Physics 2D Lecture Slides
Week of April 27th 2009

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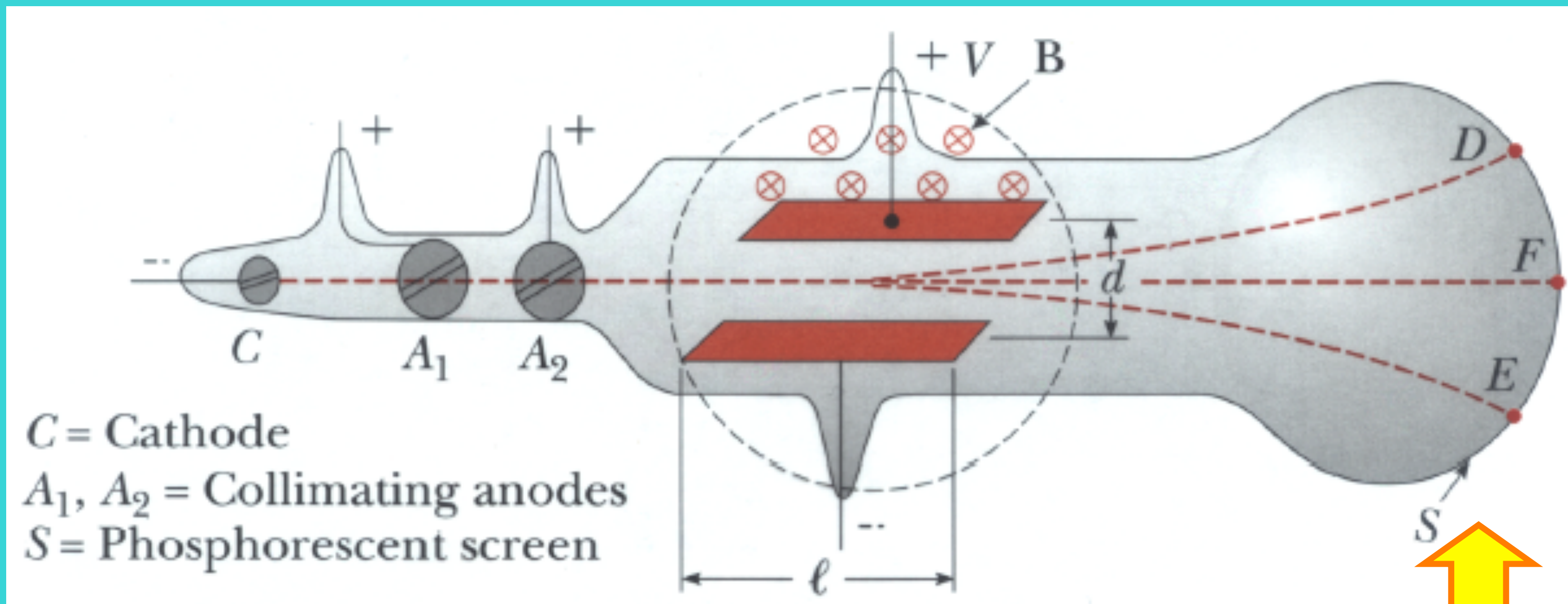
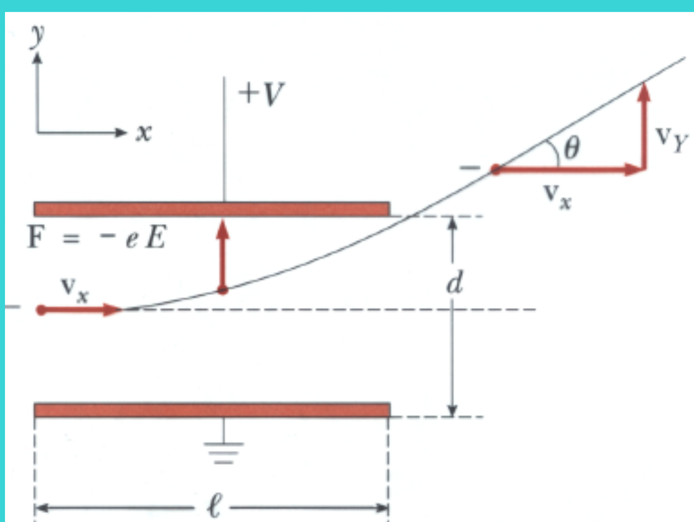
UCSD Physics

