



Physics 2D Lecture Slides

Week of April 20th 2009

(Prof. Werner Press)

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UCSD Physics

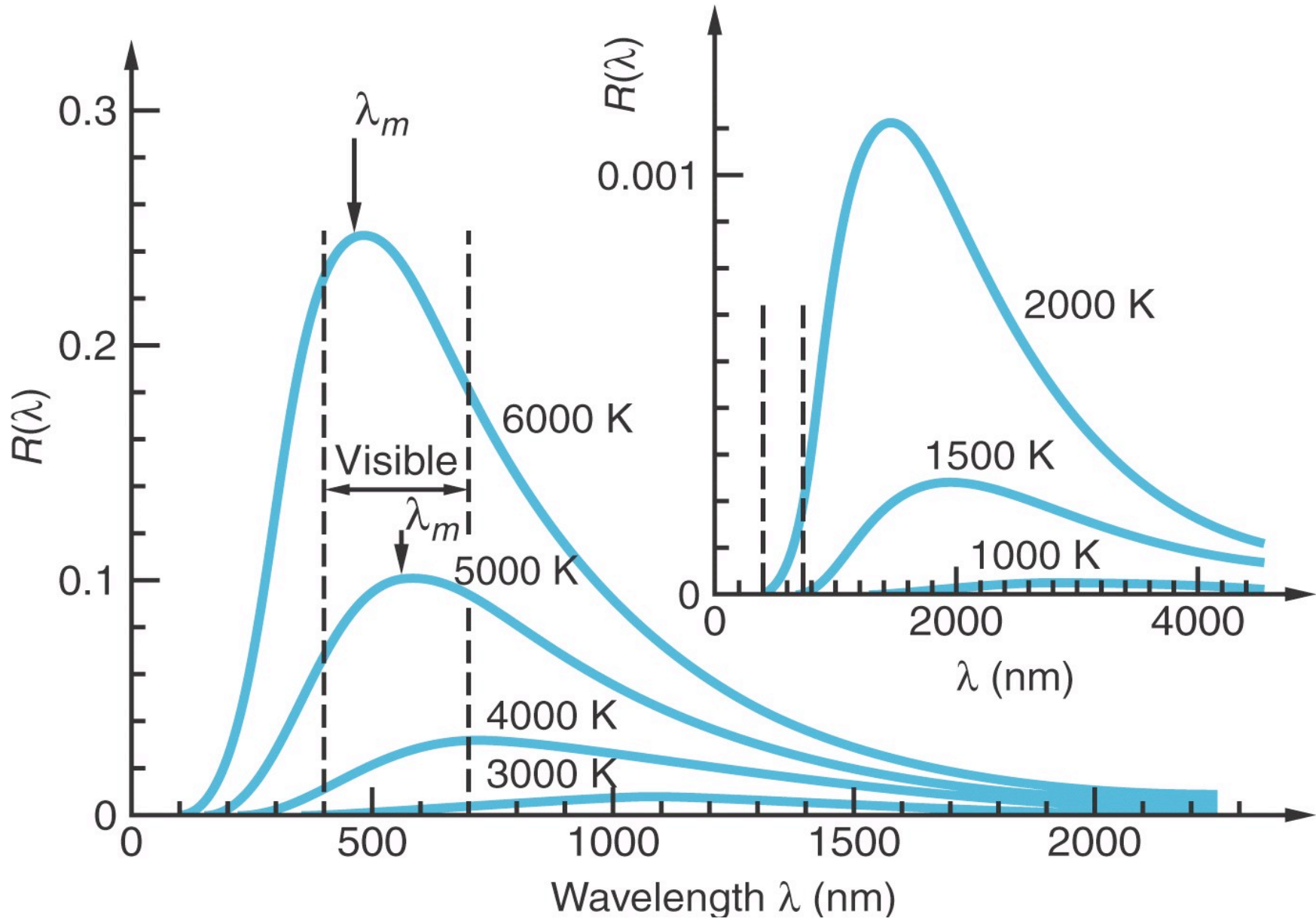
Do Maxwell's Equations always work?

- Maxwell: shows EM waves propagate @ speed of light, so light is probably a EM wave.
- Hertz: Demonstrates propagation of EM waves through space and shows they behave like light.
- BUT.....TWO major problems encountered.....

Problem 1: Black Body Radiation

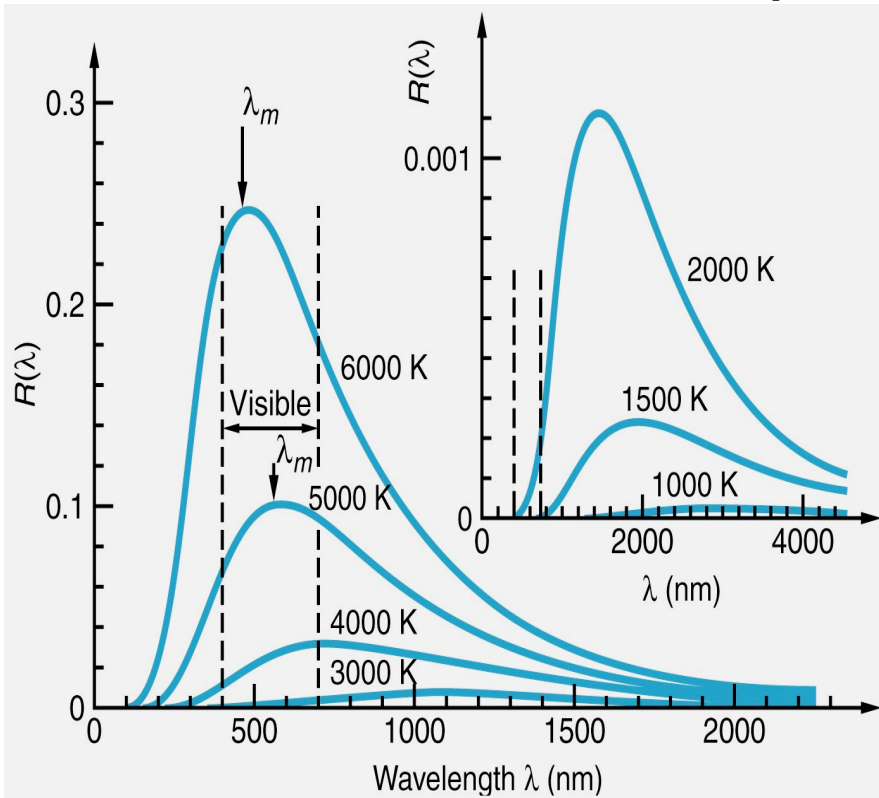
- (Thermodynamics and Statistical Mechanics): Considered radiation from ideally absorbing body in equilibrium with radiation.
- Kirchoff proved that $e_f = J(f, T)A_f$ where e_f is power emitted per unit area per unit frequency interval, and A_f is fraction of power absorbed per unit area per unit frequency interval. $J(f, T)$ is *universal* function of f and T and can be expressed as $J(f, T) = u(f, T)c/4$ where $u(f, T)$ is energy per unit volume per unit frequency interval of radiation in blackbody cavity.
- Form of $J(f, T)$ or $u(f, T)$? Empirical results (measured by Paschen and others) ---> **Wien's Law** $\lambda_m T = 2.9 \cdot 10^{-3} \text{ m.K}$
Stefan's Law $e_{\text{total}} = \sigma T^4$ for ideal BB. $\sigma = 5.7 \cdot 10^{-8} \text{ W.m}^2.\text{K}^{-4}$

Radiation from A Blackbody expressing $J(f,T)$ in terms of $R(\lambda,T)$



(a) Intensity of Radiation $I = \int R(\lambda) d\lambda \propto T^4$
 $I = \sigma T^4$ (Area under curve)

Stephan-Boltzmann Constant $\sigma = 5.67 \cdot 10^{-8} \text{ W / m}^2 \text{ K}^4$



(b) Higher the temperature of BBQ
 Lower is the λ of PEAK intensity

$$\lambda_{\text{MAX}} \propto 1/T$$

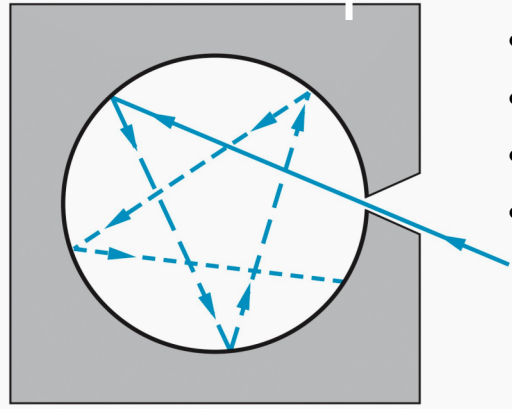
Wien's Law $\lambda_{\text{MAX}} T = \text{const} = 2.898 \cdot 10^{-3} \text{ mK}$

Reason for different shape of $R(\lambda)$ Vs λ for different temperature?
 Can one explain in on basis of Classical Physics (2A,2B,2C) ??

Can one explain functional form of $u(f,T)$?

- Rayleigh-Jeans Calculation based on calculating number of allowed frequencies in a cavity per unit volume per unit frequency interval and giving each an energy kT as per classical Statistical Mechanics.
- Another approach (empirical) by Wien
 $u(f,T) = \alpha f^3 \exp(-\beta f/T)$ α, β constants

Blackbody Radiator: An Idealization



Classical Analysis:

- Box is filled with EM standing waves
- Radiation reflected back-and-forth between walls
- Radiation in thermal equilibrium with walls of Box
- **How many waves of wavelength λ can fit inside the box ?**

Blackbody Absorbs everything

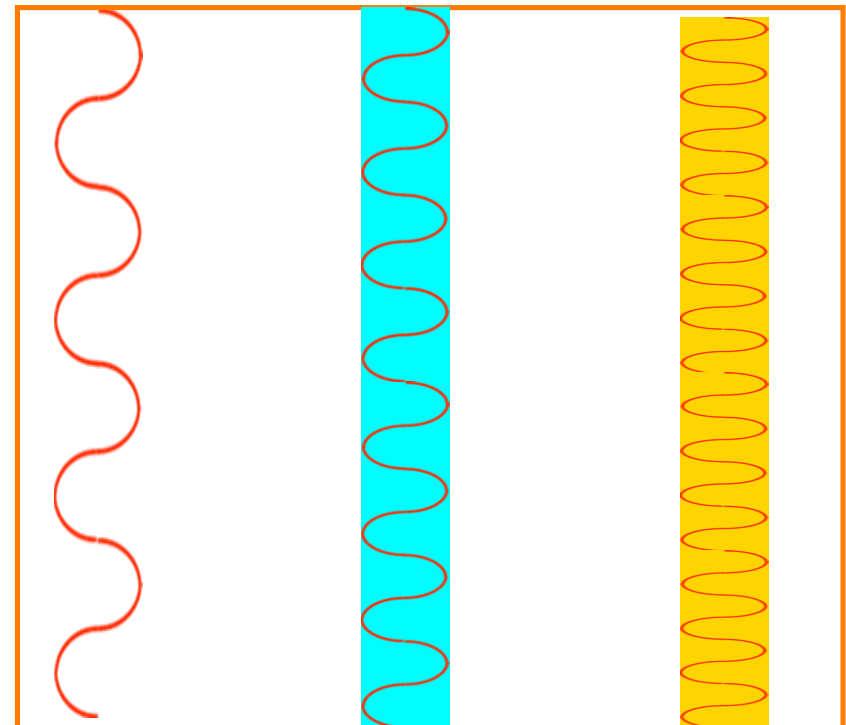
Reflects nothing

All light entering opening gets absorbed (ultimately) by the cavity wall

Cavity in equilibrium T w.r.t. surrounding. So it radiates everything it absorbs

Emerging radiation is a sample of radiation inside box at temp T

Predict nature of radiation inside Box ?

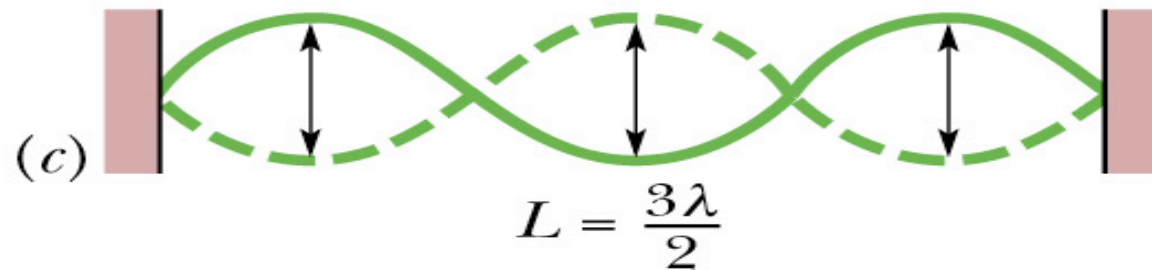
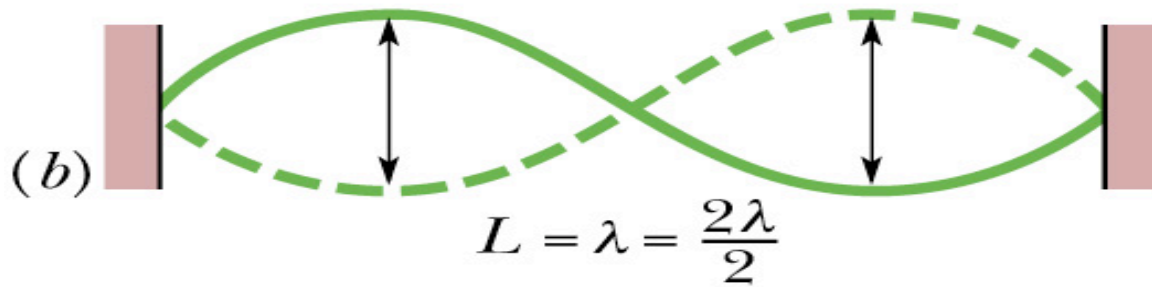
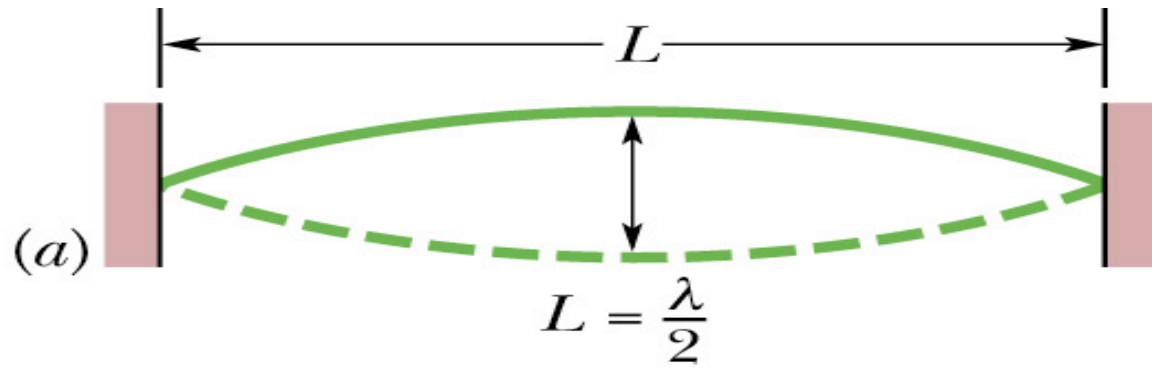


less

more

Even more

Standing Waves



Calculation of Number of Allowed modes/ Unit Volume in a Cavity

Assume cavity is cube of side L . \mathbf{E} -field = 0 at walls

Construct wave solutions out of forms $\mathbf{E} = \mathbf{E}_0 \exp(ik_x x + ik_y y + ik_z z)$

(choose all possible combinations of + or - of $k_x k_y k_z$)

Electromagnetic Wave (Soln. of Maxwell's Equations inside cavity)

Must be of form $\mathbf{E} = \mathbf{E}_0 \sin(n_1 \pi x/L) \sin(n_2 \pi y/L) \sin(n_3 \pi z/L)$

i.e. $k_x = n_1 \pi/L$, etc. ($n_1 n_2 n_3 = \text{integers} > 0$)

\mathbf{k} points lie on a cubic mesh of spacing (π/L) along k_x, k_y, k_z axes

i.e. one \mathbf{k} point per volume $(\pi/L)^3$

So density of \mathbf{k} points is $(L/\pi)^3$ per unit volume in \mathbf{k} -space

Volume of k space between \mathbf{k} vectors of magnitude k and $k + dk$

is $4\pi k^2 dk$ so no. of allowed \mathbf{k} points in that volume

$$= (1/8) \times \text{Density of } \mathbf{k} \text{ points} \times 4\pi k^2 dk = (1/8) (L/\pi)^3 4\pi k^2 dk$$

Factor of $(1/8)$ is because only positive values of k_x, k_y, k_z allowed--> positive octant of volume only.

Multiply by 2 for 2 possible polarizations of \mathbf{E} field and remember $L^3 = V$ (volume of cavity)

----> No. of allowed modes/unit volume with k between k and $k + dk$

$$= n(k)dk = (k^2/\pi^2)dk$$

Now $k = 2\pi/\lambda$ so $dk = -2\pi/\lambda^2 d\lambda$ -----> $n(\lambda)d\lambda = (8\pi/\lambda^4)d\lambda$

and $\lambda = c/f$ so $d\lambda = -c/f^2 df$ -----> $n(f) df = (8\pi f^2/c^3) df$

The above formulae give the no. of modes in k -intervals, wavelength intervals and frequency intervals for EM radiation in a cavity.

EM Energy/unit volume at Temperature T

for wavelengths between λ and $\lambda + d\lambda$ is $u(\lambda, T) d\lambda$

$$u(\lambda, T)d\lambda = \langle E(\lambda) \rangle n(\lambda)d\lambda$$

Classical Physics -----> $\langle E(\lambda) \rangle = k_B T$

So get $u(\lambda, T) d\lambda = (8\pi/\lambda^4) k_B T d\lambda$

$R(\lambda) = c/4 u(\lambda, T)$ -----> Rayleigh-Jeans Law

The Beginning of The End !

