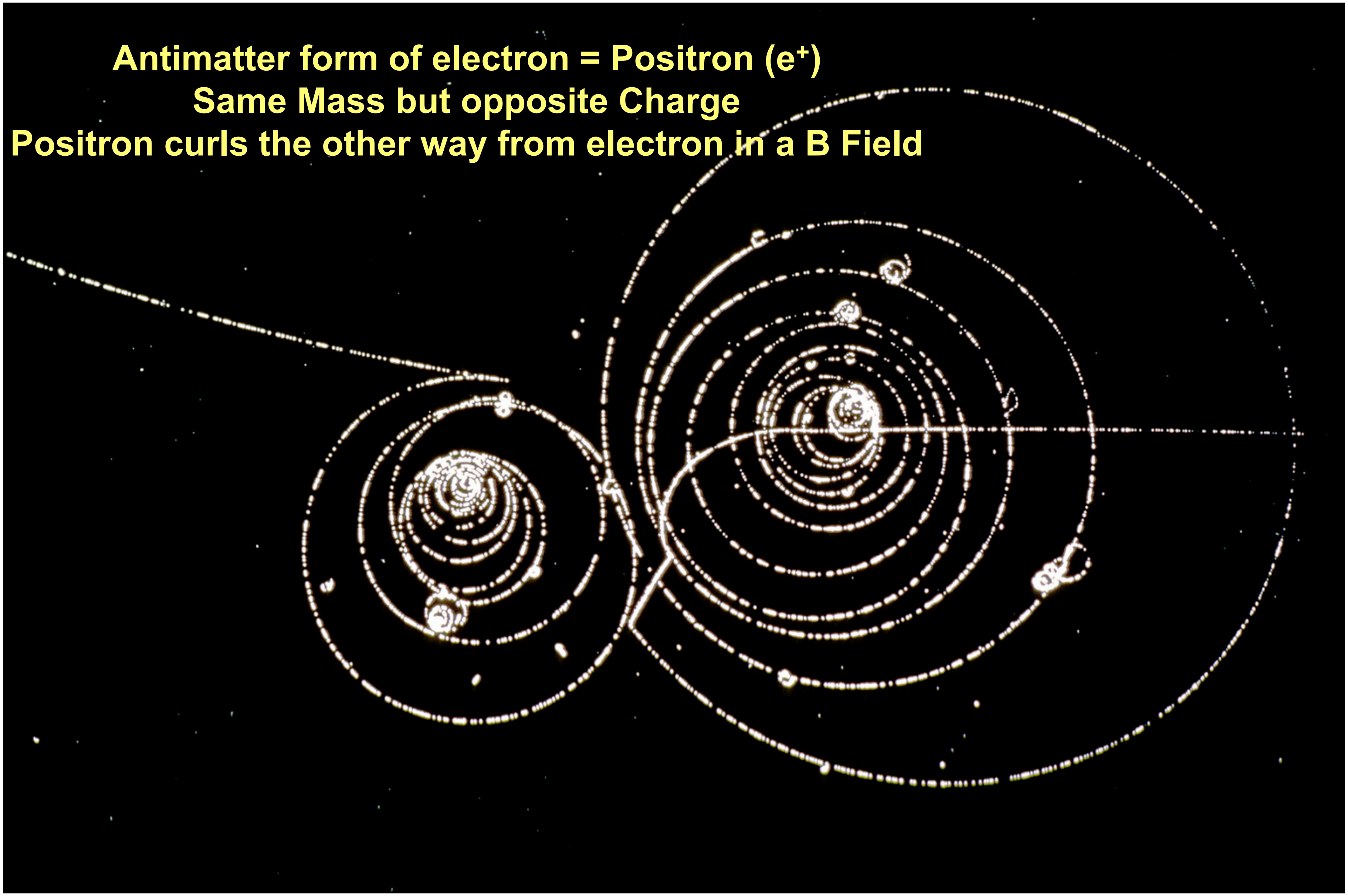


$$F = \frac{dp}{dt} = \frac{d(\gamma m u)}{dt} = \gamma m\frac{du}{dt} = quB$$

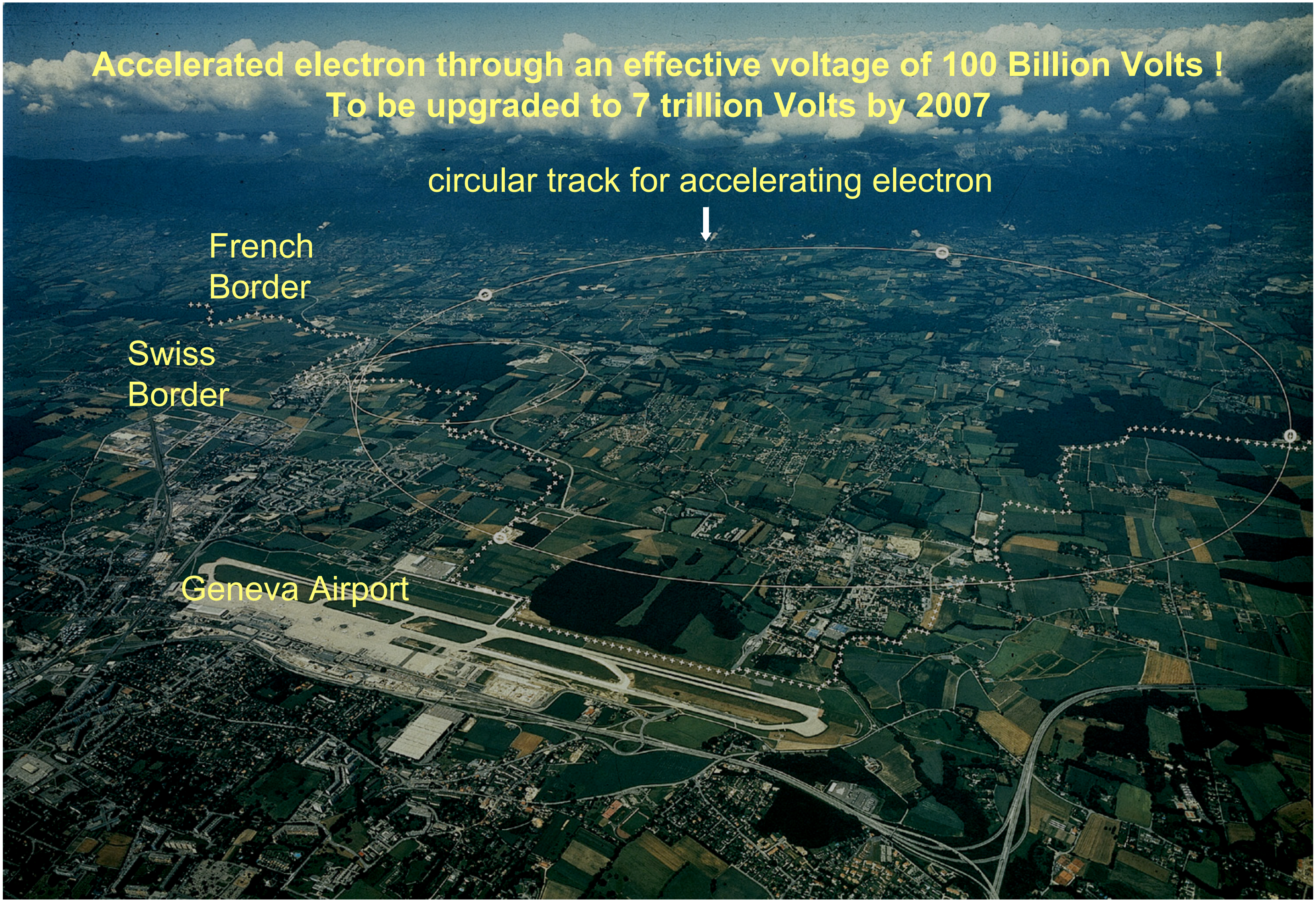
$$\frac{du}{dt} = \frac{u^2}{r} \quad \text{(Centripetal accelaration)}$$

$$\gamma m\frac{u^2}{r} = quB \quad \Rightarrow \gamma m u = qBr \Rightarrow p = qBr$$