Thomson's Determination of e/m of the Electron

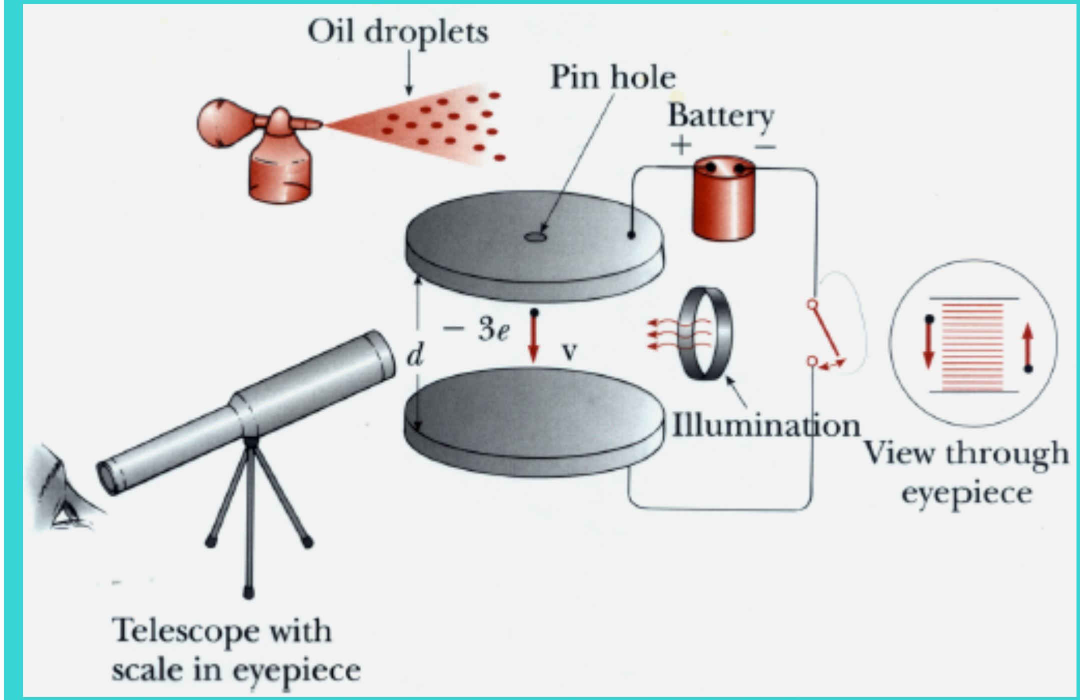


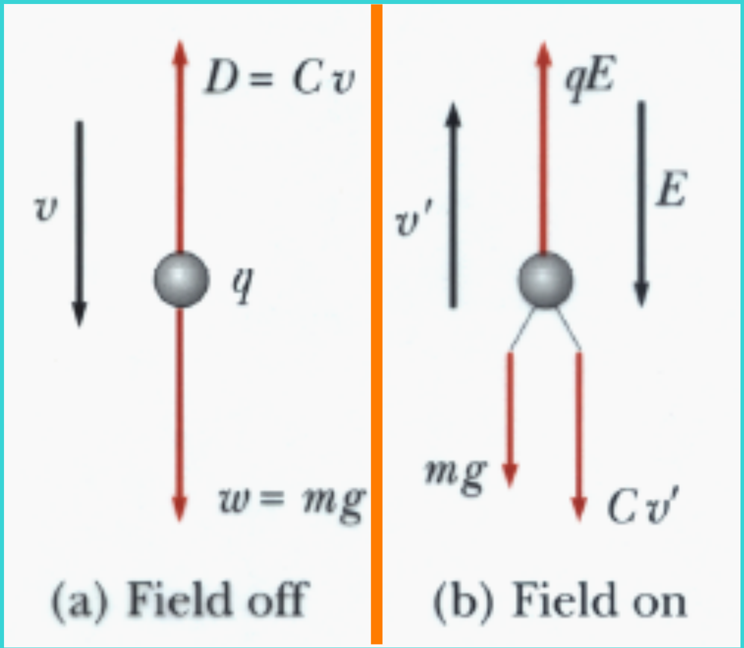


- In E Field alone, electron lands at D
- In B field alone, electron lands at E
- When E and B field adjusted to cancel each other's force → electron lands at F

→ $e/m = 1.7588 \times 10^{11}$ C/Kg

Millikan's Measurement of Electron Charge





Find charge on oil drop is always in integral multiple of some Q

$q_e = 1.688 \times 10^{-19}$ Coulombs

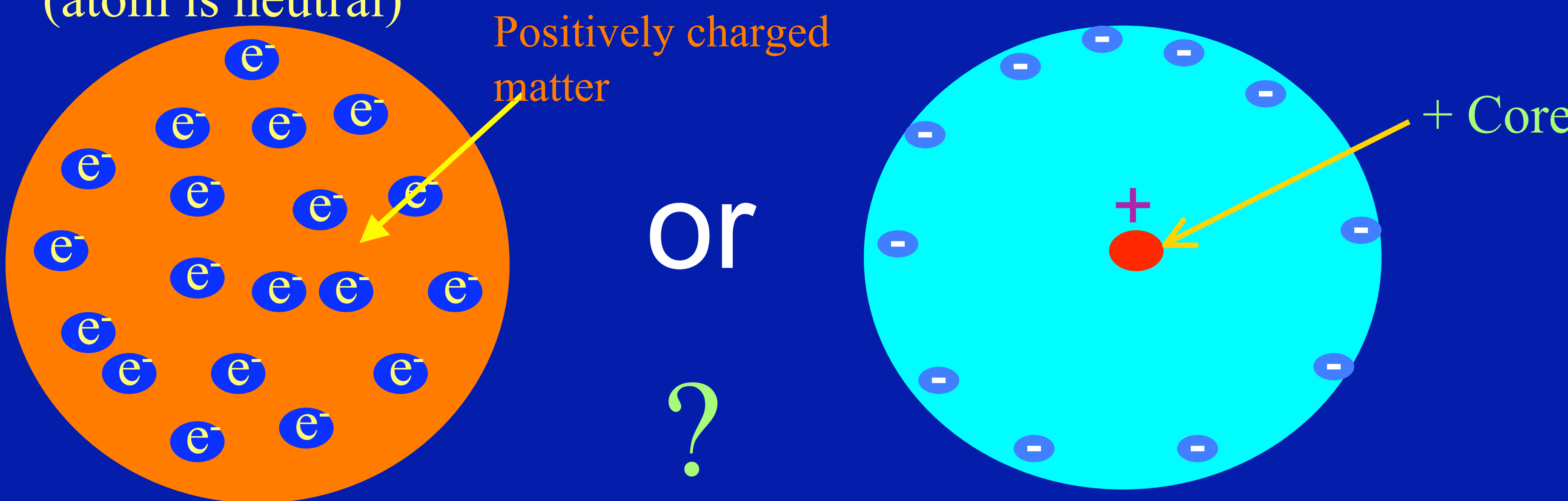
→ $m_e = 9.1093 \times 10^{-31}$ Kg

→ Fundamental properties (finger print) of electron

(similarly can measure proton properties etc)

Where are the electrons inside the atom?

Early Thought: “Plum pudding” model→ Atom has a homogenous distribution of Positive charge with electrons embedded in them (atom is neutral)