Classical Calculation

of standing waves between Wavelengths λ and $\lambda+d\lambda$ are

$$N(\lambda)d\lambda = \frac{8\pi V}{\lambda^4} \cdot d\lambda ; V = \text{Volume of box} = L^3$$

Each standing wave contributes energy $E = kT$ to radiation in Box

Energy density $u(\lambda) = [\text{\# of standing waves/volume}] \times \text{Energy/Standing Wave}$

$$= \frac{8\pi V}{\lambda^4} \times \frac{1}{V} \times kT = \frac{8\pi}{\lambda^4} kT$$

$$\text{Radiancy } R(\lambda) = \frac{c}{4} u(\lambda) = \frac{c}{4} \frac{8\pi}{\lambda^4} kT = \frac{2\pi c}{\lambda^4} kT$$

Radiancy is Radiation intensity per unit λ interval: Lets plot it

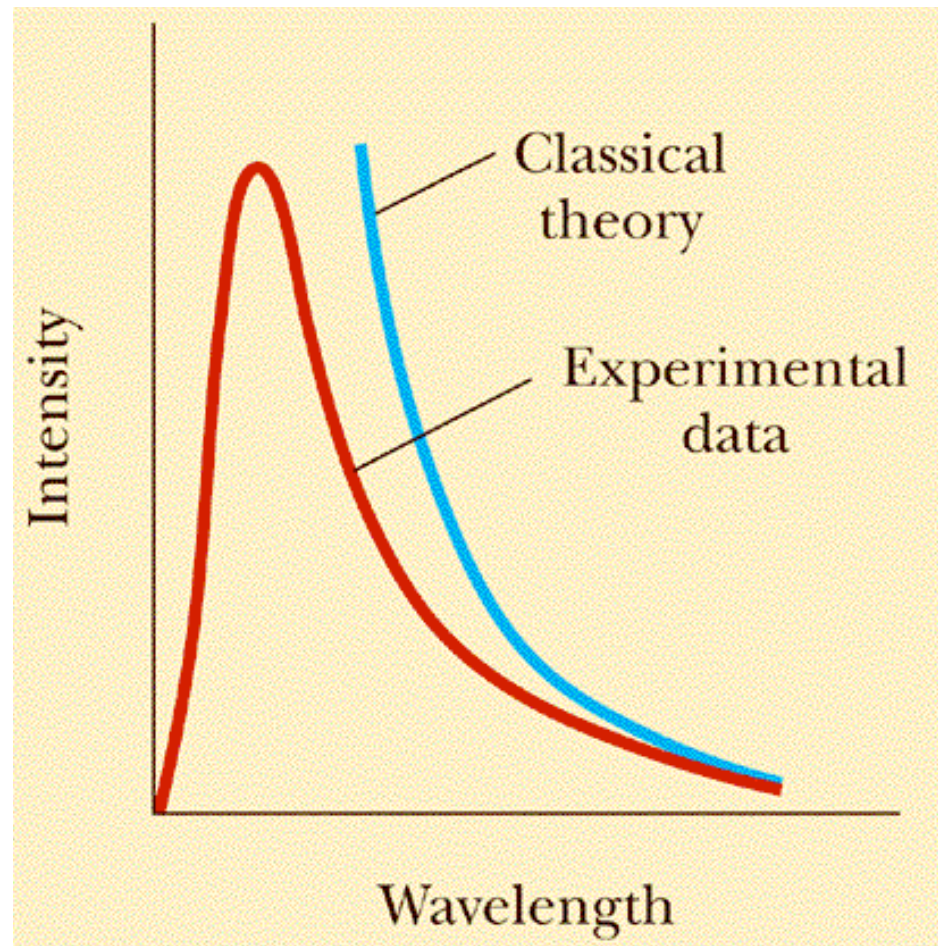
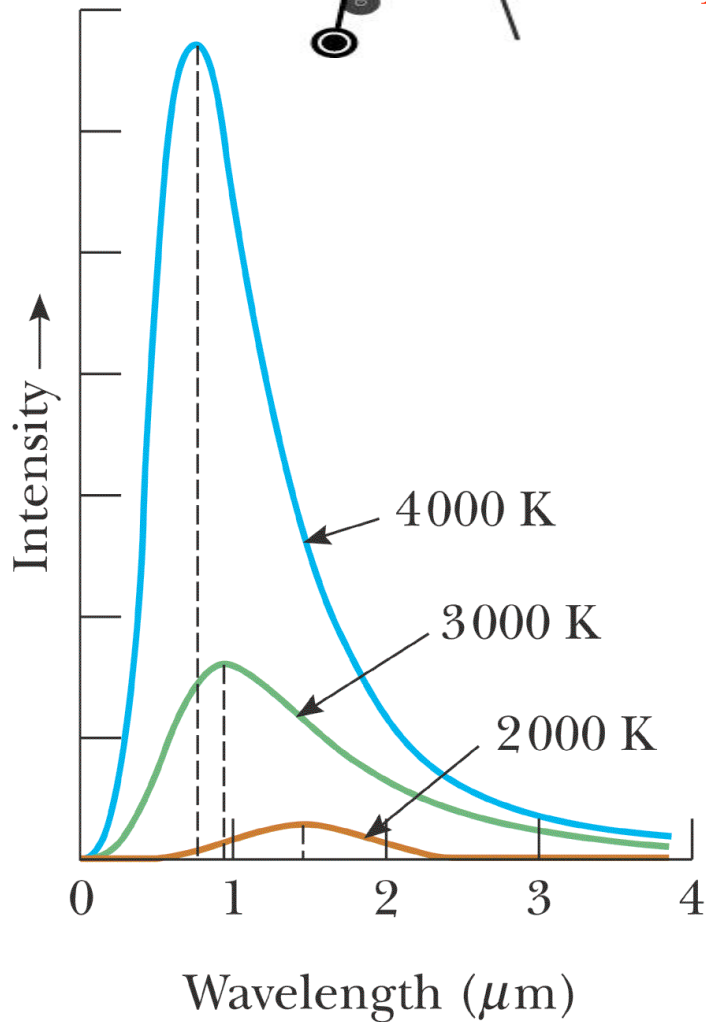
Prediction : as $\lambda \rightarrow 0$ (high frequency) $\Rightarrow R(\lambda) \rightarrow \text{Infinity}$!
Oops !

Ultra Violet (Frequency) Catastrophe



$$\text{Radiancy } R(\lambda) = \frac{c}{4} u(\lambda) = \frac{c}{4} \frac{8\pi}{\lambda^4} kT = \frac{2\pi c}{\lambda^4} kT$$

Radiancy is Radiation intensity per unit λ interval



Max Planck & Birth of Quantum Physics



Back to Blackbody Radiation Discrepancy

Planck noted the UltraViolet Catastrophe at high frequency

“Cooked” calculation with new “ideas” so as bring:

$$R(\lambda) \rightarrow 0 \text{ as } \lambda \rightarrow 0$$

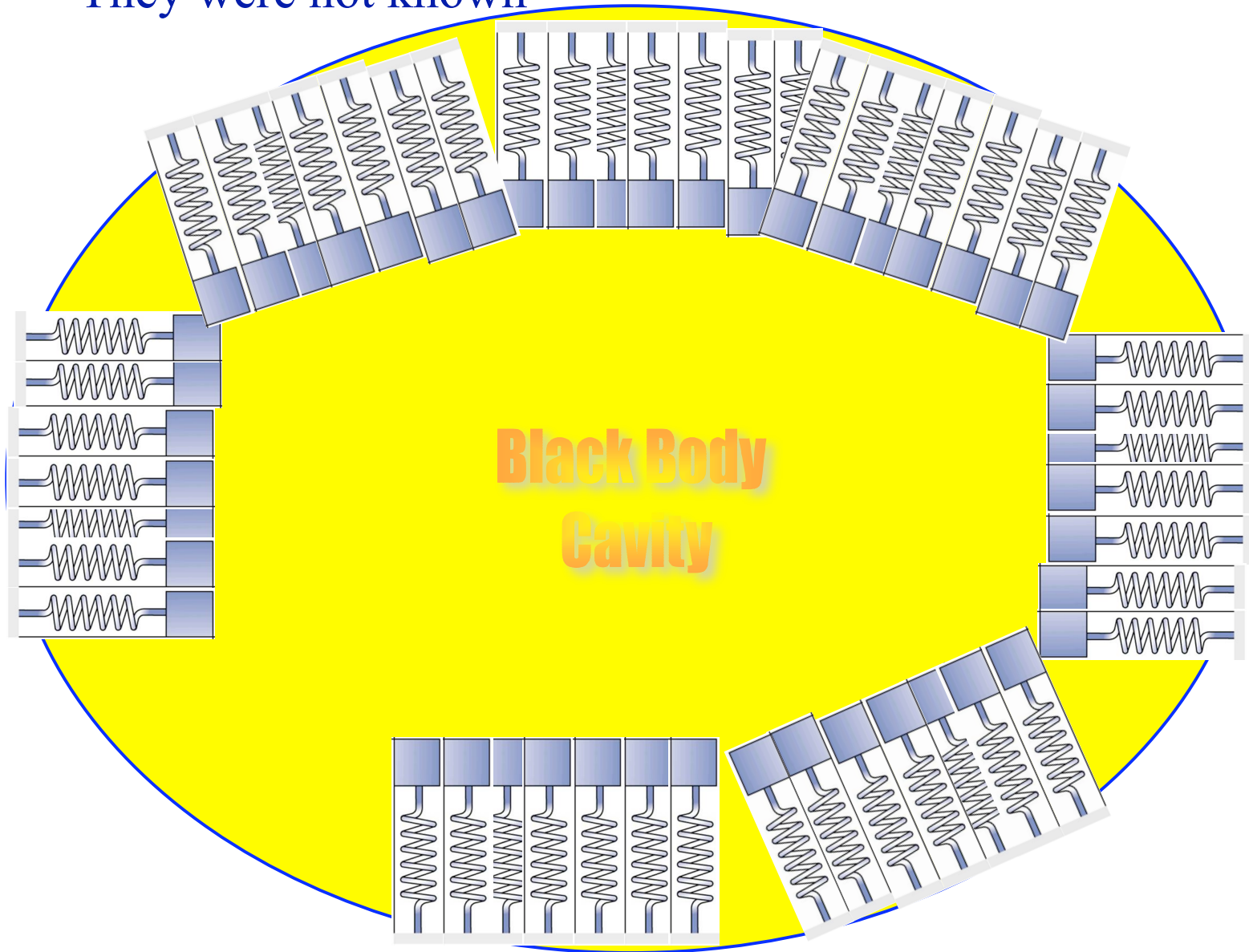
$$f \rightarrow \infty$$

- Cavity radiation as equilibrium exchange of energy between EM radiation & “atomic” oscillators present on walls of cavity
- Oscillators can have **any frequency f**
- But the Energy exchange between radiation and oscillator NOT continuous and arbitrary...it is discrete ...in **packets of same amount**
- $E = n hf$, with $n = 1, 2, 3, \dots, \infty$
 $h = \text{constant he invented, a very small number he made up}$

Planck's "Charged Oscillators" in a Black Body Cavity

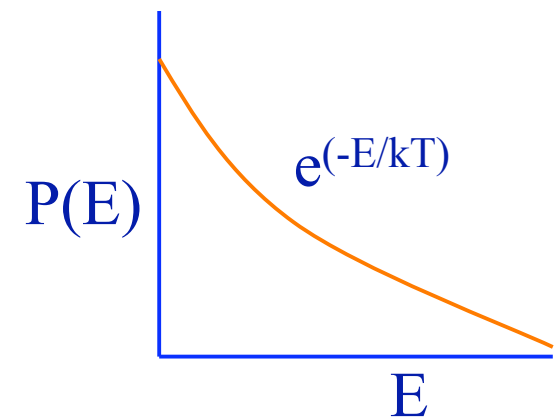
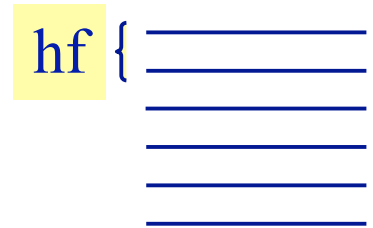
Planck did not know about electrons, Nucleus etc:

They were not known



Planck, Quantization of Energy & BB Radiation

- Keep the rule of counting how many waves fit in a BB Volume
- Radiation Energy in cavity is quantized
- EM standing waves of frequency f have energy
 - $E = n hf$ ($n = 1, 2, 3 \dots 10 \dots 1000 \dots$)
- Probability Distribution: At an equilibrium temp T , possible Energy of wave is distributed over a spectrum of states: $P(E) = e^{(-E/kT)}$
- Modes of Oscillation with :
 - Less energy $E=hf$ = favored
 - More energy $E=hf$ = disfavored



By this statistics, large energy, high f modes of EM disfavored

Planck

Difference is in calculation of $\langle E \rangle$

Consider a mode of frequency f . Planck assumed it was emitted by a set of harmonic oscillators in walls of cavity which could only have energies $E = nhf$ ($h =$ constant now known as Planck's constant).

Probability of oscillator having energy nhf by statistical mechanics

$$P(n) = (\exp - nhf/ k_B T) / \{ \sum_m (\exp - mh f/ k_B T) \}$$

Sum can be evaluated by writing $\exp(-hf/k_B T) = x$, so it can be written as

$$1 + x + x^2 + x^3 + x^4 + \dots = (1 - x)^{-1}$$

Now $E = nhf =$ Energy of oscillator so average energy

$$\text{So } \langle E(f, T) \rangle = \{ \sum_n nhf \exp - nhf/ k_B T \} / \{ \sum_m (\exp - mh f/ k_B T) \}$$

$$= \sum_m mx^m / \sum_m x^m = x(d/dx) \sum_m x^m / \sum_m x^m = x/(1-x)$$

This can be evaluated as $\langle E(f, T) \rangle = hf / [\exp(hf/k_B T) - 1]$

yields $u(f, T) = (8\pi f^3/c^3) / [\exp(hf/k_B T) - 1]$ ---> Planck's formula

Planck's Calculation

$$R(\lambda) = \left(\frac{c}{4}\right) \left(\frac{8\pi}{\lambda^4}\right) \left[\frac{hc}{\lambda} \left(\frac{1}{e^{\frac{hc}{\lambda kT}} - 1} \right) \right]$$

Odd looking form

$$\text{When } \lambda \rightarrow \text{large} \Rightarrow \frac{hc}{\lambda kT} \rightarrow \text{small}$$

$$\text{Recall } e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

$$\Rightarrow e^{\frac{hc}{\lambda kT}} - 1 = \left(1 + \frac{hc}{\lambda kT} + \frac{1}{2} \left(\frac{hc}{\lambda kT} \right)^2 + \dots \right) - 1$$

$$= \frac{hc}{\lambda kT} \quad \text{plugging this in } R(\lambda) \text{ eq:}$$

$$R(\lambda) = \left(\frac{c}{4}\right) \left(\frac{8\pi}{\lambda^4}\right) \frac{hc}{\lambda kT}$$

Graph & Compare
With BBQ data

Planck's Formula and Small λ

When λ is small (large f)

$$\frac{1}{e^{\frac{hc}{\lambda kT}} - 1} \cong \frac{1}{e^{\frac{hc}{\lambda kT}}} = e^{-\frac{hc}{\lambda kT}}$$

Substituting in $R(\lambda)$ eqn:

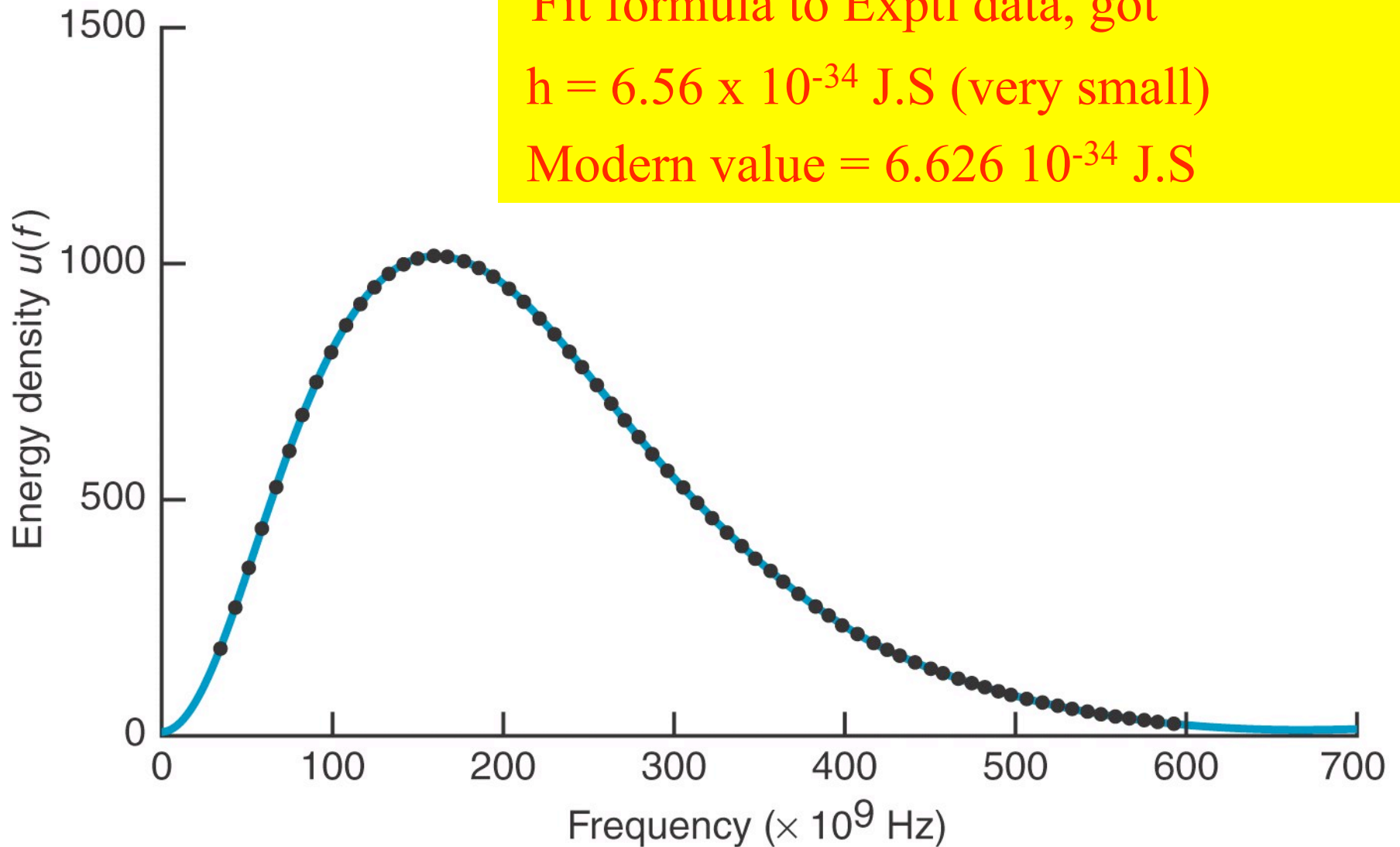
$$R(\lambda) = \left(\frac{c}{4}\right) \left(\frac{8\pi}{\lambda^4}\right) e^{-\frac{hc}{\lambda kT}}$$

