

Practice Midterm Exam

(Time: 90 minutes)

Formulae and constants

Lorentz Transformation:

$$x' = \gamma(x - vt)$$

$$y' = y$$

$$z' = z$$

$$t' = \gamma(t - vx / c^2)$$

Inverse Lorentz Transformation:

$$x = \gamma(x' + vt')$$

$$y = y'$$

$$z = z'$$

$$t = \gamma(t' + vx' / c^2)$$

Relativistic transformation of velocities

$$u'_x = (u_x - v) / (1 - u_x v / c^2)$$

$$u'_y = u_y / \gamma(1 - u_x v / c^2)$$

Inverse Transformation:

$$u_x = (u'_x + v) / (1 + u'_x v / c^2)$$

$$u_y = u'_y / \gamma(1 + u'_x v / c^2)$$

Relativistic Doppler Effect:

$$f_{obs} = \sqrt{\frac{1 + v/c}{1 - v/c}} f_{source} \text{ (approaching)}$$

Relativistic Momentum

$$p_x = \gamma m u_x \quad ; \text{ similarly for y- and z-components}$$

$$\gamma = \frac{1}{\sqrt{1 - u^2 / c^2}}$$

$$\text{Kinetic Energy } K = \gamma mc^2 - mc^2$$

$$\text{Total Energy } E = \gamma mc^2 = K + mc^2$$

$$E^2 = p^2 c^2 + m^2 c^4$$

Particle of charge q moving in magnetic field B in circle of radius r has relativistic momentum $p = qBr$

Stefan's law : $et_{\text{ot}} = \sigma T^4$; et_{ot} = power/unit area ; $\sigma = 5.67 \cdot 10^{-8} \text{ W / m}^2 \text{ K}^4$

$et_{\text{ot}} = cU / 4$, U = energy density = $\int u(\lambda, T) d\lambda$ (from 0 to ∞);

Wien's law : $\lambda_m T = (hc) / (4.96 k_B)$

Rutherford scattering : $\Delta n = C / [(1/2 mv^2)^2 \sin^4 (\phi / 2)]$

$$e = 1.6 \cdot 10^{-19} \text{ C}$$

$$m_e = 9.1 \cdot 10^{-31} \text{ Kg}$$

$$c = 3 \cdot 10^8 \text{ m / s}$$

$$1 \text{ MeV} = 1.6 \cdot 10^{-13} \text{ J}$$

$$1 \text{ MeV} / c^2 = 1.78 \cdot 10^{-30} \text{ Kg}$$

$$1 \text{ MeV} / c = 5.344 \cdot 10^{-22} \text{ Kg.m / s}$$

$$\text{Planck's Constant } h = 6.626 \cdot 10^{-34} \text{ J.s} = 4.136 \cdot 10^{-15} \text{ eV.s}$$

$$\hbar = h / 2\pi = 1.055 \cdot 10^{-34} \text{ J.s} = 6.582 \cdot 10^{-16} \text{ eV.s}$$

$$\text{Coulomb's constant } k = 1 / (4\pi\epsilon_0) = 8.99 \cdot 10^9 \text{ N.m}^2 / \text{kg}^2$$

$$\text{Velocity of light } c = 3 \cdot 10^8 \text{ m/s}$$

$$\text{Bohr's quantization for Angular momentum } mvr = n\hbar$$

$$\text{Bohr radius } a_0 = 0.529 \cdot 10^{-10} \text{ m}$$

$$1 \text{ Rydberg (Energy required to ionize hydrogen atom)} = 13.6 \text{ eV}$$

$$\text{Rydberg Constant } R = 1.097 \cdot 10^7 \text{ m}^{-1}$$

$$\text{Energy of photon } E = hf$$

$$\text{For photon } \lambda f = c$$

$$\text{Force due to Electric field : } \mathbf{F} = q\mathbf{E}$$

$$\text{Force due to Magnetic Field: } \mathbf{F} = q\mathbf{v} \times \mathbf{B}$$