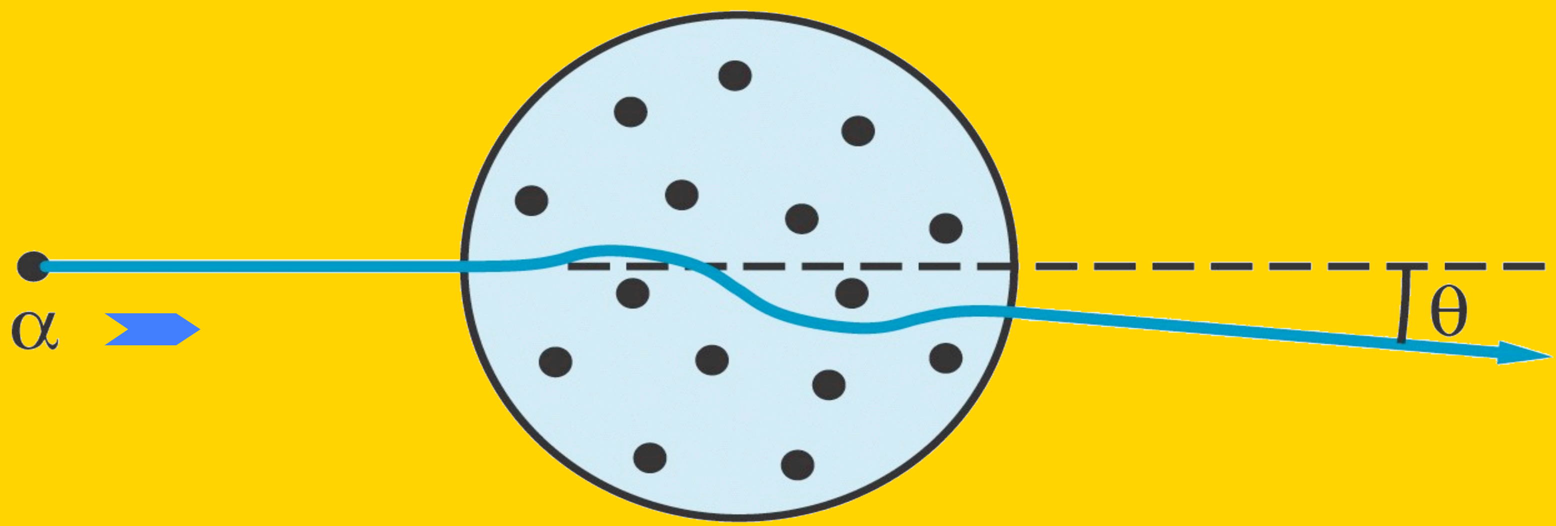


- How to test these hypotheses? → Shoot “bullets” at the atom and watch their trajectory. What Kind of bullets ?
  - •Indestructible charged bullets → Ionized He^{++} atom = α^{++} particles
  - •Q = +2e , Mass M_α=4amu >> m_e , V_α= 2 x 10 ^{7} m/s (non-relavistic)

  [charged to probe charge & mass distribution inside atom]

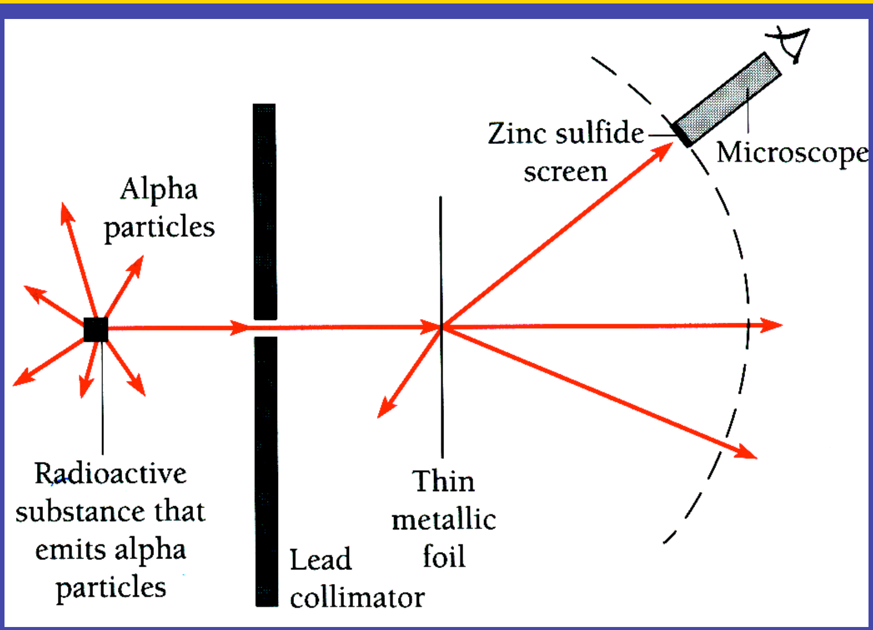
Plum Pudding Model of Atom

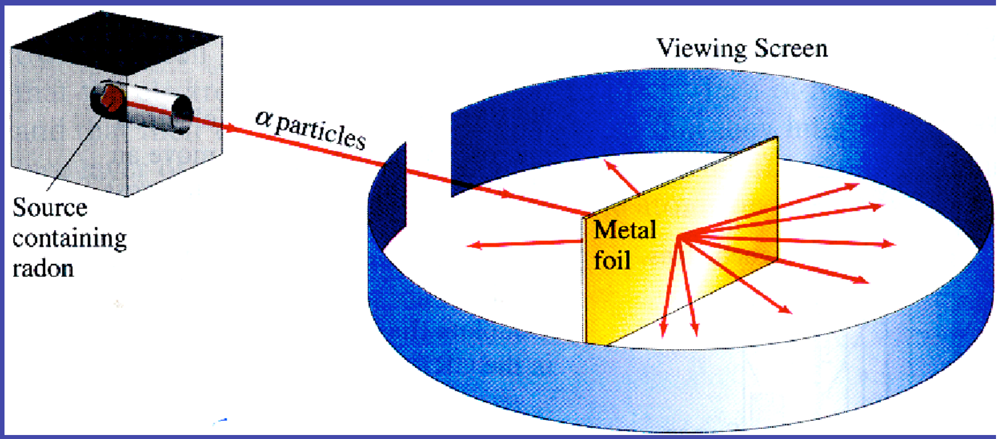
- Non-relativistic mechanics ($V_\alpha/c = 0.1$)
- In Plum-pudding model, α-rays hardly scatter because
  - Positive charge distributed over size of atom (10^{-10}m)
  - $M_\alpha >> M_e$ (like moving truck hits a bicycle)
  - → predict α-rays will pass thru array of atoms with little scatter (~1°)



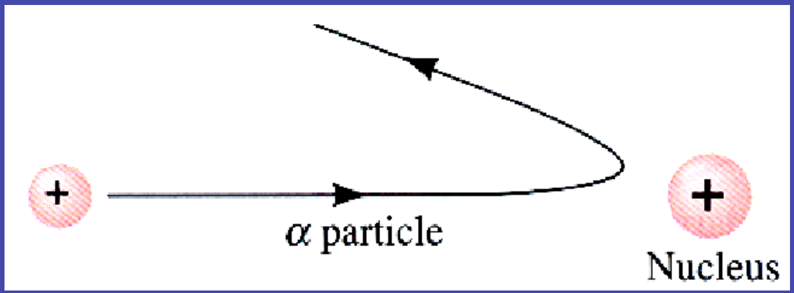
Need to test this hypothesis → Ernest Rutherford