$$\text{As } \lambda \rightarrow 0, e^{-\frac{hc}{\lambda kT}} \rightarrow 0$$

$$\Rightarrow R(\lambda) \rightarrow 0$$

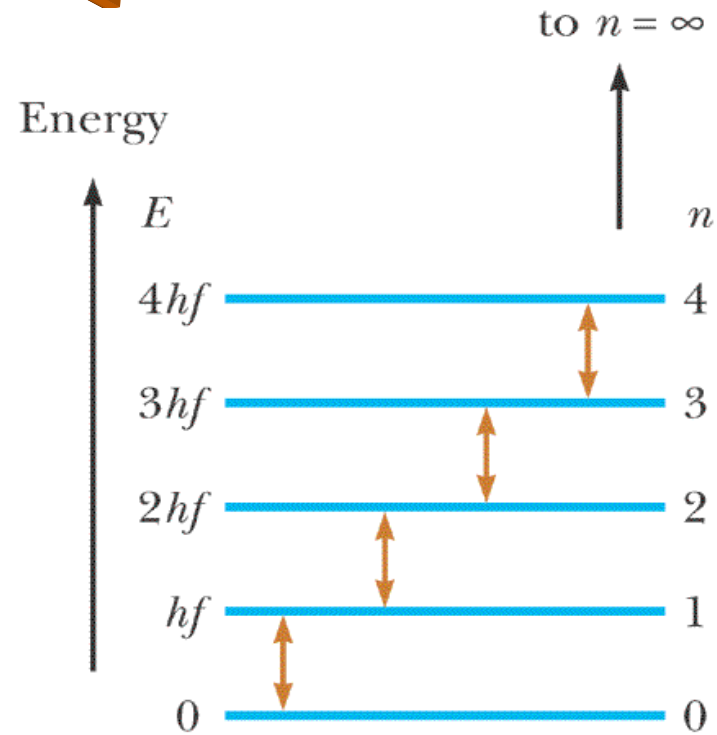
Just as seen in the experimental data

Planck's Explanation of BB Radiation



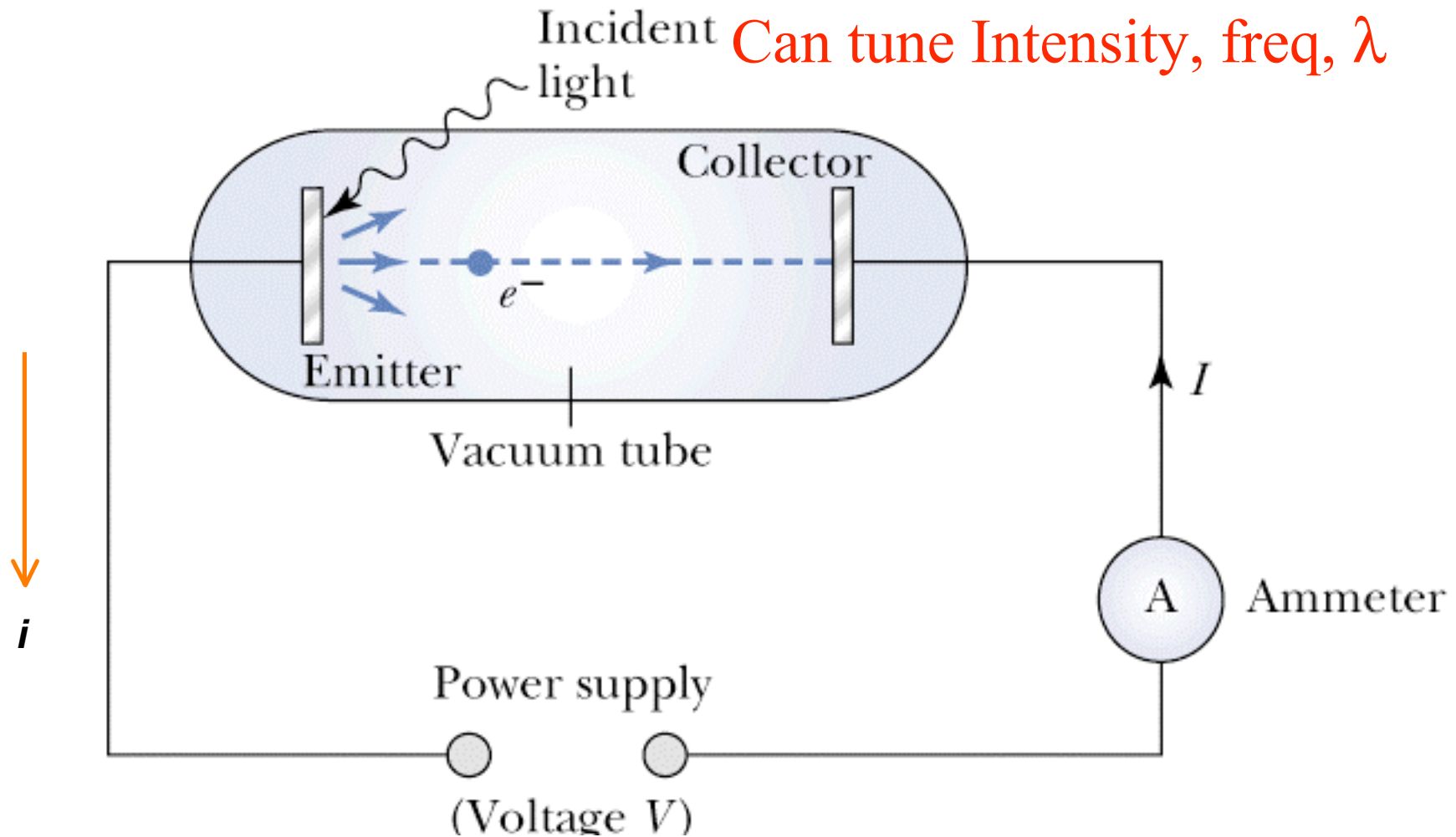
Major Consequence of Planck's Formula

Quantization of Energy!



Problem # 2 : Photo-Electric Effect

Light of intensity I , wavelength λ and frequency ν incident on a photo-cathode



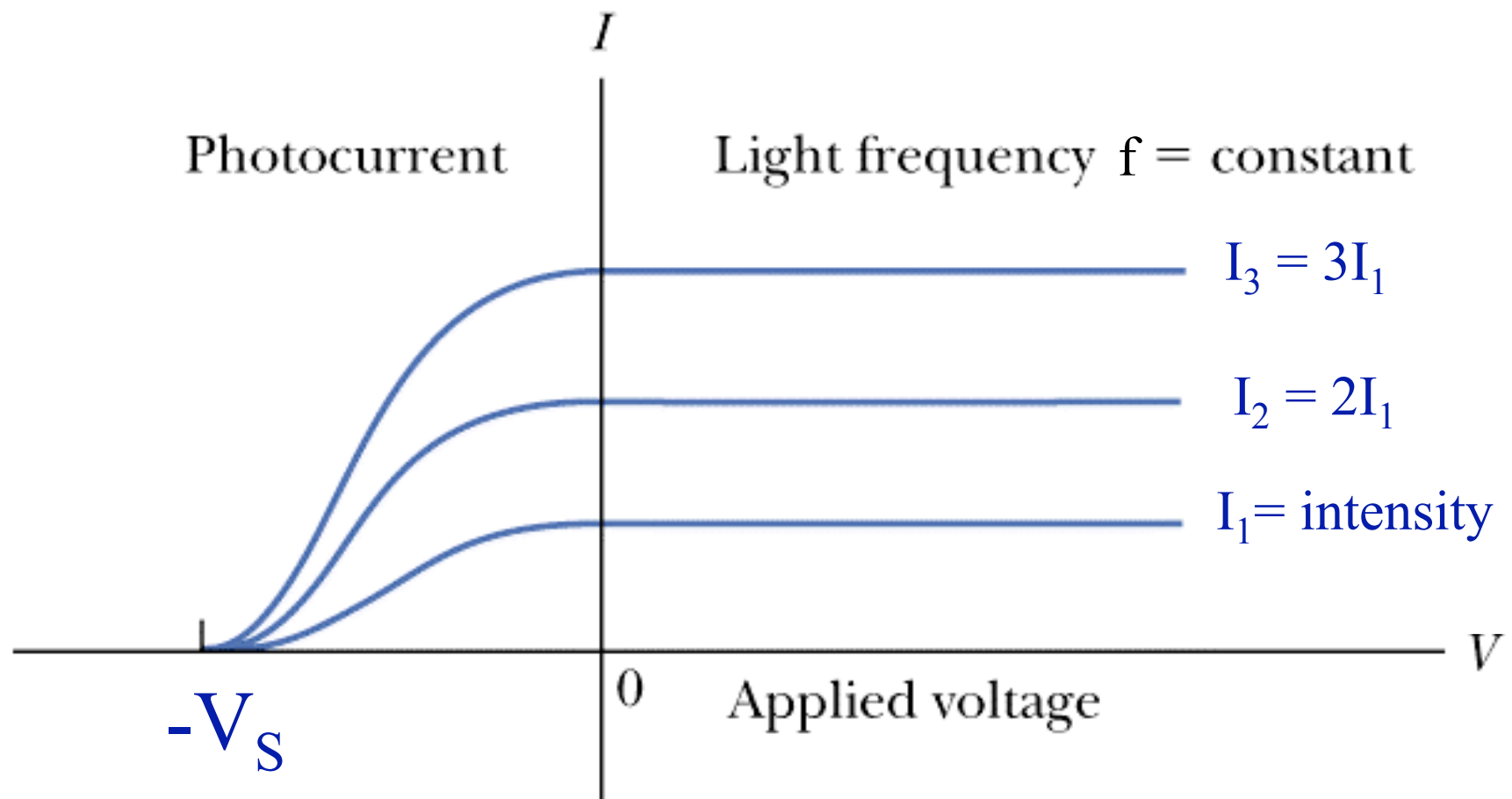
Measure characteristics of current in the circuit as a fn of I , f , λ

Photo Electric Effect: Measurable Properties

- Rate of electron emission from cathode
 - From current i seen in ammeter
- Maximum kinetic energy of emitted electron
 - By applying retarding potential on electron moving towards Collector plate
 - » $K_{MAX} = eV_S$ ($V_S =$ Stopping voltage)
 - » Stopping voltage \rightarrow no current flows
- Effect of different types of photo-cathode metal
- Time **between** shining light and first sign of photo-current in the circuit

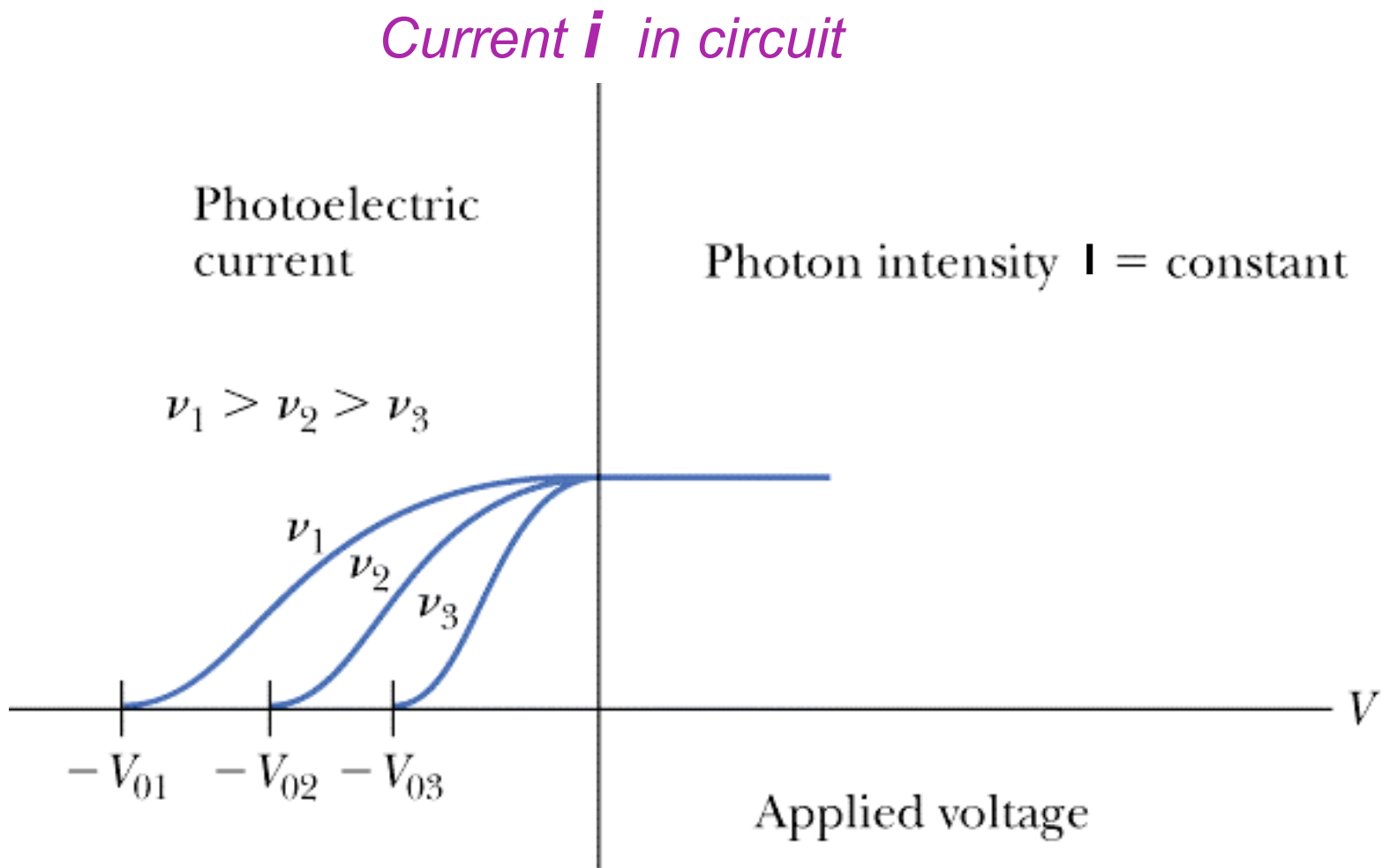
Observation: Photo-Current Vs Frequency of Incident Light

Stopping voltage V_s is a measure of the **Max kinetic energy** of the electron_

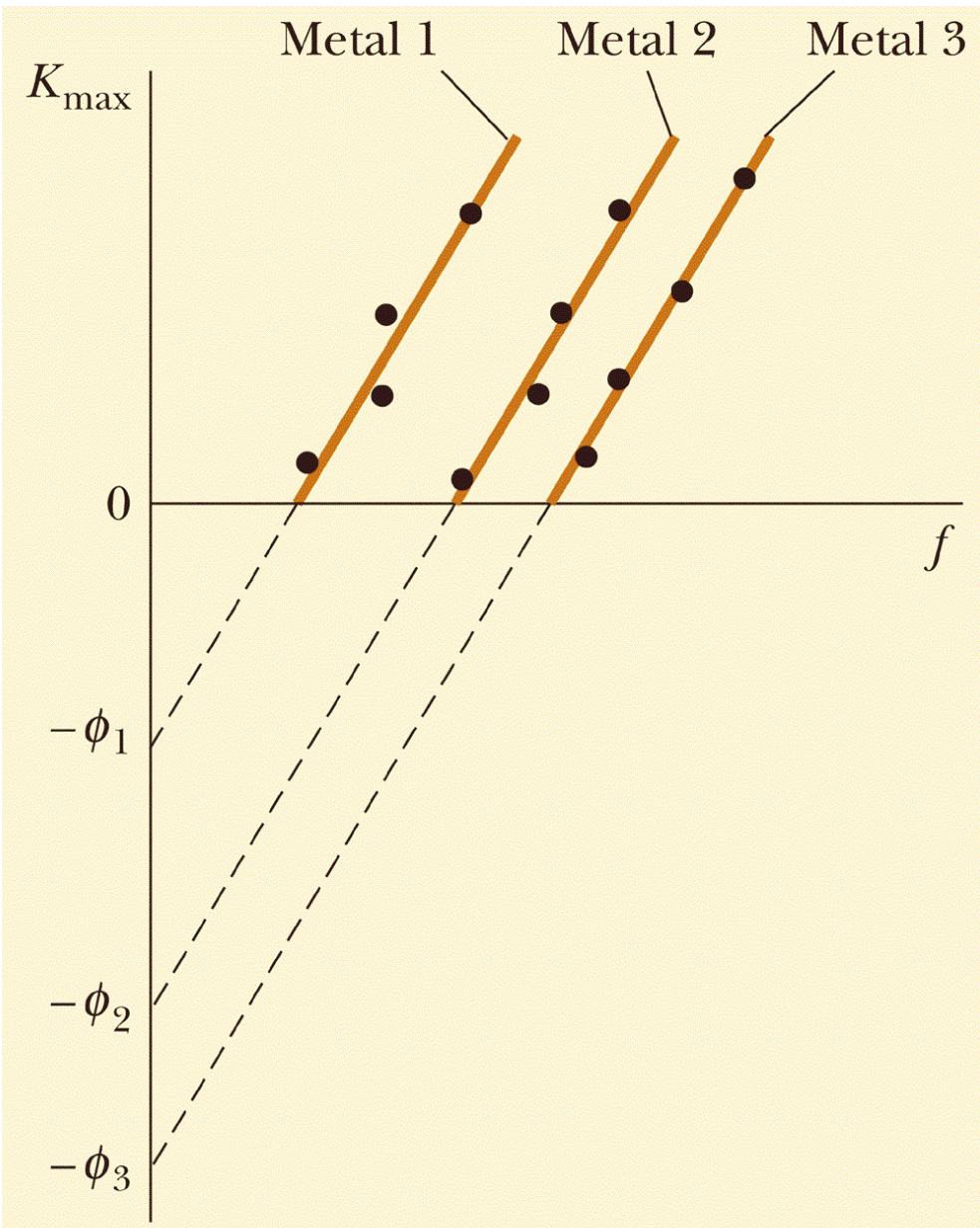


Retarding Potential Vs Light Frequency (f)

Shining Light With Constant Intensity But different frequencies
Larger the frequency of light, larger is the stopping voltage (and thus the kinetic energy of the “photoelectrons”)

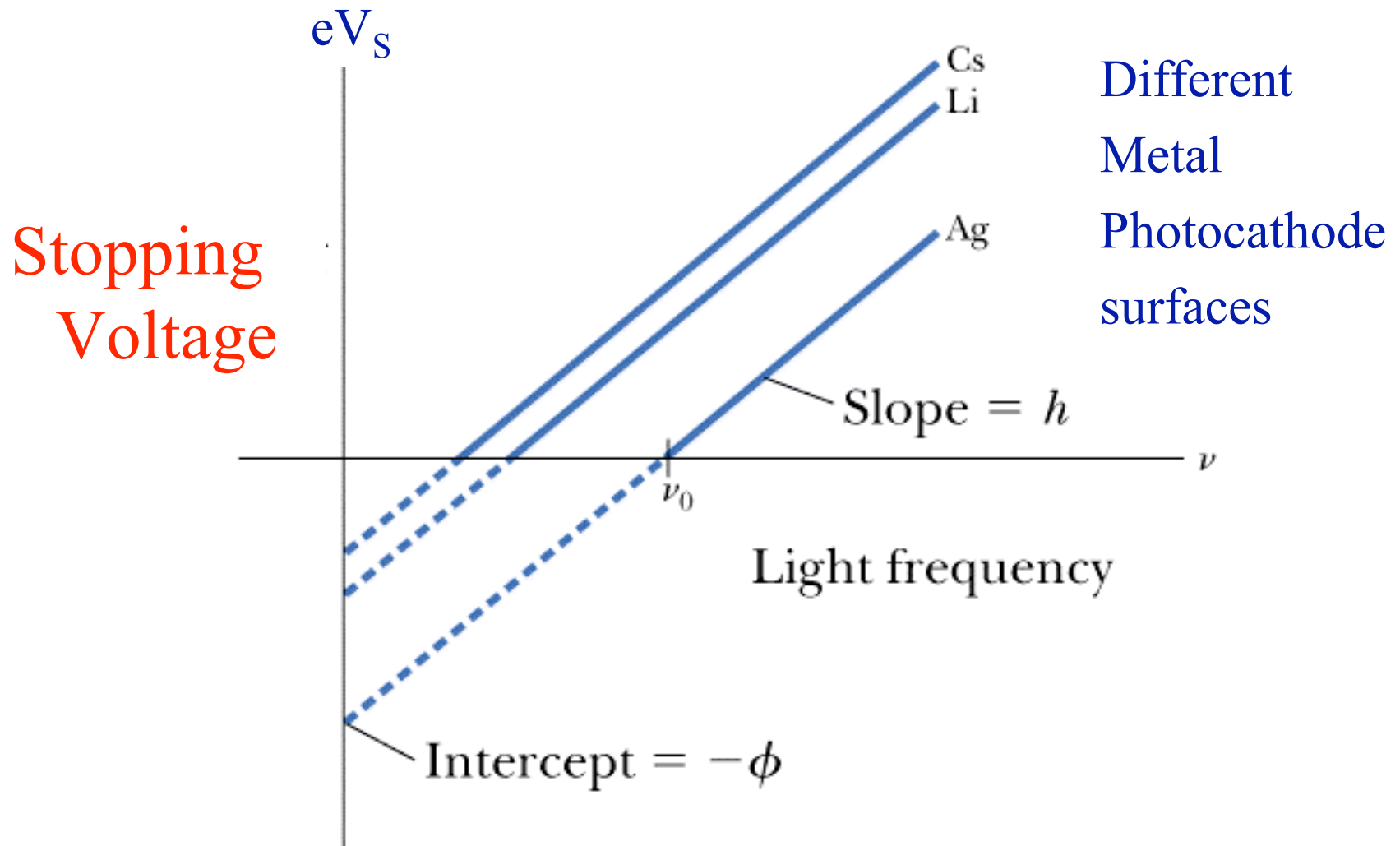


Stopping Voltage V_s For Different Photocathode Surfaces



$$eV_s = K_{\text{MAX}} = \max \text{ KE}$$

Stopping Voltage V_s Vs Incident Light Frequency



Conclusions from the Experimental Observation

- Max Kinetic energy K_{MAX} **independent** of Intensity I for light of same frequency
- **No** photoelectric effect occurs if light frequency f is below a threshold no matter how high the intensity of light
- For a particular metal, light with $f > f_0$ causes photoelectric effect **IRRESPECTIVE** of light intensity.
 - f_0 is characteristic of that metal
- Photoelectric effect is instantaneous !...not time delay

Can one Explain all this Classically ?