Charged Form of Matter & Anti-Matter in a B Field



Circular Particle Accelerator: LEP @ CERN, Geneve



Test of Relativistic Momentum In Circular Accelerator

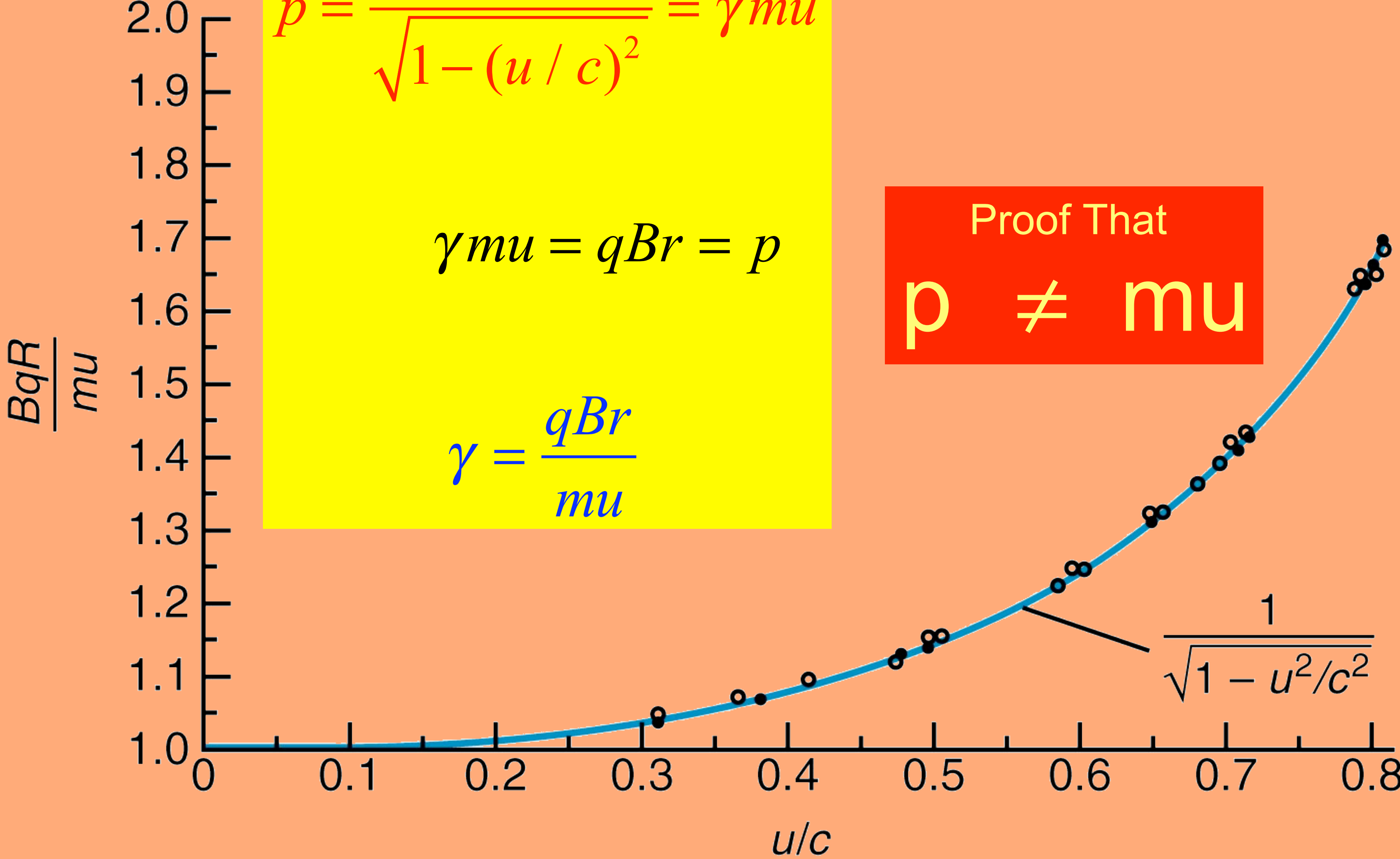
$$\vec{p} = \frac{m\vec{u}}{\sqrt{1-(u/c)^2}} = \gamma m\vec{u}$$