Probing Within an Atom with α Particles

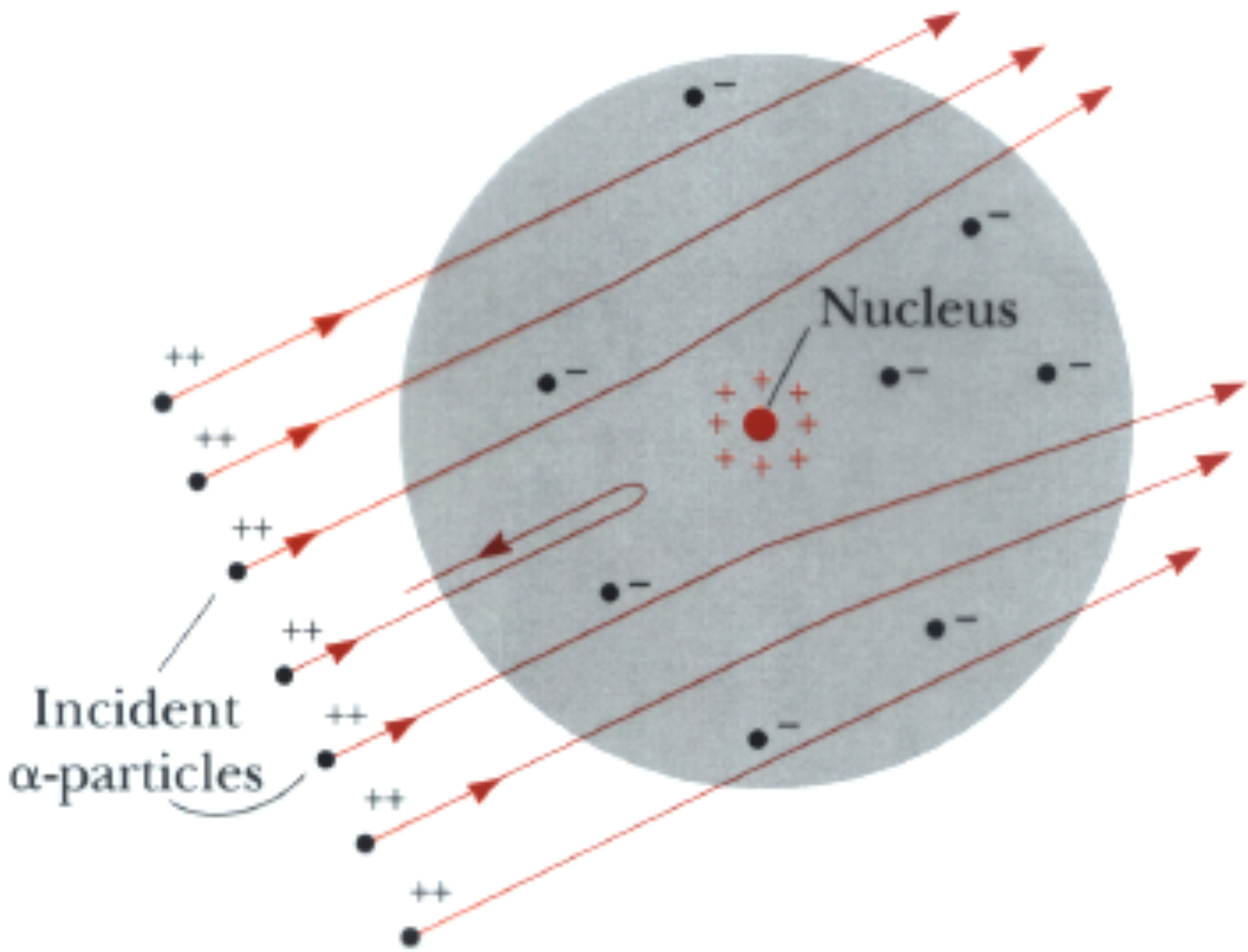




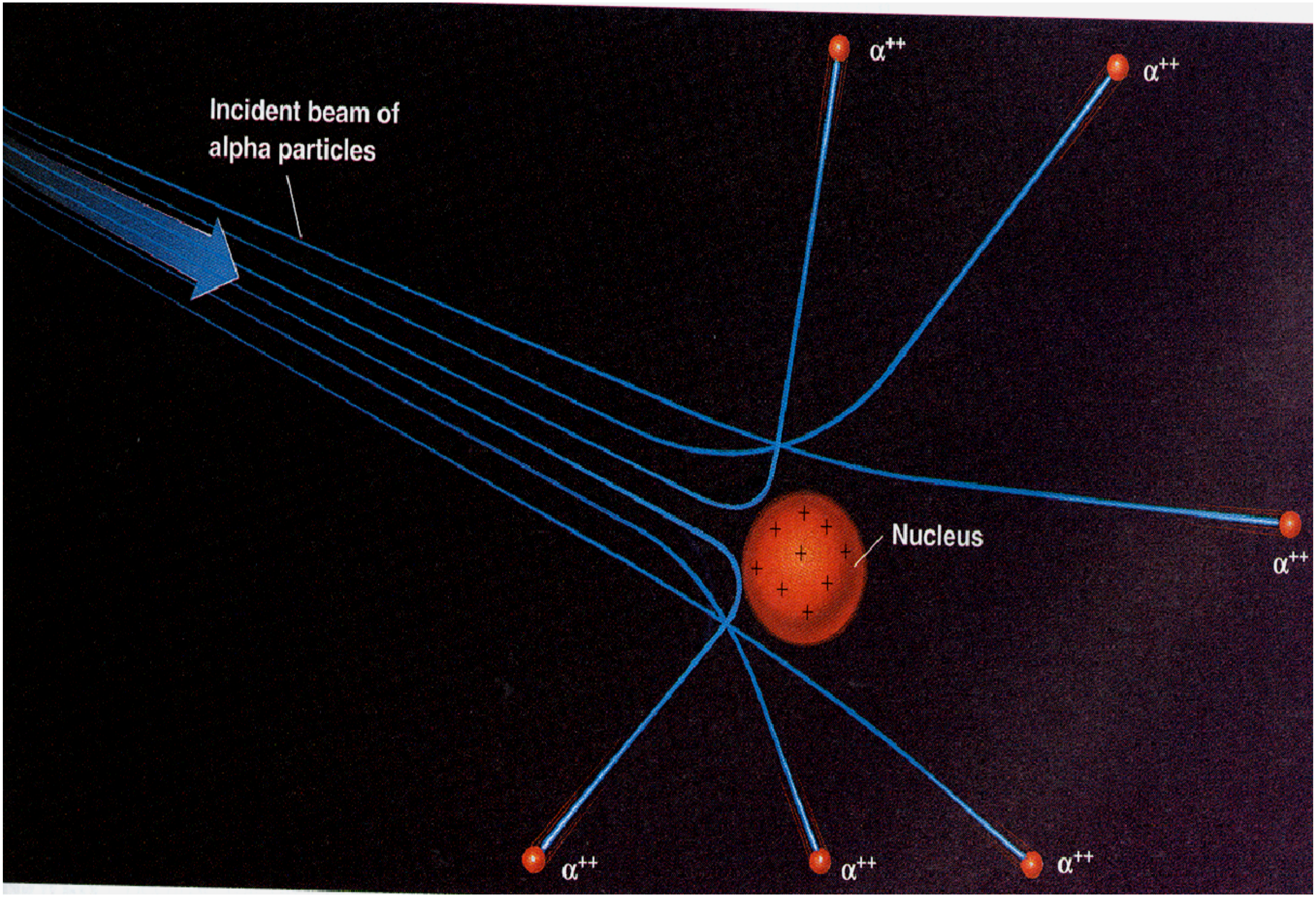
- Most α particles pass thru gold foil with no deflection
- SOME ($\cong 10^{-4}$) scatter at LARGE angles Φ
- Even fewer scatter almost backwards → Why