Classical Explanation of Photo Electric Effect

- As light Intensity increased $\Rightarrow \vec{E}$ field amplitude larger
 - E field and electrical force seen by the “charged subatomic oscillators” Larger
 - $\vec{F} = e\vec{E}$
 - More force acting on the subatomic charged oscillator
 - \Rightarrow More energy transferred to it
 - \Rightarrow Charged particle “hooked to the atom” should leave the surface with more Kinetic Energy KE !! The intensity of light shining rules !
- As long as light is intense enough , light of **ANY** frequency f should cause photoelectric effect
- Because the Energy in a Wave is uniformly distributed over the Spherical wavefront incident on cathode, should be a **noticeable time lag ΔT** between time is incident & the time a photo-electron is ejected : Energy absorption time
 - How much time ? Lets calculate it classically.

Classical Physics: Time Lag in Photo-Electric Effect

- Electron absorbs energy incident on a surface area where the **electron is confined \cong size of atom** in cathode metal
- Electron is **“bound” by attractive Coulomb force in the atom**, so it must absorb a minimum amount of radiation before its stripped off
- Example : Laser light Intensity $I = 120\text{W}/\text{m}^2$ on Na metal
 - Binding energy = 2.3 eV= “Work Function”
 - Electron confined in Na atom, size $\cong 0.1\text{nm}$..how long before ejection ?
 - Average Power Delivered $P_{AV} = I \cdot A$, $A = \pi r^2 \cong 3.1 \times 10^{-20} \text{m}^2$
 - If all energy absorbed then $\Delta E = P_{AV} \cdot \Delta T \Rightarrow \Delta T = \Delta E / P_{AV}$

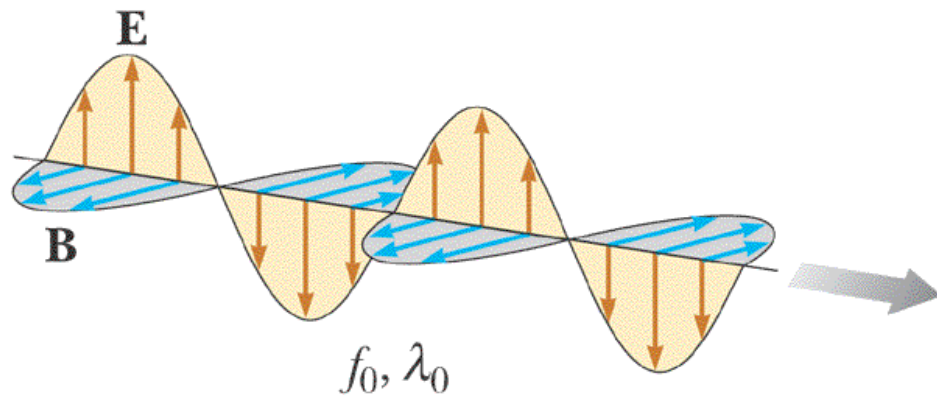
$$\Delta T = \frac{(2.3\text{eV})(1.6 \times 10^{-19} \text{J} / \text{eV})}{(120\text{W} / \text{m}^2)(3.1 \times 10^{-20} \text{m}^2)} = 0.10 \text{ S}$$

- Classical Physics predicts Measurable delay even by the primitive clocks of 1900
- But in experiment, the effect was observed to be instantaneous !!
- **Classical Physics fails in explaining all results**

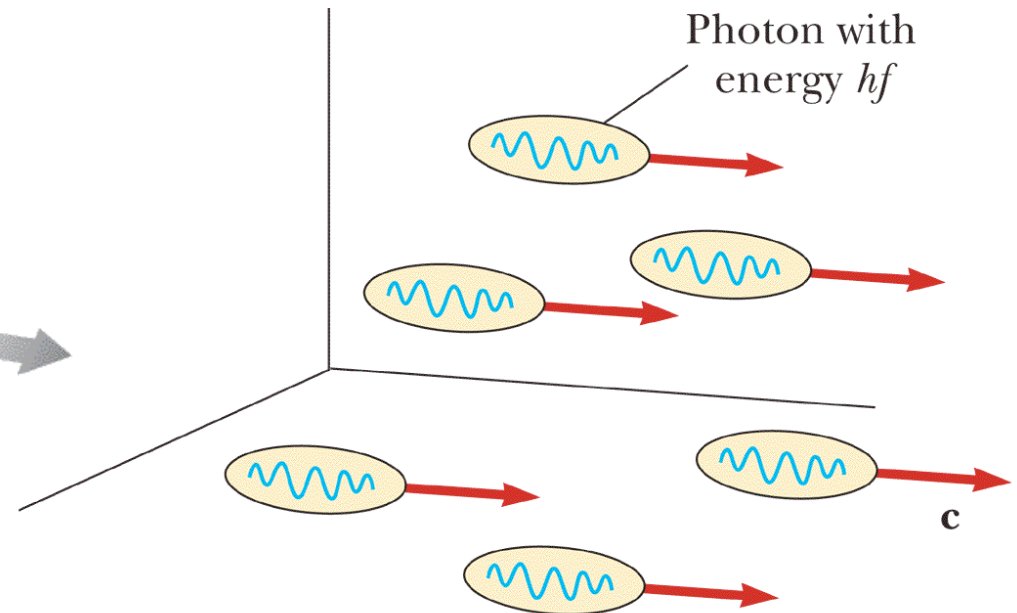
That's Disaster # 2 !

Einstein's Explanation of PhotoElectric Effect

What Maxwell Saw of EM Waves



What Einstein Saw of EM Waves



Light as bullets of "photons"
Energy concentrated in photons
Energy exchanged instantly
Energy of EM Wave $E = hf$

Einstein's Explanation of Photoelectric Effect

- Energy associated with EM waves is not uniformly distributed over wave-front, rather is contained in packets of “stuff” \Rightarrow PHOTON
- $E = hf = hc/\lambda$ [but is it the same h as in Planck's th.??]
- Light shining on metal emitter/cathode is a stream of photons of energy which depends on frequency f
- Photons knock off electron from metal instantaneously
 - Transfer all energy to electron
 - Energy gets used up to pay for Work Function Φ (Binding Energy)
 - Rest of the energy shows up as KE of electron $KE = hf - \Phi$
- Cutoff Frequency $hf_0 = \Phi$ (pops an electron, $KE = 0$)
- Larger intensity $I \rightarrow$ more photons incident
- Low frequency light $f \rightarrow$ not energetic enough to overcome work function of electron in atom

Photo Electric & Einstein (Nobel Prize 1921)

Light shining on metal cathode is made of photons

Energy E , depends on frequency f , $E = hf = h(c/\lambda)$

This QUANTUM of energy is used to knock off electron

$$E = hf = \phi + KE_{electron}$$

$$eV_s = KE = hf - \phi$$

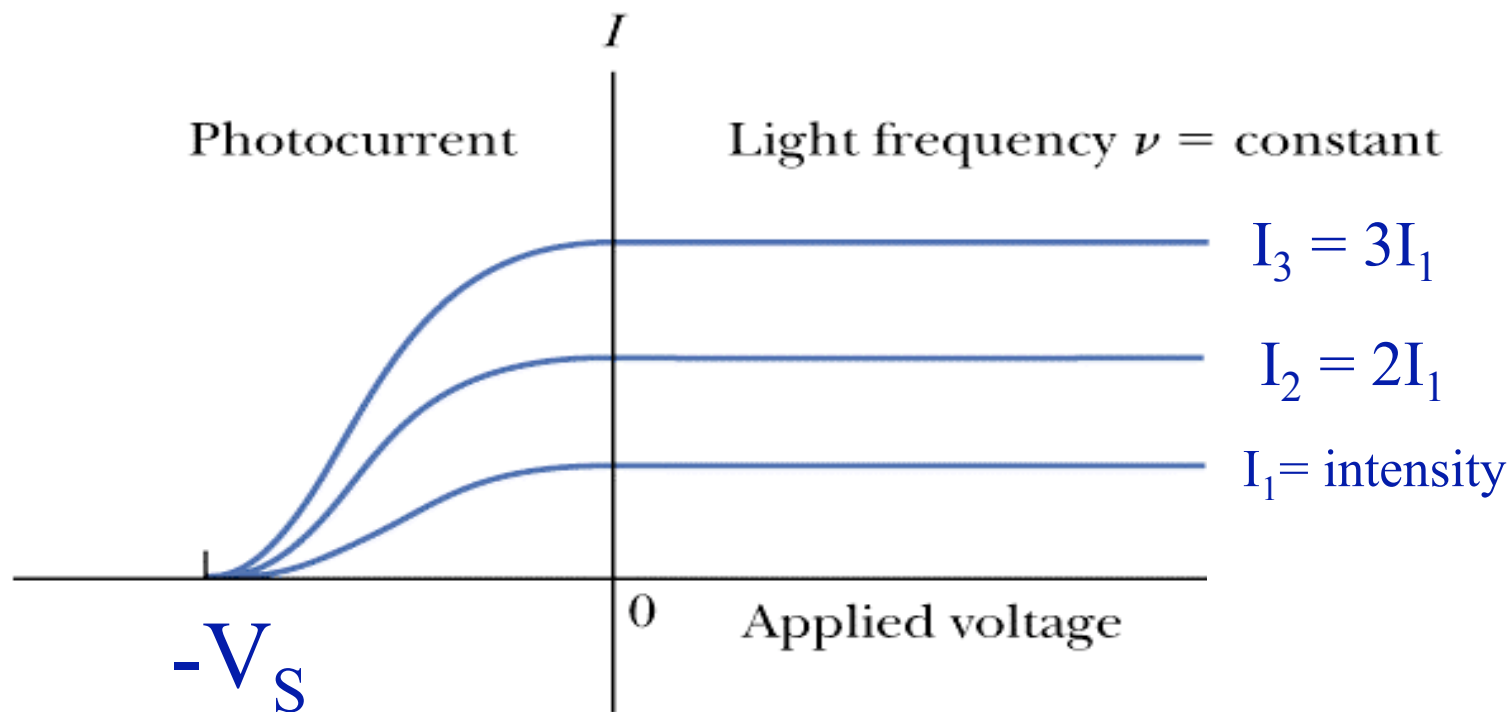
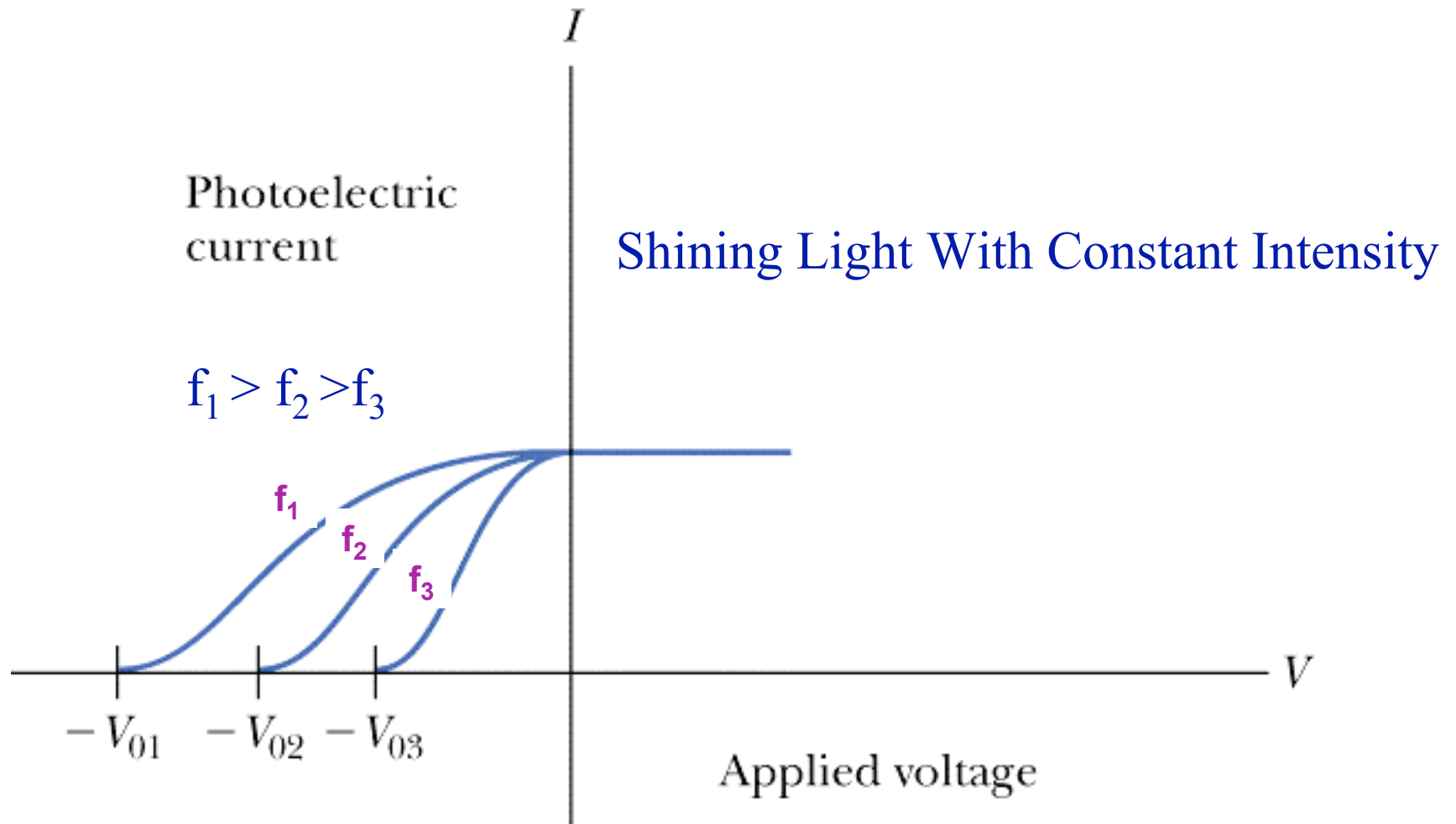


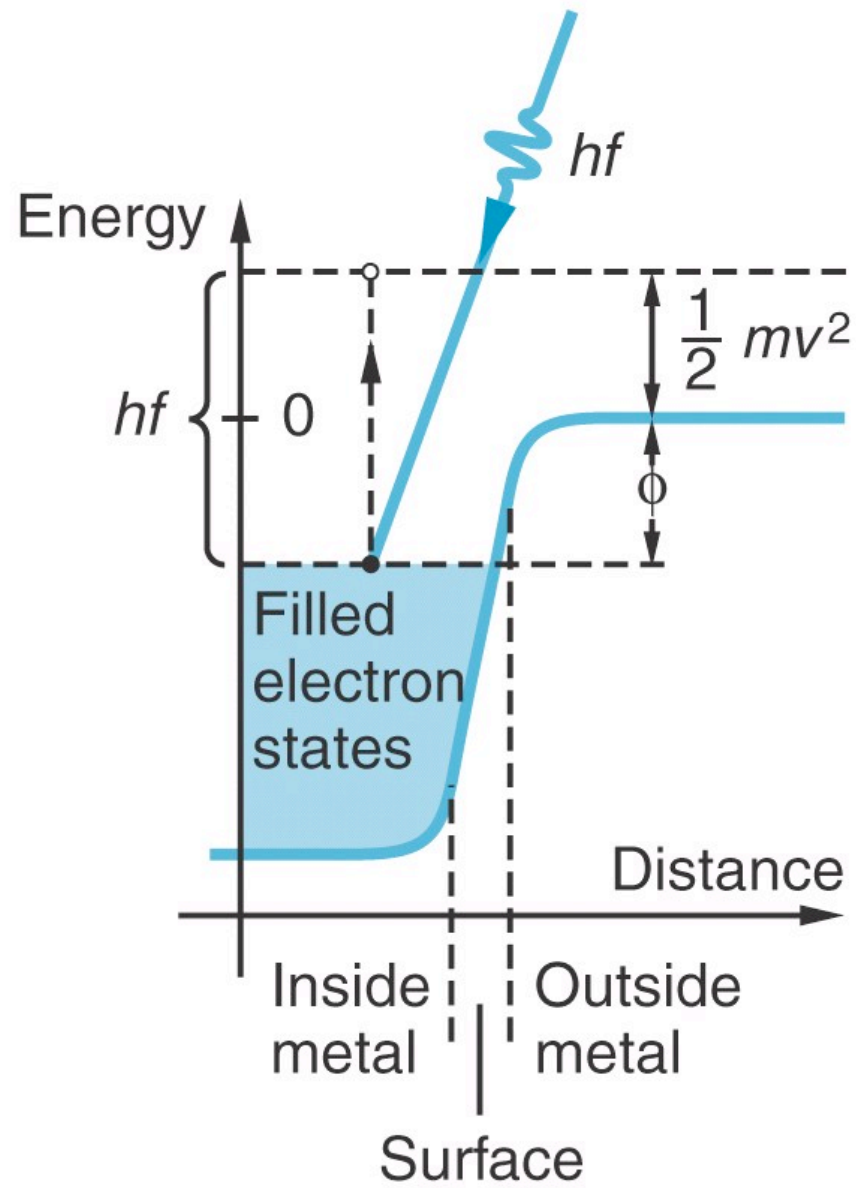
Photo Electric & Einstein (Nobel Prize 1921)