Drag Force on drop of radius α and velocity v in medium of viscosity η

(always opposite to direction of v) : $D = 6\pi\alpha\eta v = Cv$

$$\text{Mass of electron } m_e = 9.1 \cdot 10^{-31} \text{ kg}$$

$$\text{Charge on electron} = 1.6 \cdot 10^{-19} \text{ C}$$

$$\text{Planck's Constant } h = 6.626 \cdot 10^{-34} \text{ J.s} = 4.136 \cdot 10^{-15} \text{ eV.s}$$

$$\hbar = h / 2\pi = 1.055 \cdot 10^{-34} \text{ J}\cdot\text{s} = 6.582 \cdot 10^{-16} \text{ eV}\cdot\text{s}$$

$$1 \text{ eV} = 1.6 \cdot 10^{-19} \text{ J}$$

$$\text{Coulomb's constant } k = 1 / (4\pi\epsilon_0) = 8.99 \cdot 10^9 \text{ N}\cdot\text{m}^2 / \text{kg}^2$$

$$\text{Velocity of light } c = 3 \cdot 10^8 \text{ m/s}$$

$$\text{Energy of photon } E = hf$$

$$\text{For photon } \lambda f = c$$

$$\text{Compton formula } \lambda' - \lambda = (h/m_e c)(1 - \cos \varphi)$$

$$\text{Bragg's Law } n\lambda = 2d \sin \theta \quad (n=1,2,\dots)$$

$$\text{Bohr's quantization for Angular momentum } mvr = n\hbar$$

$$\text{Bohr radius } a_0 = 0.529 \cdot 10^{-10} \text{ m}$$

$$1 \text{ Rydberg (Energy required to ionize hydrogen atom)} = 13.6 \text{ eV}$$

$$\text{Rydberg Constant } R = 1.097 \cdot 10^7 \text{ m}^{-1}$$

$$\text{Force due to Electric field : } \mathbf{F} = q\mathbf{E}$$

$$\text{Force due to Magnetic Field: } \mathbf{F} = q\mathbf{v}\times\mathbf{B}$$

(1) A spaceship moves at speed $v=0.6c$ with respect to the ground. The proper length of the spaceship is $L_p=300\text{m}$. There is a flashlight at the back end of the spaceship and a mirror at the front end.

(a) As measured on the spaceship: how much time, in seconds, elapses from the moment the flashlight is turned on to the moment when the light reaches the mirror?

(b) As measured on the spaceship: how much time, in seconds, elapses from the moment the flashlight is turned on to the moment the light reaches the back end of the spaceship after having been reflected off the mirror?

(c) As measured on the ground: how much time, in seconds, elapses from the moment the flashlight is turned on to the moment the light reaches the back end of the spaceship after having been reflected off the mirror?

(d) As measured on the ground: how much time, in seconds, elapses from the moment the flashlight is turned on to the moment when the light reaches the mirror?

Explain your reasoning.

Hint: (d) is more difficult than (a), (b), and (c); it can be done with or without using the

Lorentz transformation formulas.

(2) A particle of mass M at rest decays into two particles of mass m_1 and m_2 that fly off in opposite directions with velocities of $0.4c$ and $0.6c$ respectively in a frame of reference at rest with the original mass.

(a) Find m_1 and m_2 expressed in terms of M .

- (b) Find what fraction of mass was converted into kinetic energy.
- (c) Find the kinetic energies of the two fragments and show that they add up to the amount of mass loss times c^2

(3) α particles (i.e. He nuclei) are produced far from a target of Fe nuclei ($Z=26$). The kinetic energy K of the α particles incident in head-on collisions with stationary Fe nuclei are raised until $K = 15$ MeV at which point the rate of scattering deviates from the Rutherford formula. What is the radius of the Fe nucleus?

(4) For an electron in the $n=2$ level of He^+ :

- (a) Give the radius of the orbit, in nm.
- (b) Give the potential and kinetic energy of this electron, in eV.
- (c) Give the speed of the electron, expressed as v/c .

(5).(a) An electron has de Broglie wavelength $\lambda=1\text{\AA}$. What is its kinetic energy, in eV? ($1 \text{\AA} = 10^{-10} \text{ m}$)

(b) An electron has de Broglie wavelength $\lambda=10^{-7}\text{\AA}$. What is its kinetic energy, in MeV?

(c) An electron is described by a wavepacket containing wavelengths in the range $\lambda = 9.9\text{\AA}$ to $\lambda = 10.1\text{\AA}$. Estimate the uncertainty in the position of this electron, Δx , in \AA .