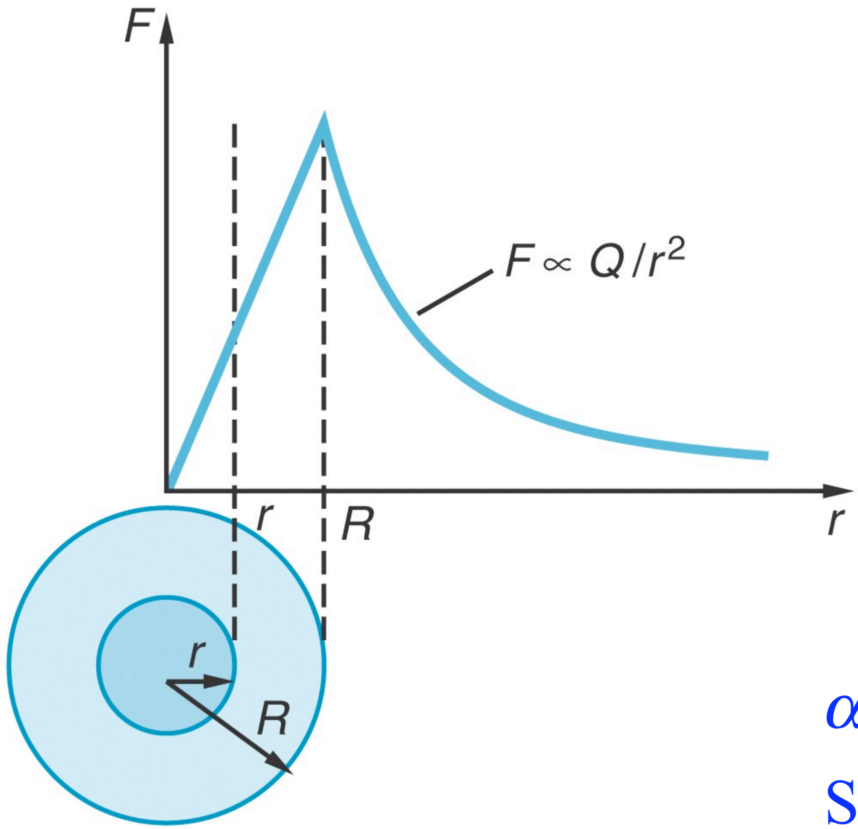
“Rutherford Scattering” discovered by his PhD Student (Marsden)

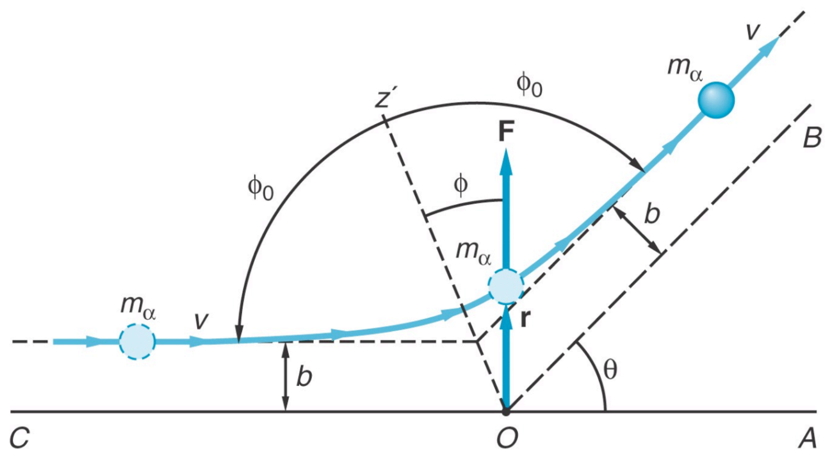


Rutherford Discovers Nucleus (Nobel Prize)



Force on α-particle due to heavy Nucleus





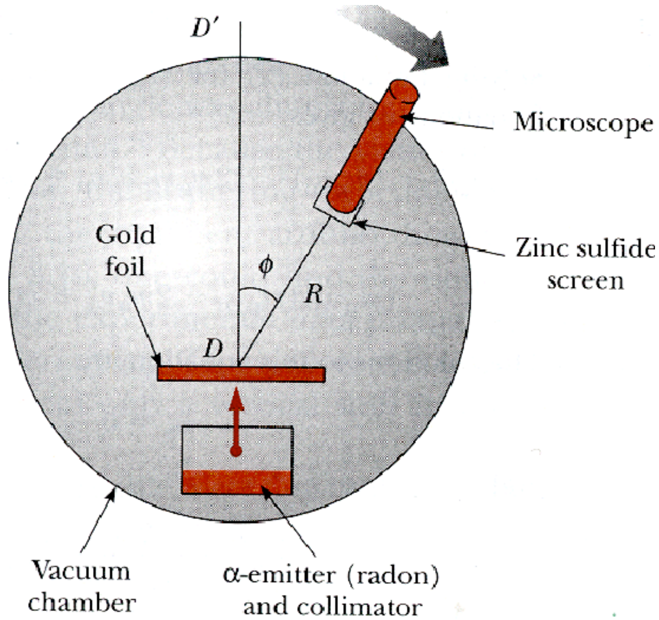
α particle trajectory is hyperbolic

Scattering angle is related to impact par.

Impact Parameter $b = \left(\frac{kq_\alpha Q}{m_\alpha v_\alpha^2}\right)\left(\cot\frac{\theta}{2}\right)$