Light shining on metal cathode is made of photons

$$\text{Quantum of Energy } E = hf = KE + \phi \Rightarrow KE = hf - \phi$$



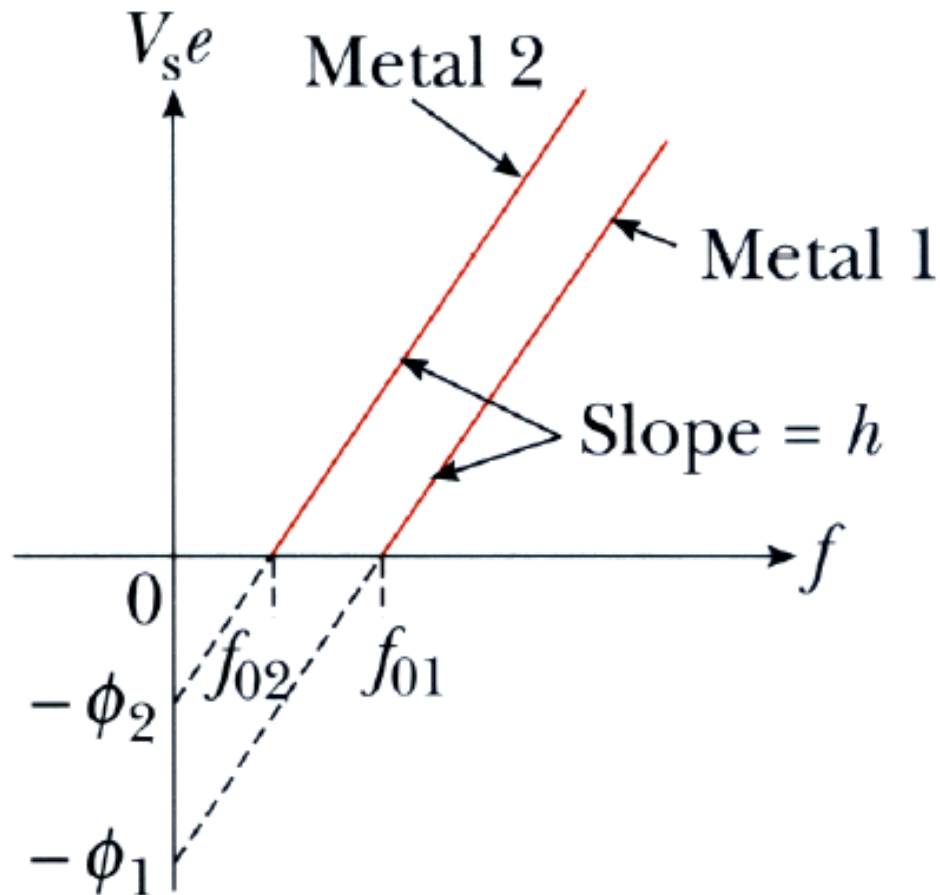
Modern View of Photoelectric Effect



Is “h” same in Photoelectric Effect as BB Radiation?

Slope $h = 6.626 \times 10^{-34} \text{ JS}$

Einstein \rightarrow Nobel Prize!



No matter where you travel
in the galaxy and beyond...
..no matter what experiment
You do

h : Planck's constant is same

NOBEL PRIZE FOR PLANCK

Work Function (Binding Energy) In Metals

TABLE 3-1 Photoelectric work functions

Element	ϕ (eV)
Na	2.28
C	4.81
Cd	4.07
Al	4.08
Ag	4.73
Pt	6.35
Mg	3.68
Ni	5.01
Se	5.11
Pb	4.14

Photoelectric Effect on An Iron Surface:

Light of Intensity $I = 1.0 \mu\text{W}/\text{cm}^2$ incident on 1.0cm^2 surface of Fe

Assume Fe reflects 96% of light

further only 3% of incident light is Violet region ($\lambda = 250\text{nm}$)

barely above threshold frequency for Ph. El effect

(a) Intensity available for Ph. El effect $I = 3\% \times 4\% \times (1.0 \mu\text{W}/\text{cm}^2)$

(b) how many photo-electrons emitted per second ?

$$\begin{aligned} \# \text{ of photoelectrons} &= \frac{\text{Power}}{h f} = \frac{3\% \times 4\% \times (1.0 \mu\text{W}/\text{cm}^2) \lambda}{hc} \\ &= \frac{(250 \times 10^{-9} \text{ m})(1.2 \times 10^{-9} \text{ J/s})}{(6.6 \times 10^{-34} \text{ J}\cdot\text{s})(3.0 \times 10^8 \text{ m/s})} \\ &= 1.5 \times 10^9 \end{aligned}$$

(c) Current in Ammeter : $i = (1.6 \times 10^{-19} \text{ C})(1.5 \times 10^9) = 2.4 \times 10^{-10} \text{ A}$

(d) Work Function $\Phi = hf_0 = (4.14 \times 10^{-15} \text{ eV}\cdot\text{s})(1.1 \times 10^{15} \text{ s}^{-1})$
 $= 4.5 \text{ eV}$

Facts

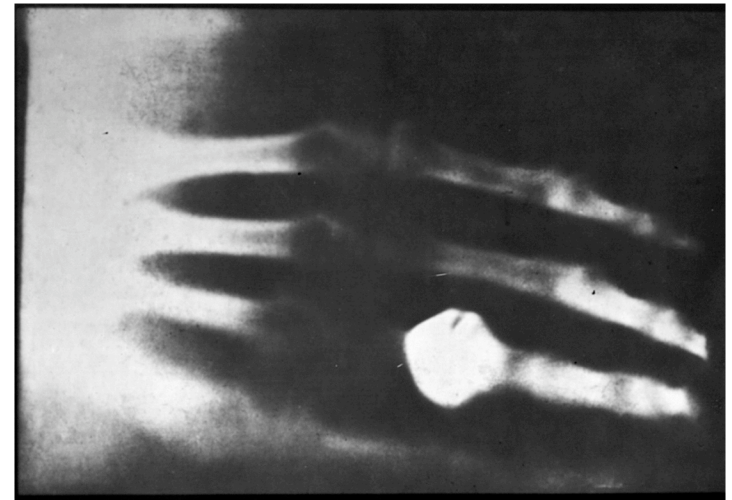
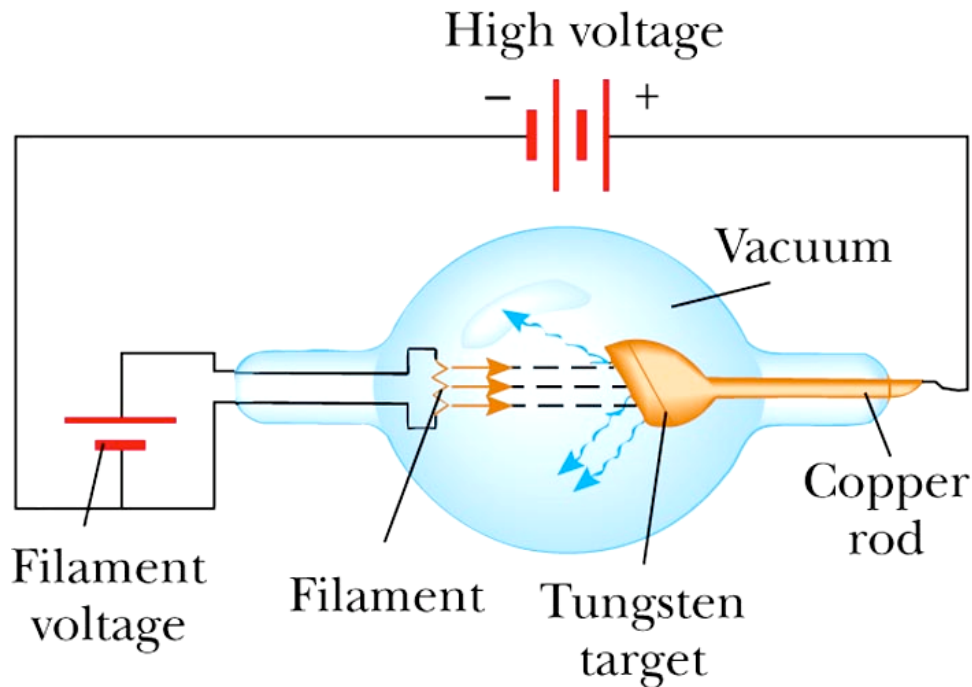
- The human eye is a sensitive photon detector at visible wavelengths: Need >5 photons of $\cong 550\text{nm}$ to register on your optical sensor
- The Photographic process :
 - Energy to Dissociate an AgBr molecule = 0.6eV
- Photosynthesis Process : 9 sunlight photon per reaction cycle of converting CO_2 and water to carbohydrate & O_2
 - chlorophyll absorbs best at $\lambda \cong 650\text{-}700\text{ nm}$
- Designing Space Shuttle “skin” : Why Platinum is a good thing
- designing Solar cells : picking your metal cathode

Photon & Relativity: Wave or a Particle ?

- Photon associated with EM waves, travel with speed =c
- For light ($m = 0$) : Relativity says $E^2 = (pc)^2 + (mc^2)^2$
- $\Rightarrow E = pc$
- But Planck tells us : $E = hf = h (c/\lambda)$
- Put them together : $hc /\lambda = pc$
 - $\Rightarrow p = h/\lambda$
 - Momentum of the photon (light) is inversely proportional to λ
- But we associate λ with waves & p with particles
....what is going on??
 - A new paradigm of conversation with the subatomic particles : **Quantum Physics**

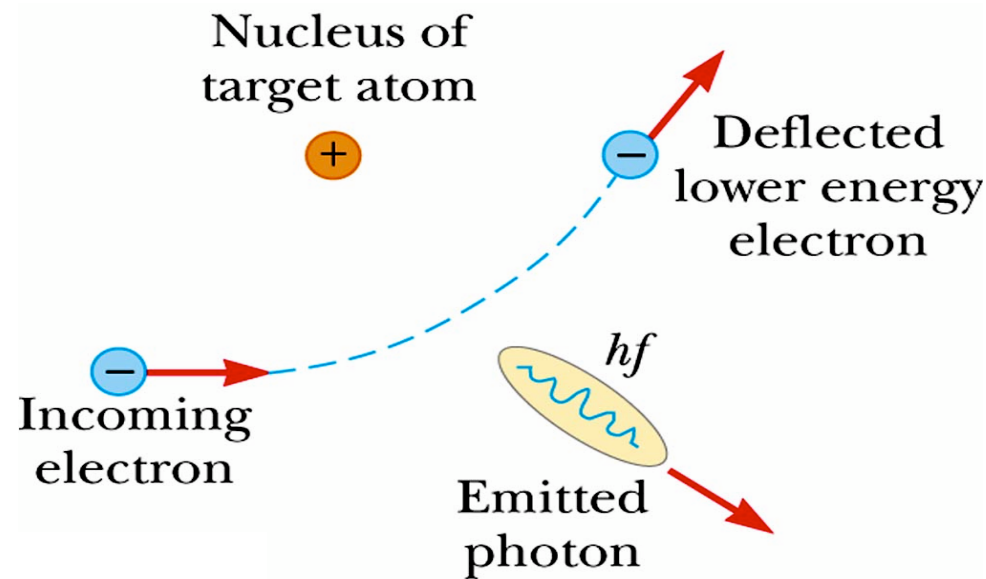
X Rays “Bremsstrahlung”: The Braking Radiation

- EM radiation, produced by bombarding a metal target with energetic electrons.
- Produced in general by ALL decelerating charged particles
- X rays : very short $\lambda \cong 60\text{-}100 \text{ pm}$ (10^{-12}m), large frequency f
- Very penetrating because very energetic $E = hf$!!



Useful for probing structure of sub-atomic Particles
(and your teeth)

X Ray Production Mechanism

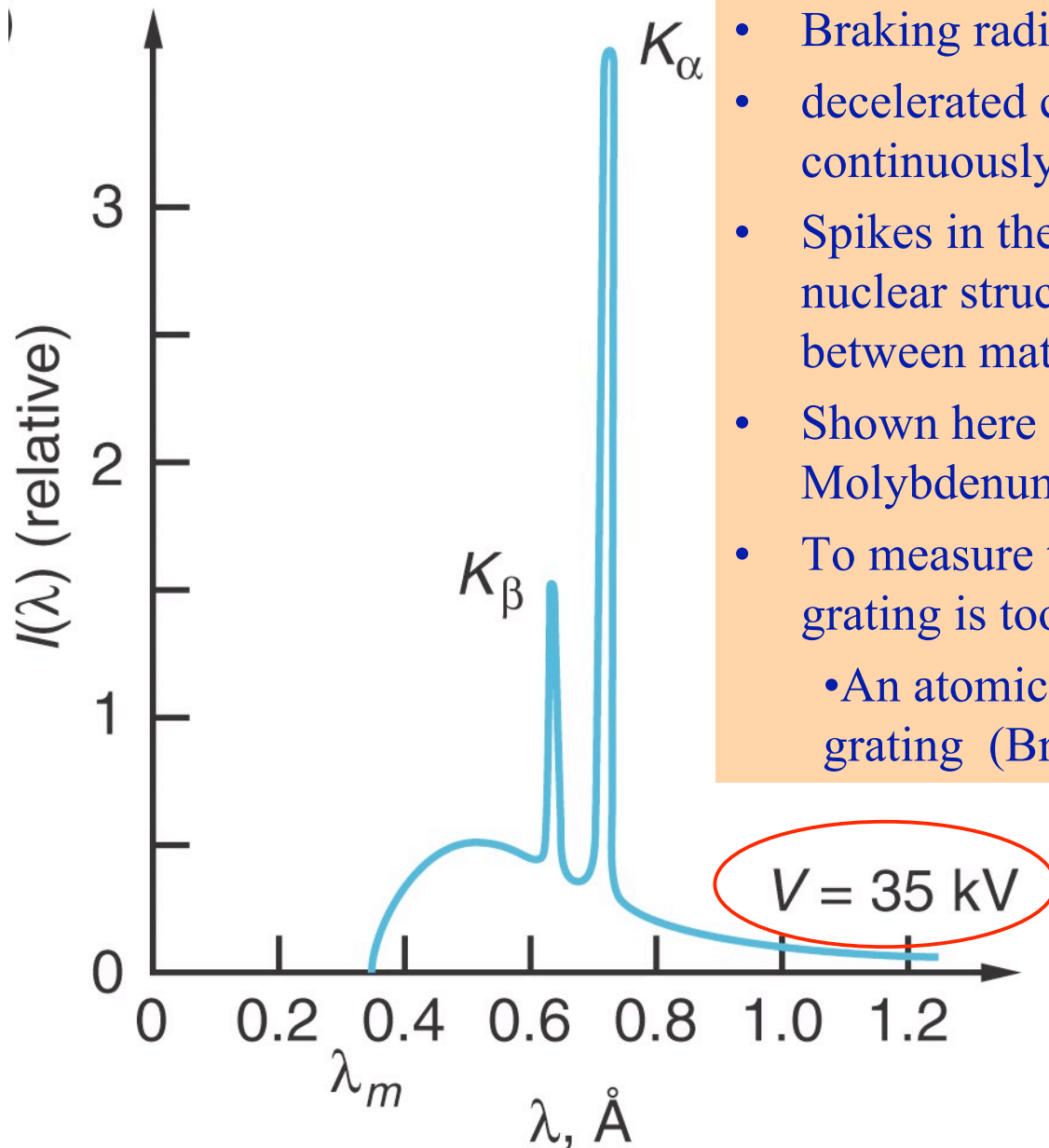


when electron passes near a positively charged target nucleus contained in target material, its deflected from its path because of its electrical attraction, experiences acceleration.

Rules of E&M say that any charged particle will emit radiation when accelerated. This EM radiation “appears” as photons. Since photon carries energy and momentum, the electron must lose same amount. If all of electron’s energy is lost in just one single collision then

$$e \Delta V = hf_{\max} = \frac{hc}{\lambda_{\min}} \quad \text{or} \quad \boxed{\lambda_{\min} = \frac{hc}{e \Delta V}}$$

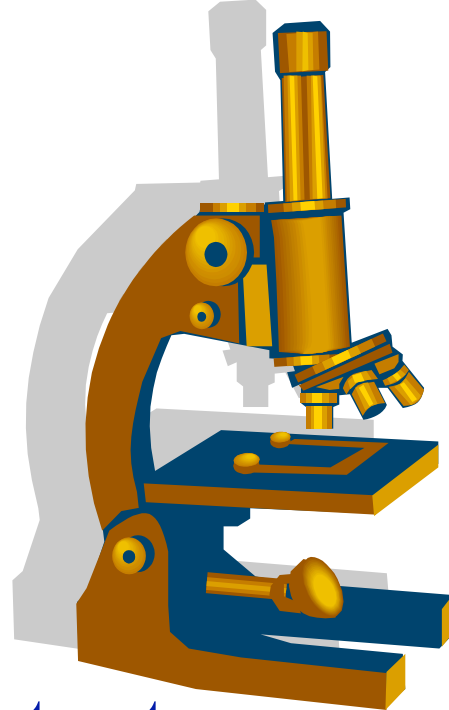
X Ray Spectrum in Molybdenum (Mo)



- Braking radiation predicted by Maxwell's eqn
- decelerated charged particle will radiate continuously
- Spikes in the spectrum are characteristic of the nuclear structure of target material and varies between materials
- Shown here are the α and β lines for Molybdenum (Mo)
- To measure the wavelength, diffraction grating is too wide, need smaller slits
 - An atomic crystal lattice as diffraction grating (Bragg)

- X rays are EM waves of low wavelength, high frequency (and energy) and demonstrate **characteristic features of a wave**