$$\gamma mu = qBr = p$$

$$\gamma = \frac{qBr}{mu}$$

Proof That

$p \neq mu$



Relationship between P and E

$$\boxed{E = \gamma mc^2} \Rightarrow E^2 = \gamma^2 m^2 c^4$$

$$\boxed{p = \gamma mu} \Rightarrow \boxed{p^2 c^2 = \gamma^2 m^2 u^2 c^2}$$

$$\Rightarrow E^2 - p^2c^2 = \gamma^2 m^2 c^4 - \gamma^2 m^2 u^2 c^2 = \gamma^2 m^2 c^2 (c^2 - u^2)$$

$$= \frac{m^2c^2}{1 - \frac{u^2}{c^2}}(c^2 - u^2) = \frac{m^2 c^4}{c^2 - u^2}(c^2 - u^2) = m^2 c^4$$

$$\boxed{E^2 = p^2c^2 + (mc^2)^2}$$ ....important relation

For particles with zero rest mass like photon (EM waves)

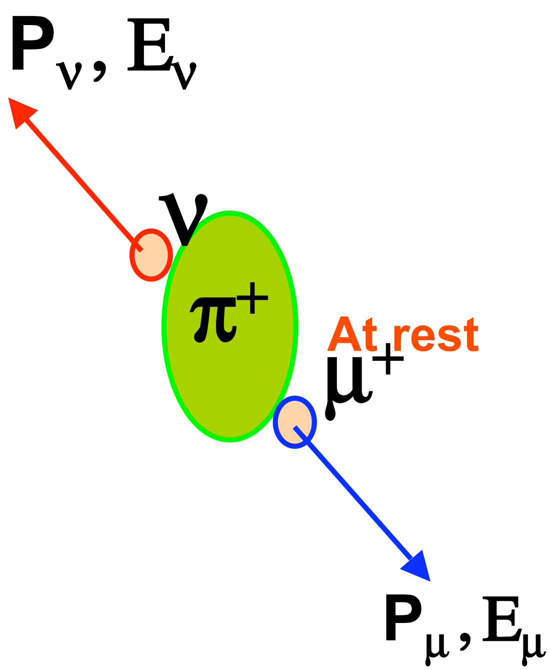
$$\boxed{E = pc \text{ or } p = \frac{E}{c}}$$ (light has momentum!)

Relativistic Invariance : $\boxed{E^2 - p^2c^2 = m^2c^4}$: In all Ref Frames

Rest Mass is a "finger print" of the particle

Relativistic Kinematics of Subatomic Particles

Reconstructing Decay of a π Meson



The decay of a stationary $\pi^+ \rightarrow \mu^+ \upsilon$ happens quickly,

υ is invisible, has m $\cong 0$; μ^+ leaves a trace in a B field

μ^+ mass=106 MeV/c^2, KE = 4.6 MeV

What was mass of the fleeting π^+?

Energy Conservation:

$$\mathrm{E}_\pi = E_\mu + E_\nu \ \Rightarrow \ \mathrm{m}_\pi c^2 = \sqrt{(m_\mu c^2)^2 + p_\mu^2 c^2} + p_\upsilon c$$

Momentum Conservation : $p_\mu = p_\nu$

$$\Rightarrow m_\pi c^2 = \sqrt{(m_\mu c^2)^2 + p_\mu^2 c^2} + p_\mu c$$