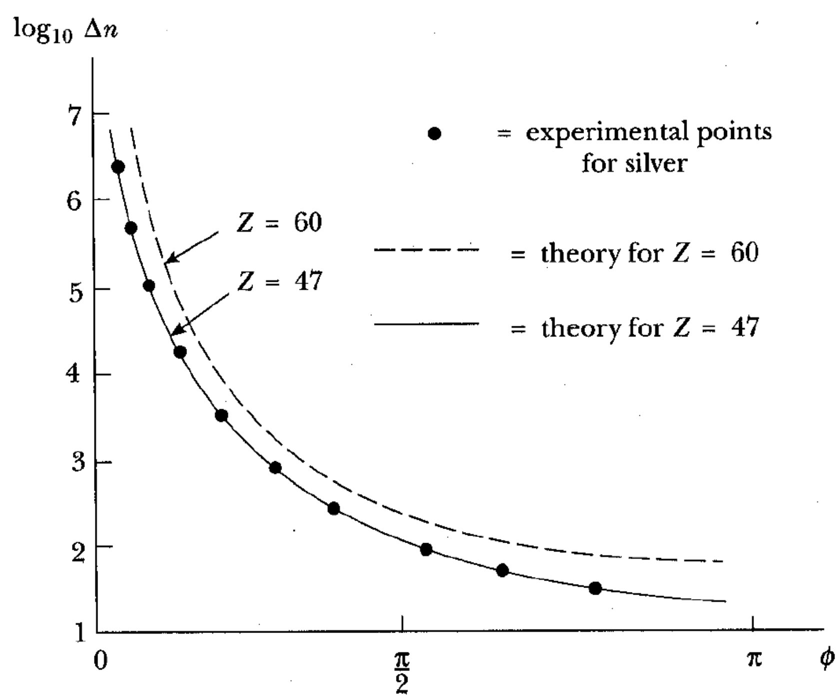
- Outside radius r =R, F ∝ Q/r^2
- Inside radius r < R, F ∝ $q/r^2 = Qr/R^2$
- Maximum force at radius r = R

Rutherford Scattering: Prediction and Experimental Result

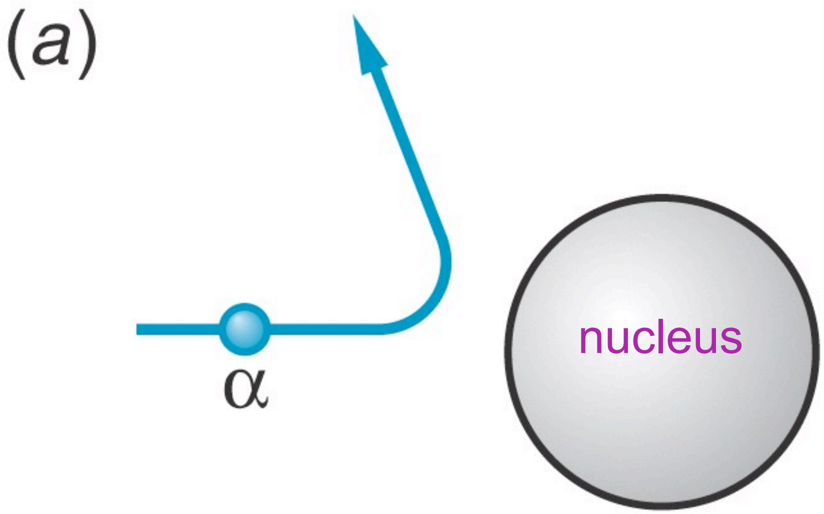


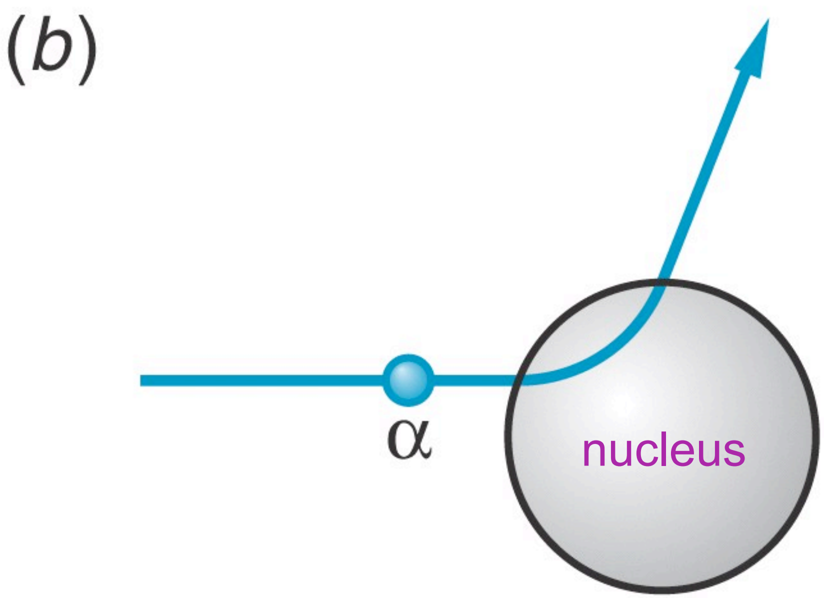
$$\Delta n = \frac{k^2 Z^2 e^4 N n A}{4R^2 \left(\frac{1}{2} m_\alpha v_\alpha^2\right)^2 Sin^4(\varphi/2)}$$

- # scattered Vs ϕ depends on :
  - n = # of incident alpha particles
  - N = # of nuclei/area of foil
  - Ze = Nuclear charge
  - K_α of incident alpha beam
  - A= detector area



Rutherford Scattering & Size of Nucleus





distance of closest appoach ∝ r size of nucleus

Kinetic energy of $\alpha = \mathrm{K}_\alpha = \frac{1}{2} m_\alpha v_\beta^2$

α particle will penetrate thru a radius r until all its kinetic energy is used up to do work AGAINST the Coulomb potential of the Nucleus:

$$\mathrm{K}_\alpha = \frac{1}{2} m_\alpha v_\beta^2 = 8MeV = k\frac{(Ze)(2e)}{r}$$