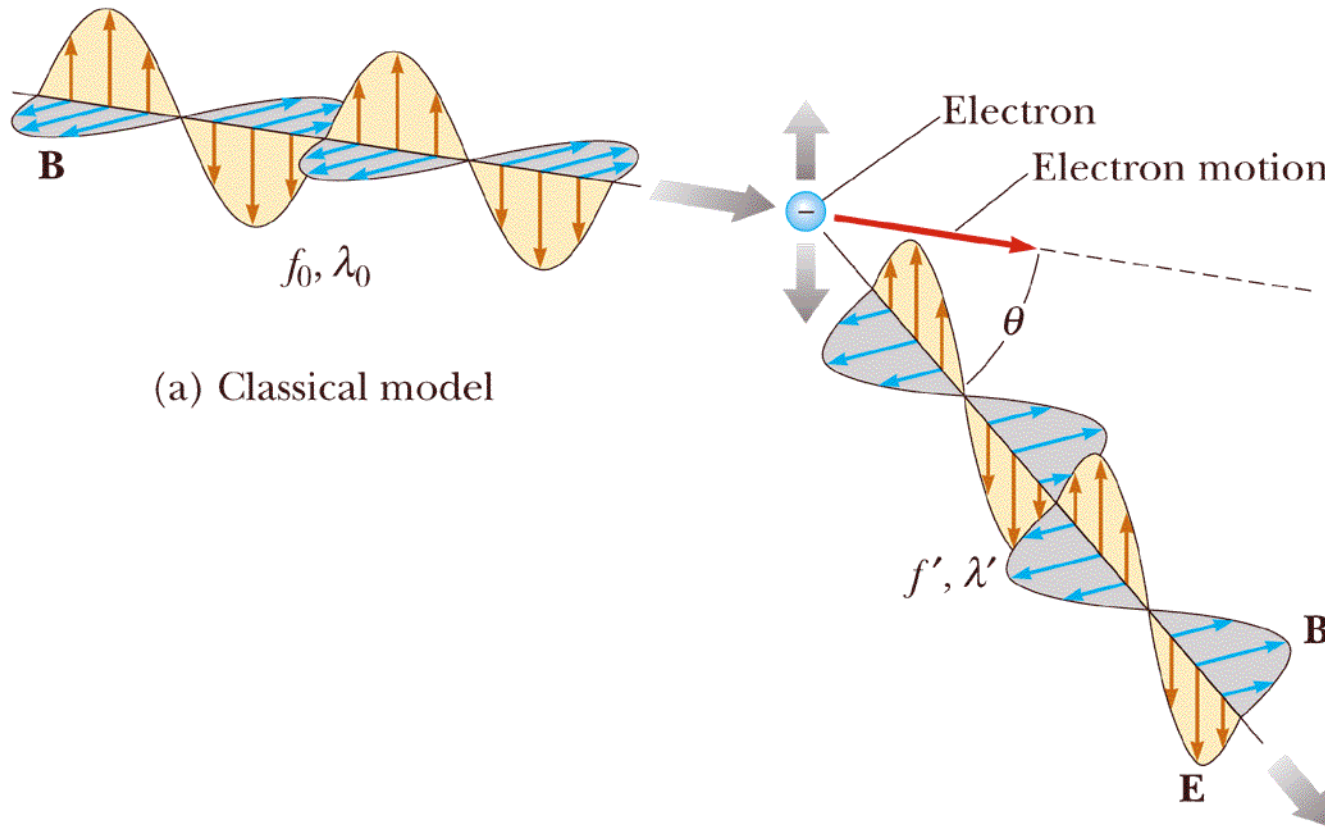
- Interference
- Diffraction



- To probe into a structure you need a light source with wavelength much smaller than the features of the object being probed
 - Good Resolution $\rightarrow \lambda \ll \Delta$
- X rays allows one probe at atomic size (10^{-10})m

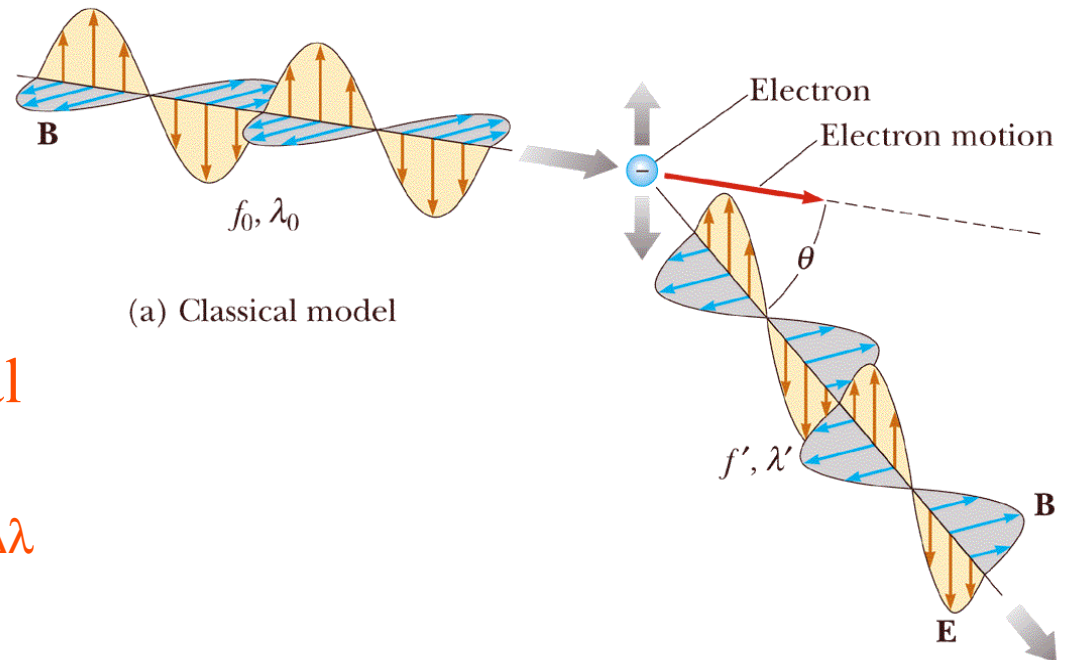
Compton Scattering : Quantum Pool !

- 1922: Arthur Compton (USA) proves that X-rays (EM Waves) have particle like properties (acts like photons)
 - Showed that classical theory failed to explain the scattering effect of
 - X rays on to free (not bound, barely bound electrons)
- Experiment : shine X ray EM waves on to a surface with “almost” free electrons
 - Watch the scattering of light off electron : measure time + wavelength of scattered X-ray

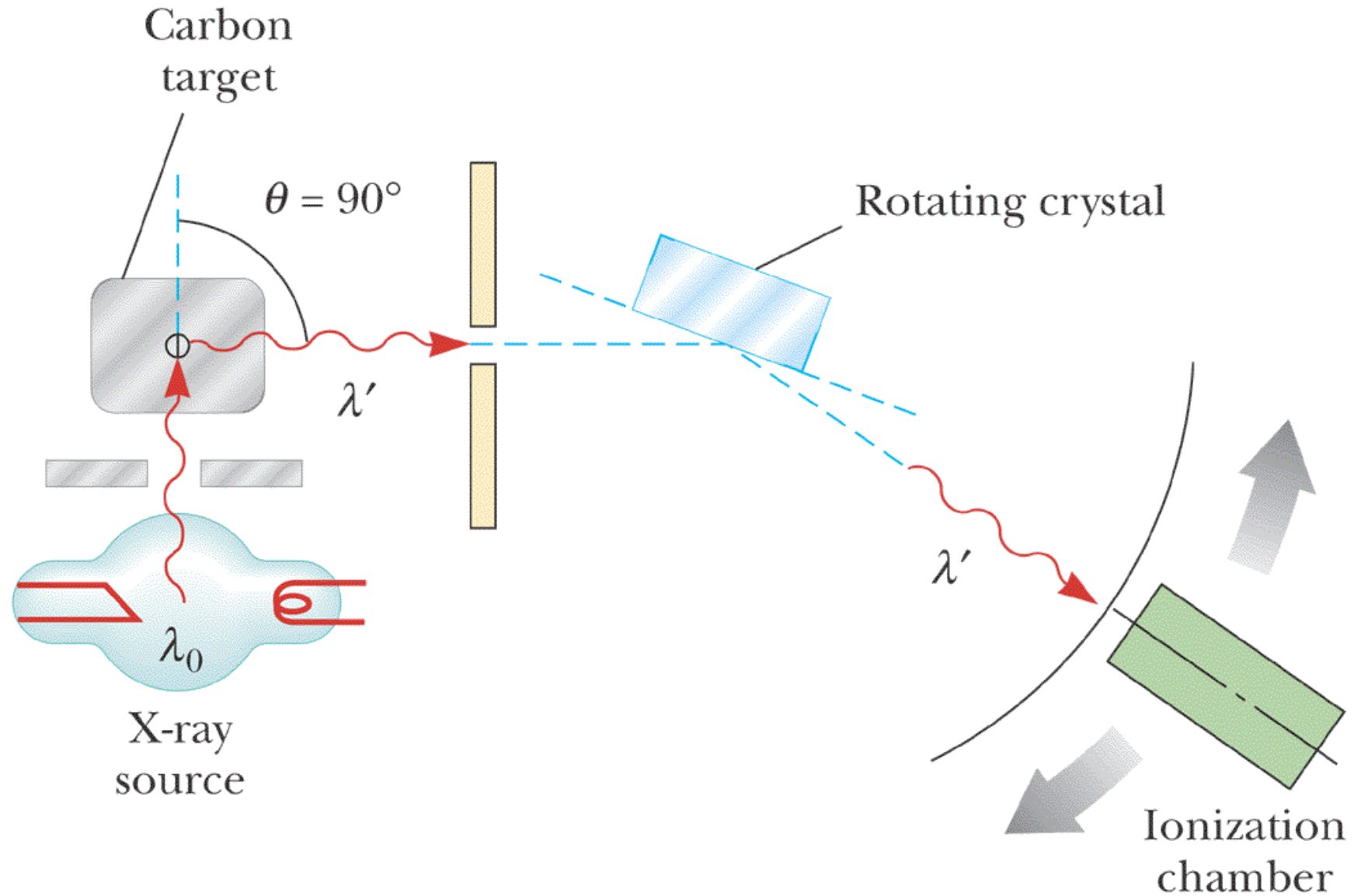


Compton Effect: what should Happen Classically?

- Plane wave $[f, \lambda]$ incident on a surface with loosely bound electrons \rightarrow interaction of E field of EM wave with electron: $\mathbf{F} = e\mathbf{E}$
- Electron oscillates with $f = f_{\text{incident}}$
- Eventually radiates **spherical waves** with $f_{\text{radiated}} = f_{\text{incident}}$
 - At all scattering angles, Δf & $\Delta \lambda$ must be zero
- Time delay while the electron gets a “tan” : soaks in radiation

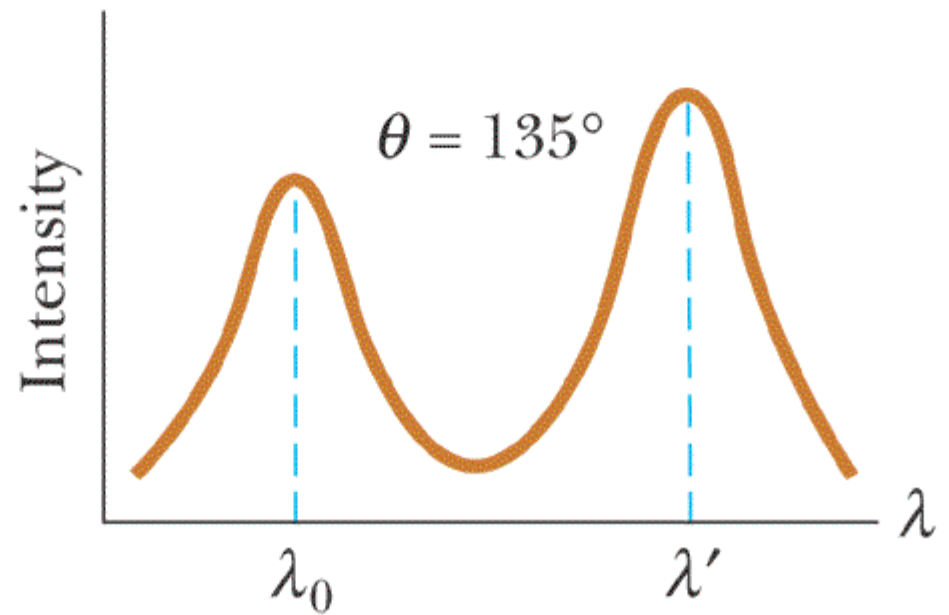
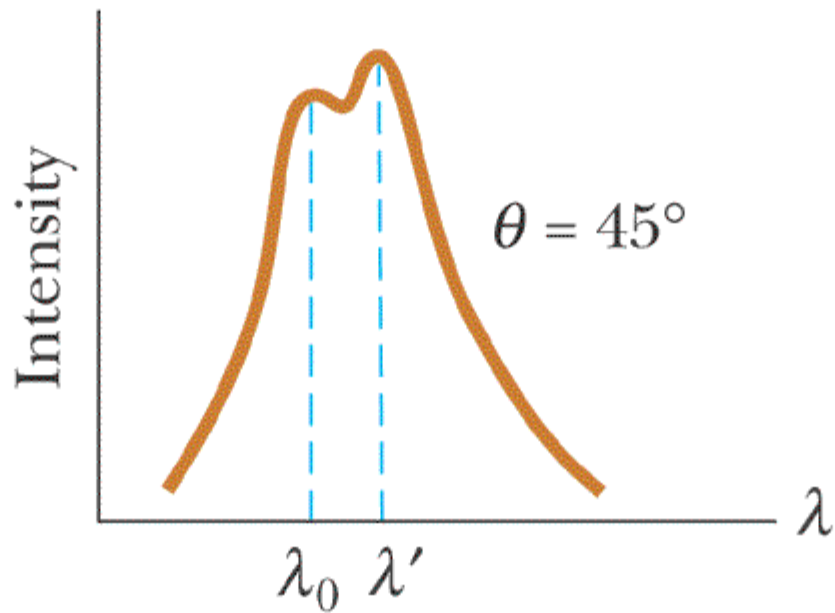
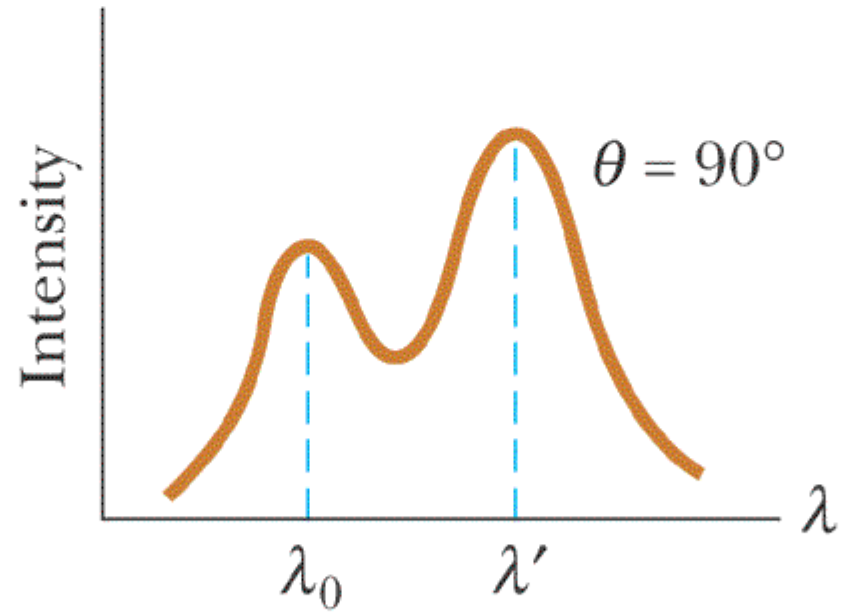
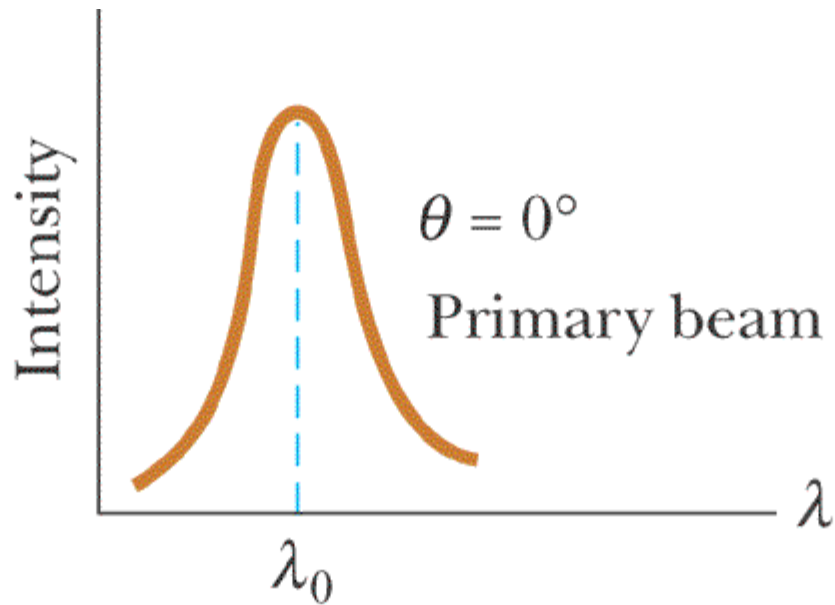