$$\Rightarrow \quad \boxed{r = \frac{2kZe^2}{K_\alpha}}$$

For K_α =7.7.MeV, $\mathrm{Z}_{\mathrm{Al}} = 13$

$$\Rightarrow \quad r = \frac{2kZe^2}{K_\alpha} = 4.9\times 10^{-15} m$$

Size of Nucleus = $10^{-15} m$

Size of Atom = $10^{-10} m$

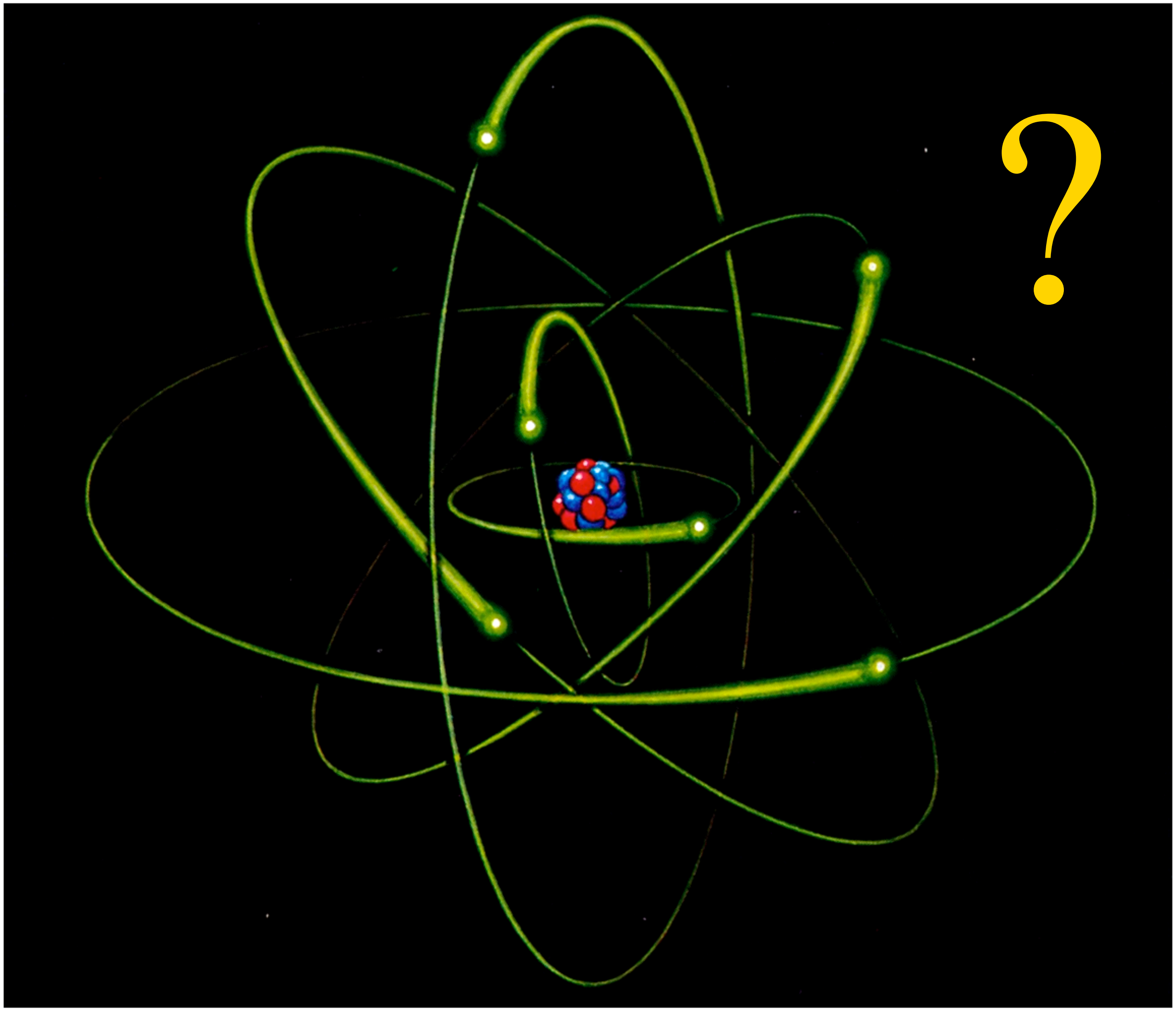
Dimension Matters !

$$\text{Size of Nucleus} = 10^{-15}m$$

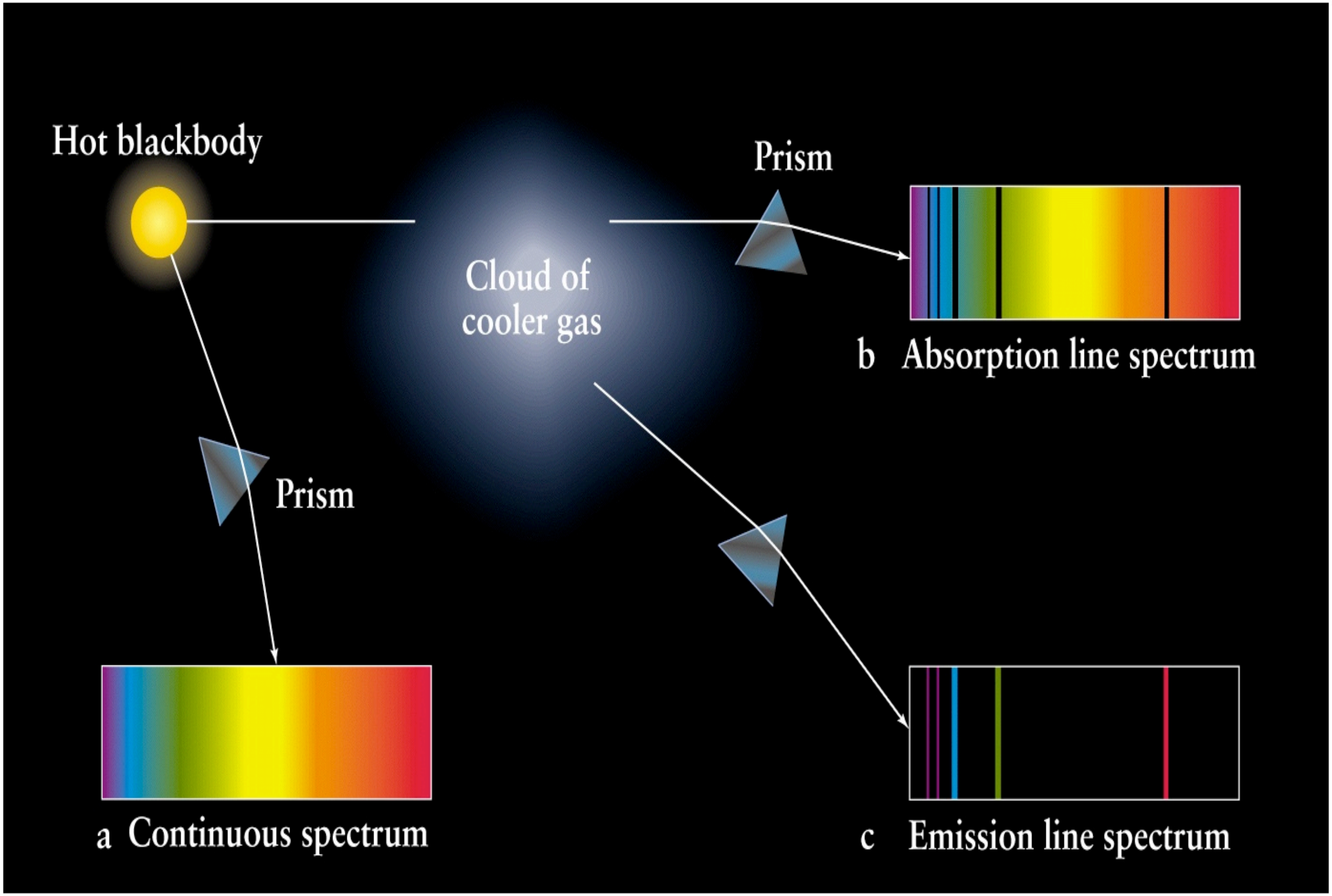
$$\text{Size of Atom} = 10^{-10}m$$

- how are the electrons located inside an atom?
- How are they held in a stable fashion? (necessary condition for us to exist)
  - What makes up the rest of the mass of the nucleus?
  - Why doesn’t Coulomb repulsion blow the nucleus apart‘
  - All these discoveries will require new experiments and observations