Compton Scattering : Setup & Results

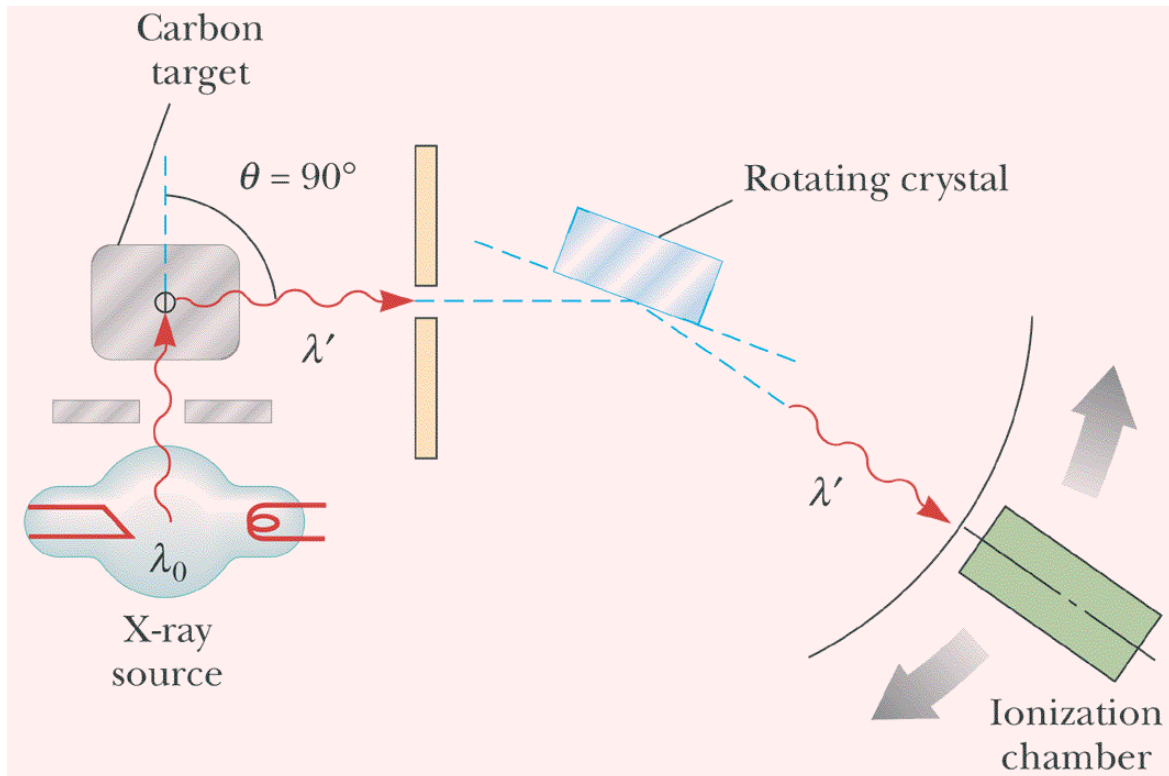


Scattered λ' larger than incident

Compton Scattering Observations



Compton Scattering : Summary of Observations

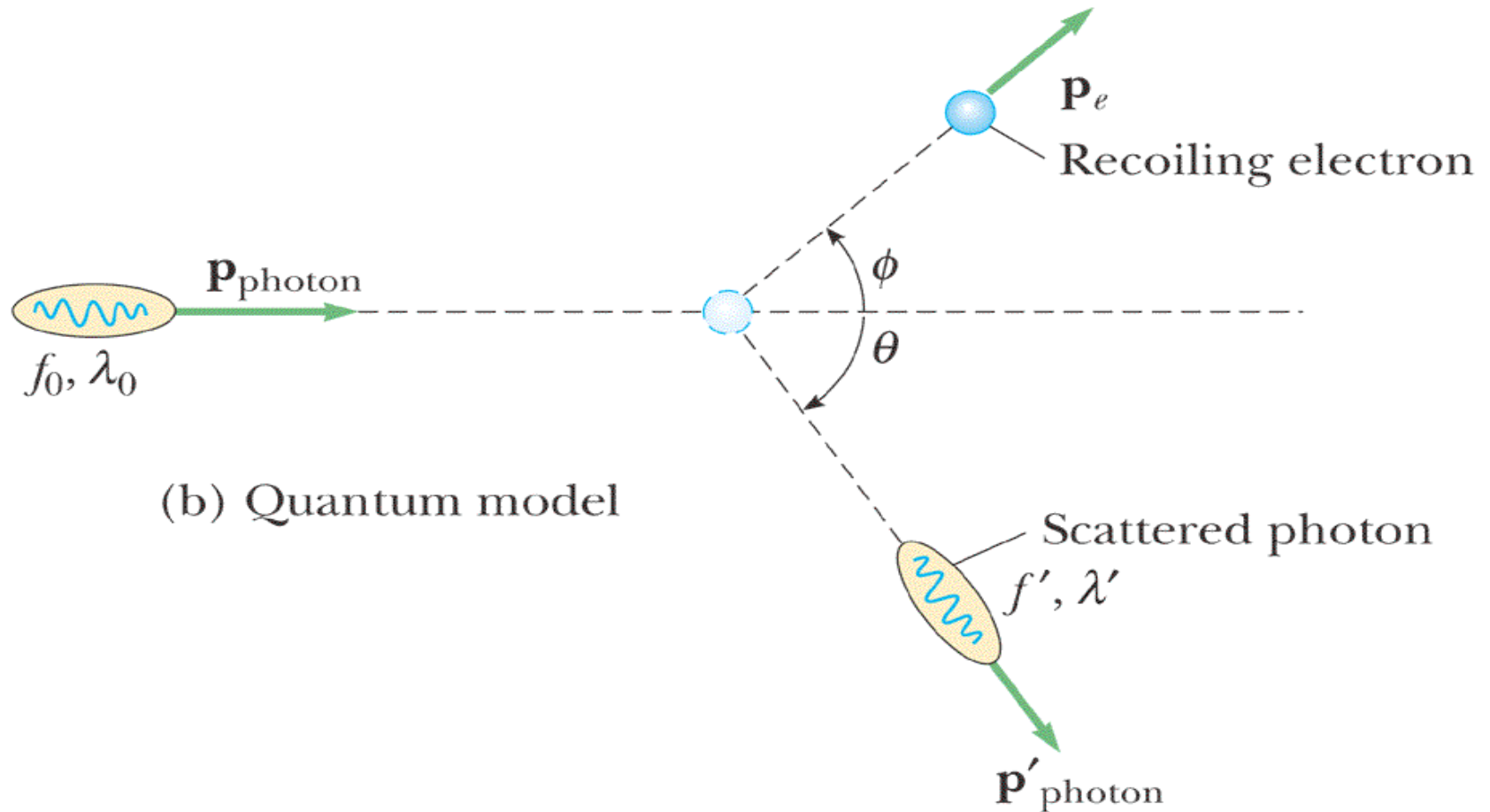


$$\Delta\lambda = (\lambda' - \lambda) \propto (1 - \cos \theta) !$$

Not isotropy in distribution of scattered radiation

How does one explain this startling anisotropy?

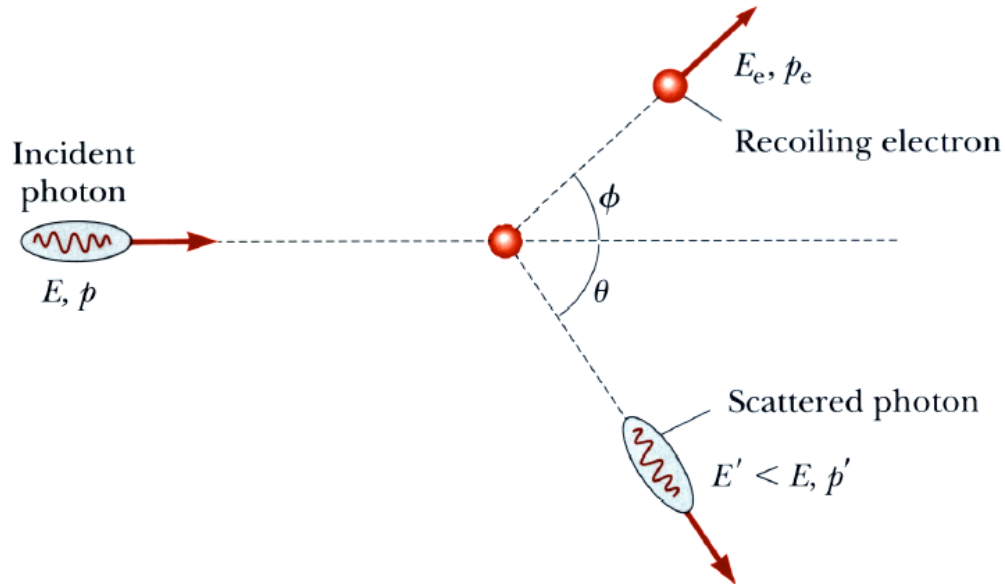
Compton Effect : Quantum (Relativistic) Pool



(b) Quantum model



Compton Scattering: Quantum Picture



Energy Conservation:

$$E + m_e c^2 = E' + E_e$$

Momentum Conserv:

$$p = p' \cos \theta + p_e \cos \phi$$

$$0 = p' \sin \theta - p_e \sin \phi$$

Use these to **eliminate**
electron deflection

angle (not measured)

$$p_e \cos \phi = p - p' \cos \theta$$

$$p_e \sin \phi = p' \sin \theta$$

Square and add \Rightarrow

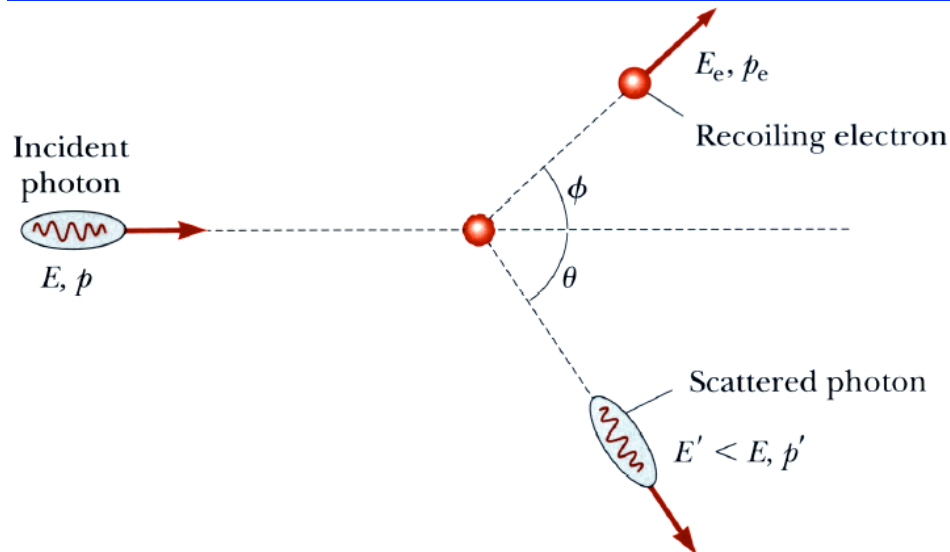
$$p_e^2 = p^2 - 2pp' \cos \theta + p'^2$$

Eliminate p_e & E_e using

$$E_e^2 = p_e^2 c^2 + m_e^2 c^4 \quad \&$$

$$E_e = (E - E') + m_e c^2$$

Compton Scattering: The Quantum Picture



Energy Conservation:

$$E + m_e c^2 = E' + E_e$$

Momentum Conserv:

$$p = p' \cos \theta + p_e \cos \phi$$

$$0 = p' \sin \theta - p_e \sin \phi$$

Use these to **eliminate** electron deflection angle (not measured)

$$\left((E - E') + m_e c^2 \right)^2 = \left[p^2 - 2pp' \cos \theta + p'^2 \right] + (m_e c^2)^2$$

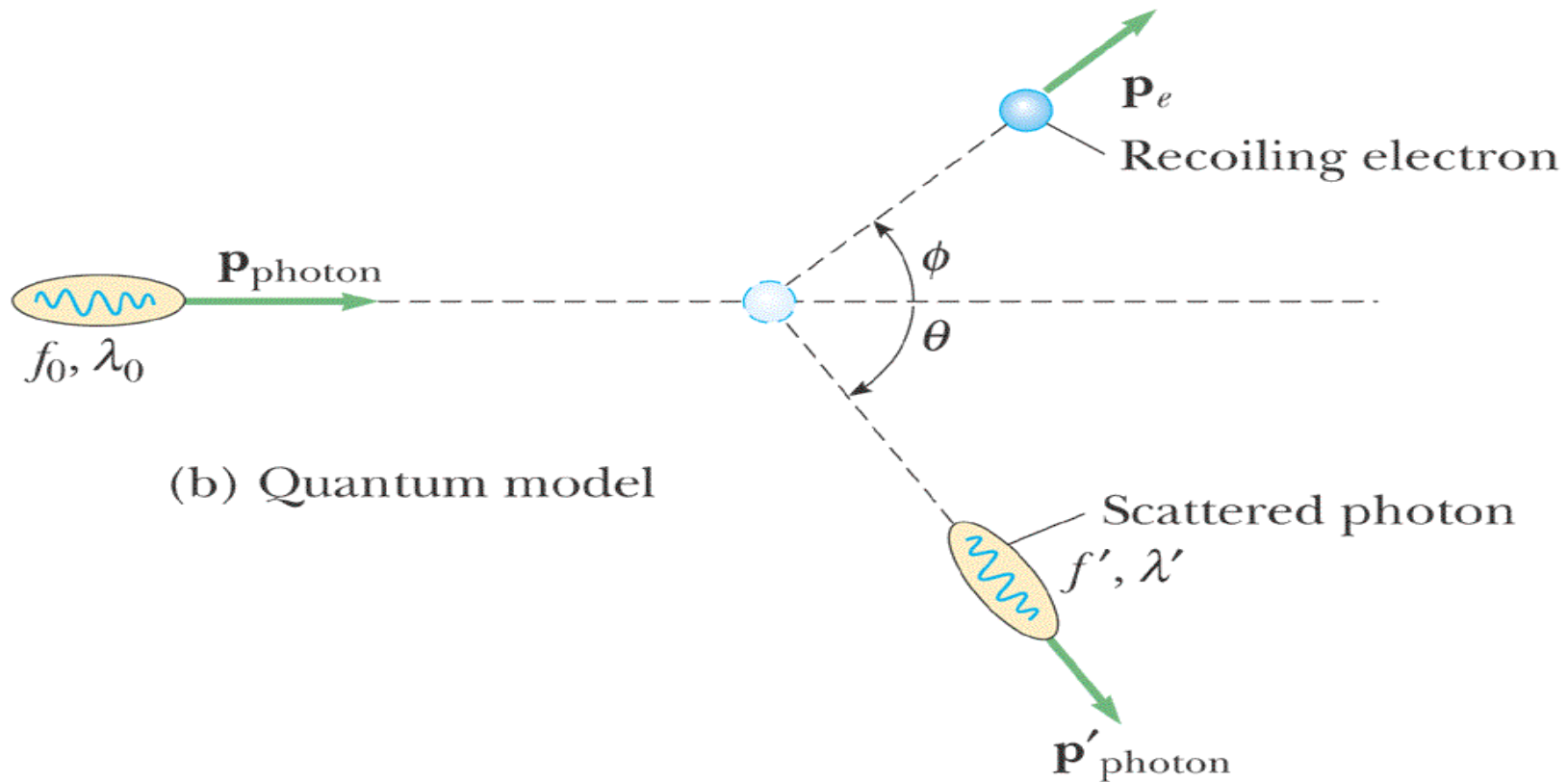
For light $p = \frac{E}{c} \Rightarrow$

$$E^2 + E'^2 - 2EE' + 2(E - E')mc^2 = \left[\frac{E^2}{c^2} - 2\frac{EE'}{c^2} \cos \theta + \frac{E'^2}{c^2} \right] c^2$$

$$\Rightarrow -EE' + (E - E')mc^2 = -EE' \cos \theta$$

$$\Rightarrow \frac{E - E'}{EE'} = -\frac{1}{m_e c^2} (1 - \cos \theta) \Rightarrow \boxed{(\lambda' - \lambda) = \left(\frac{h}{m_e c} \right) (1 - \cos \theta)}$$

Rules of Quantum Pool between Photon and Electron

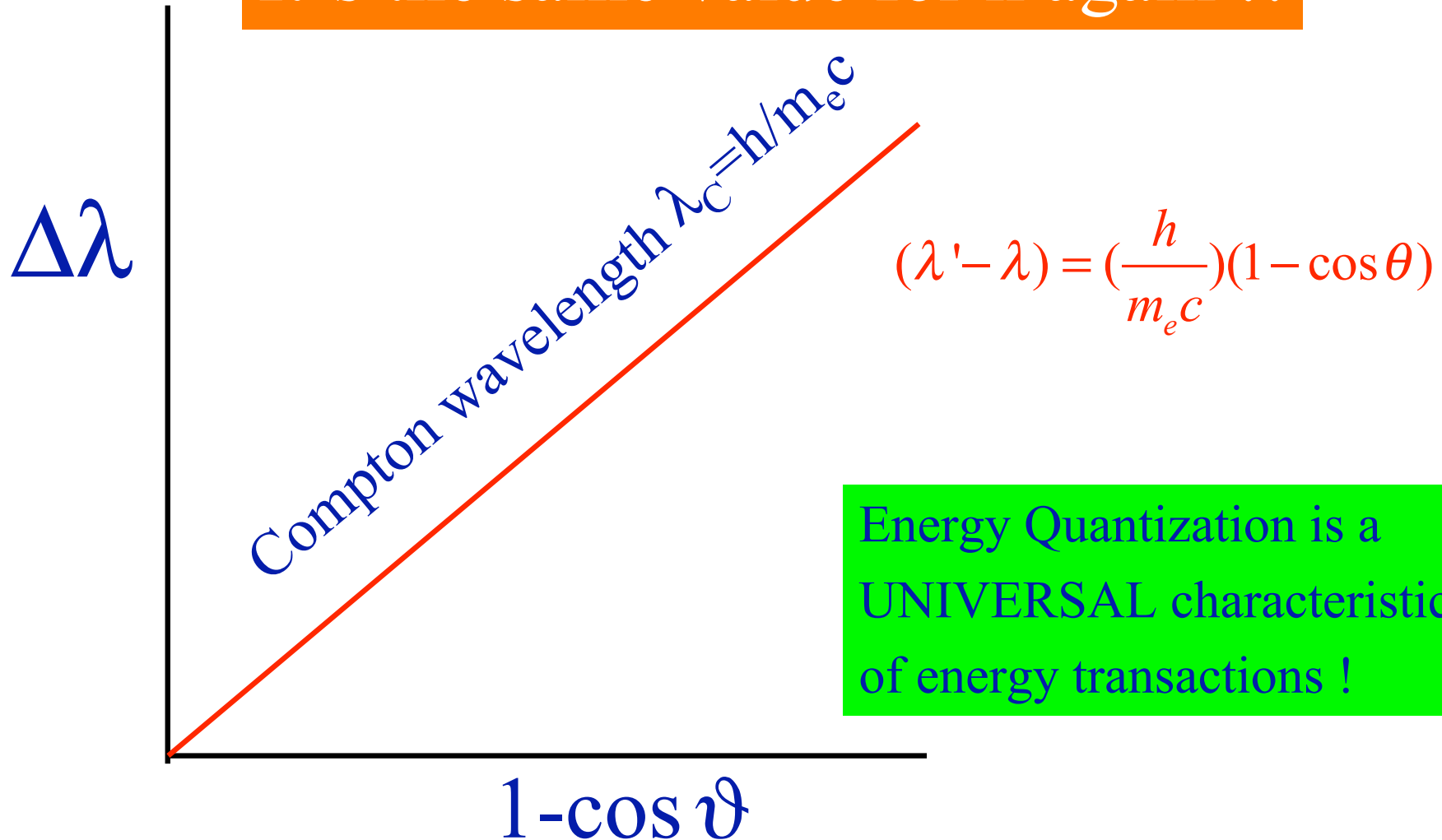


$$(\lambda' - \lambda) = \left(\frac{h}{m_e c} \right) (1 - \cos \theta)$$

Checking for h in Compton Scattering

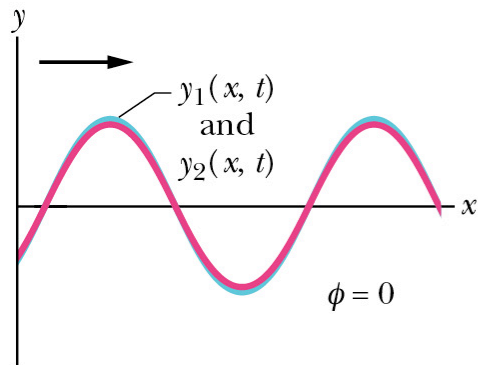
Plot scattered photon data, calculate slope and measure “h”

It's the same value for h again !!

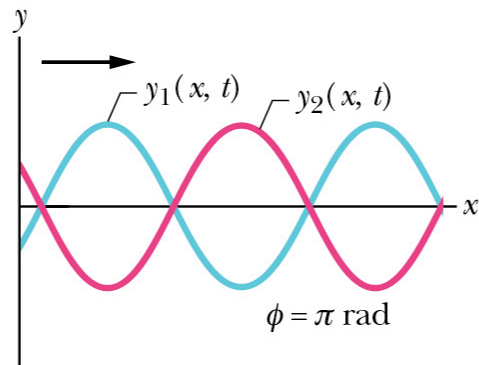
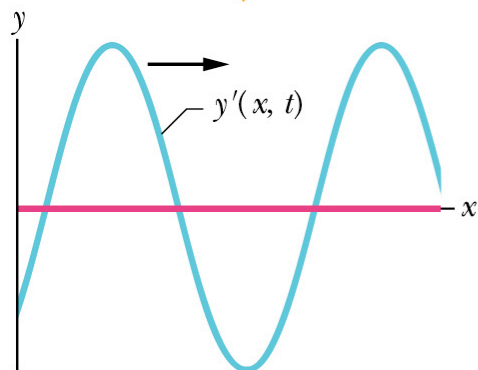


Interference of Waves: A Reminder

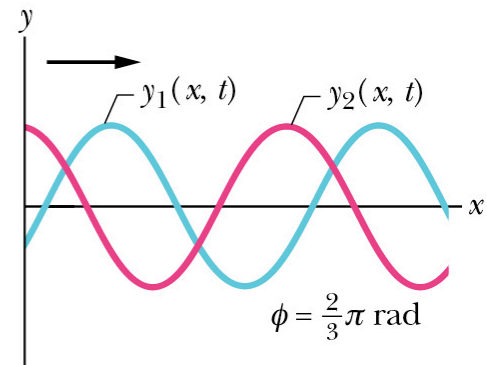
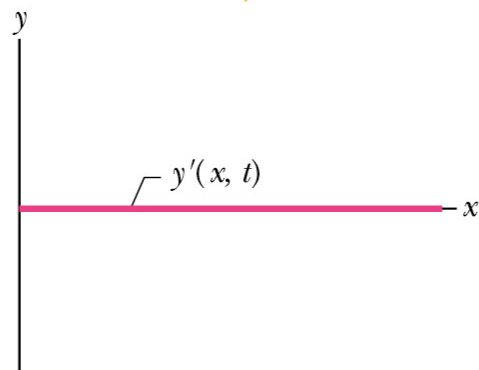
Two Identical waves $y_i(x, t) = y_{\max} \sin(k_i x - \omega_i t + \phi_i)$ travel along +x and interfere to give a resulting wave $y'(x, t)$. The resulting wave form depends on relative phase difference between 2 waves. Shown for $\Delta\phi = 0, \pi, \frac{2}{3}\pi$



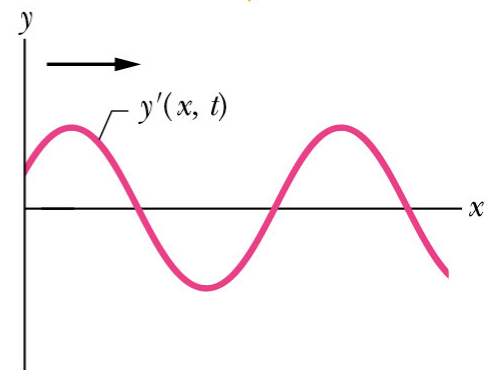
(a)



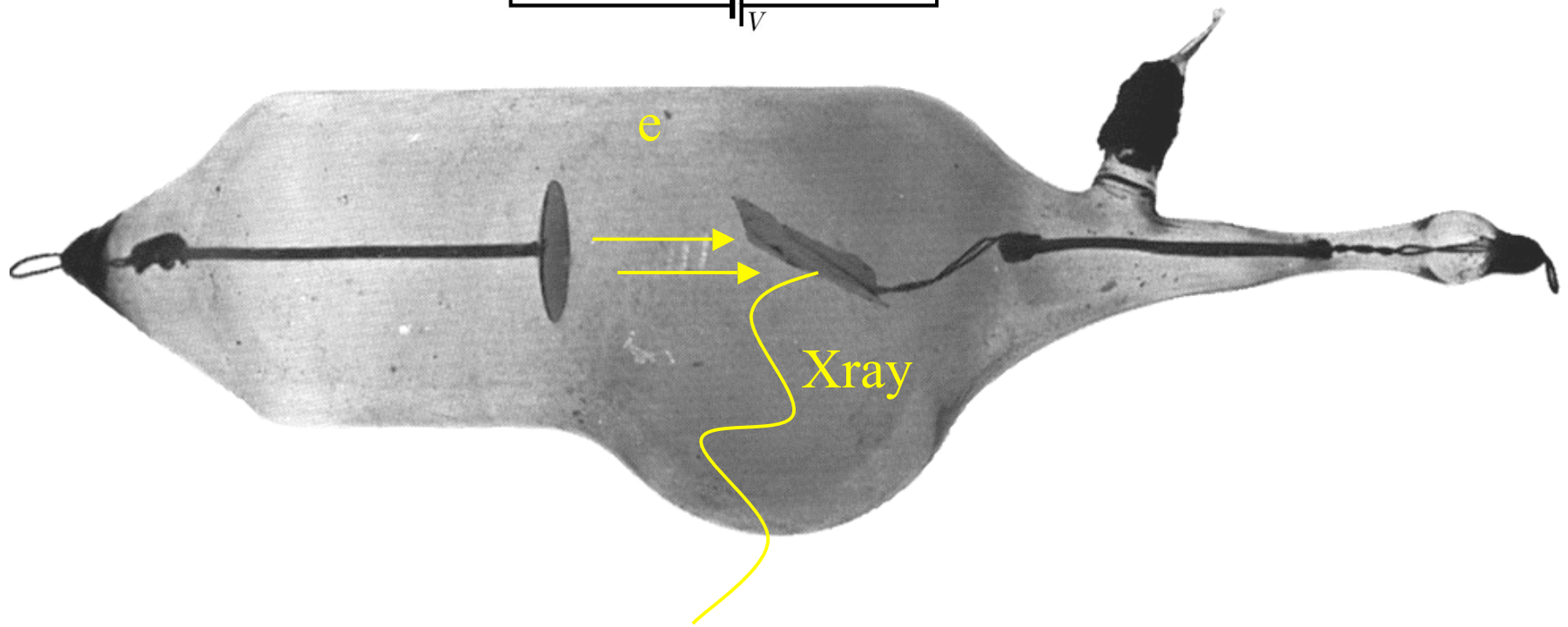
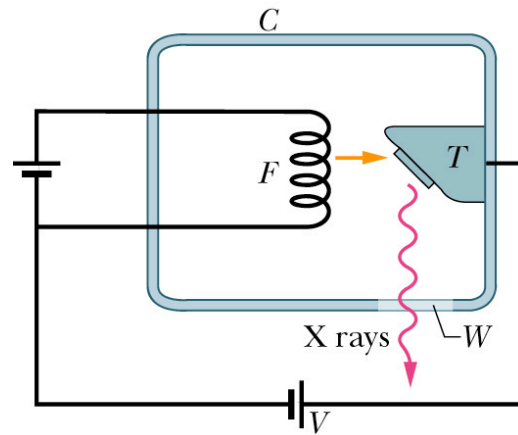
(b)



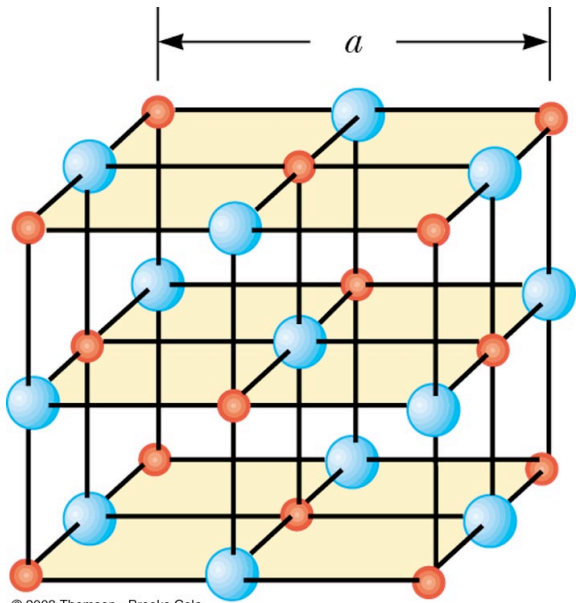
(c)



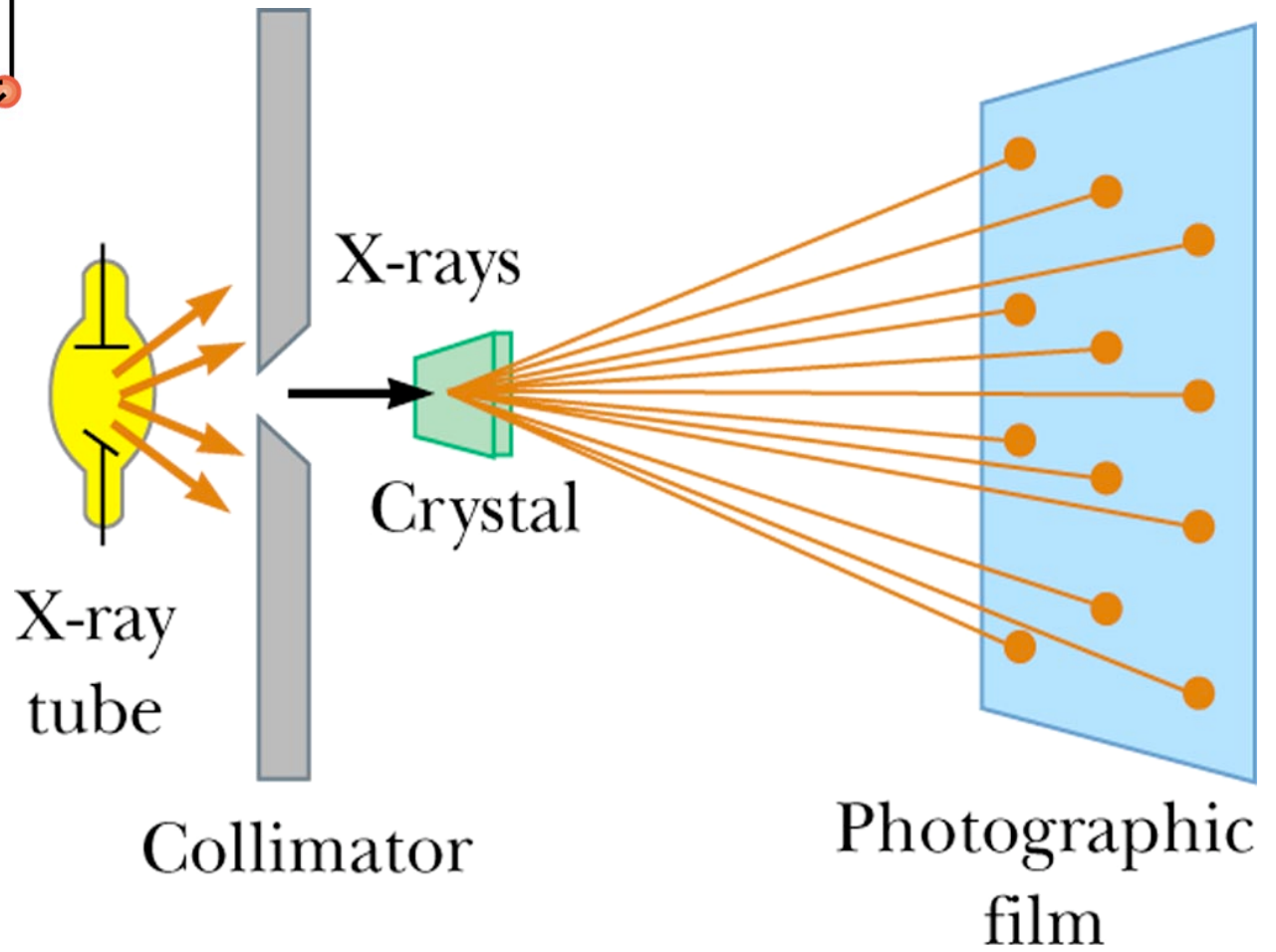
An X-ray Tube from 20th Century



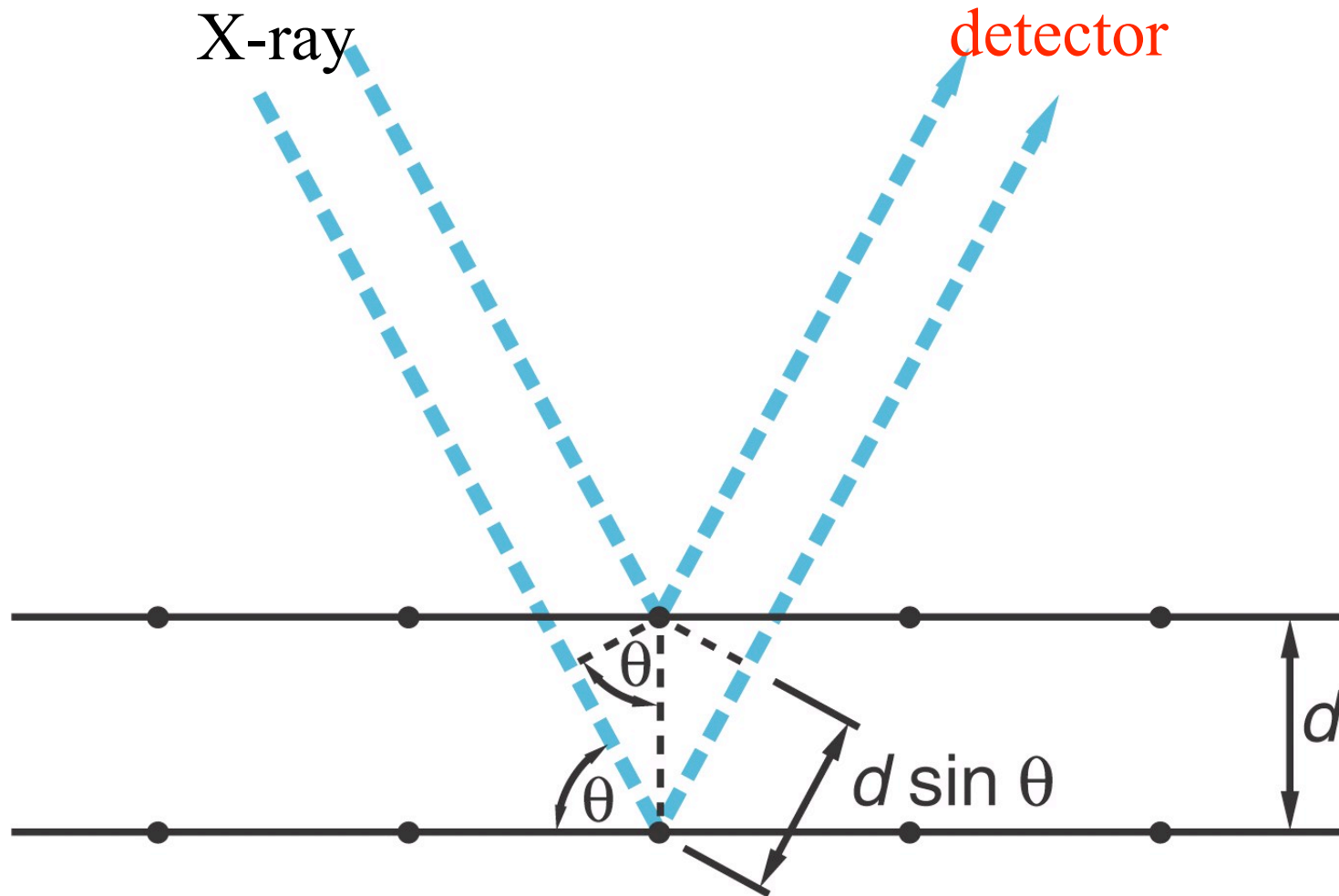
The “High Energy Accelerator” of 1900s: produced energetic light : X Ray , gave new optic to subatomic phenomena



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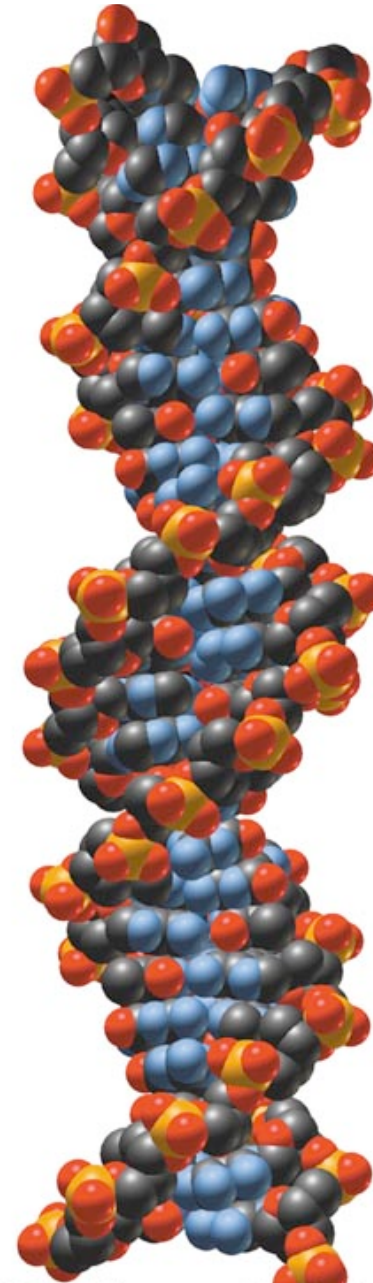
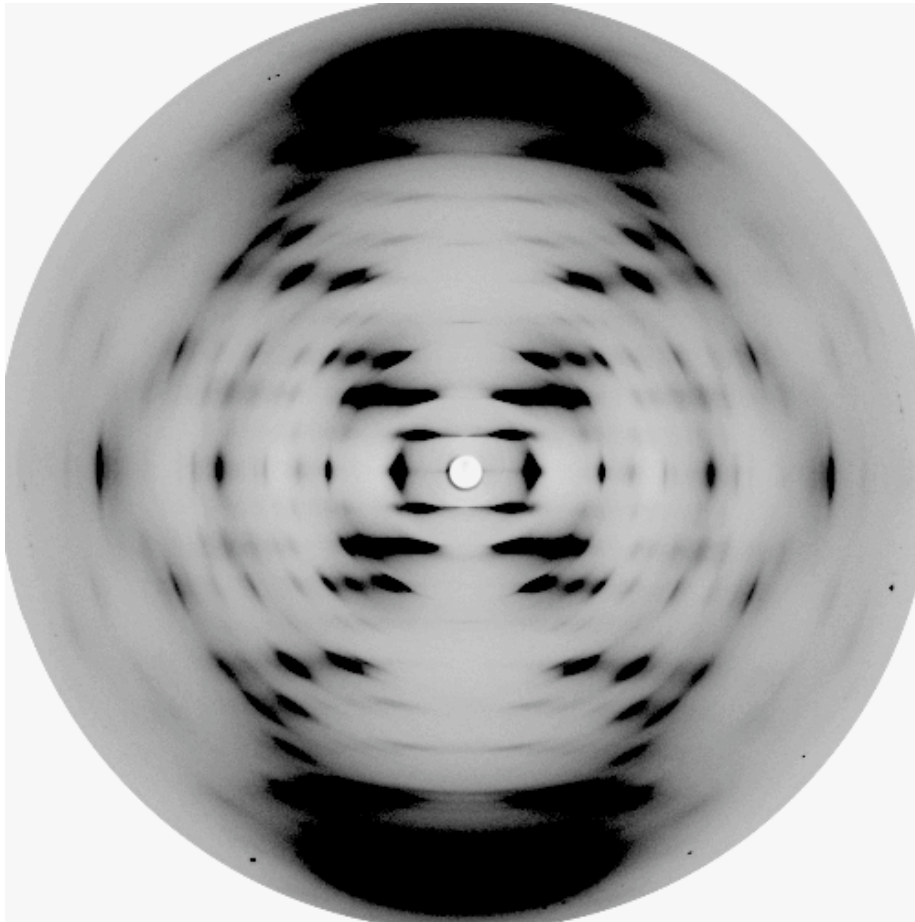


Bragg Scattering: Probing Atoms With X-Rays



Constructive Interference when net phase difference is $0, 2\pi$ etc
This implied path difference traveled by two waves must be integral multiple of wavelength : $n\lambda = 2d \sin \theta$

X-Ray Picture of a DNA Crystal



Proteins inside Rhinovirus reconstructed by x-ray diffraction