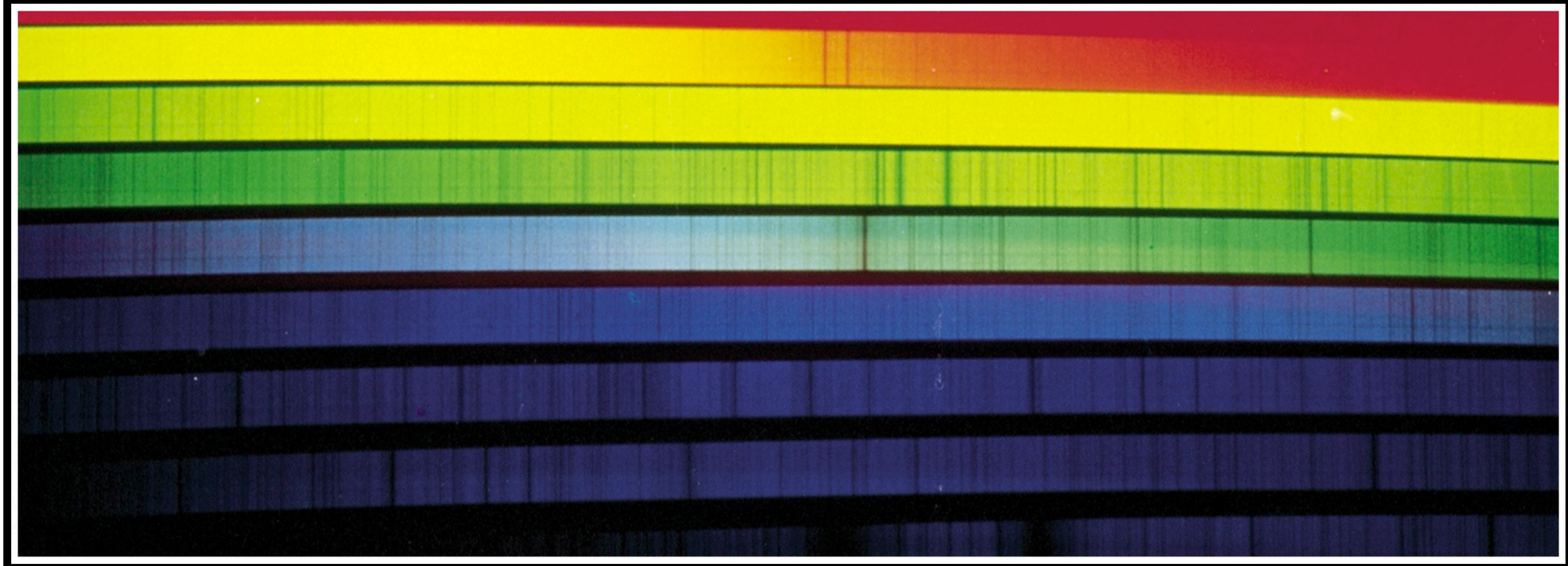
Rutherford Atom & Classical Physics



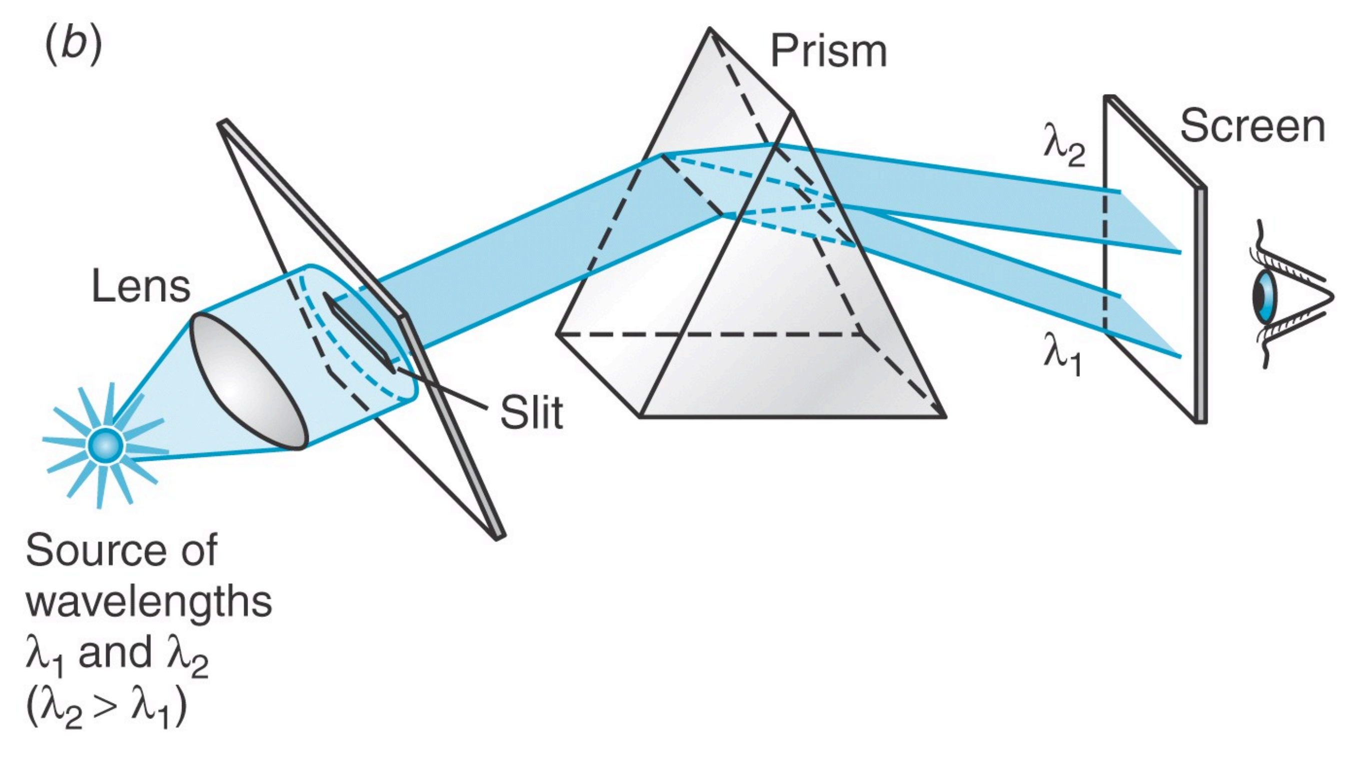
Continuous & Discrete spectra of Elements

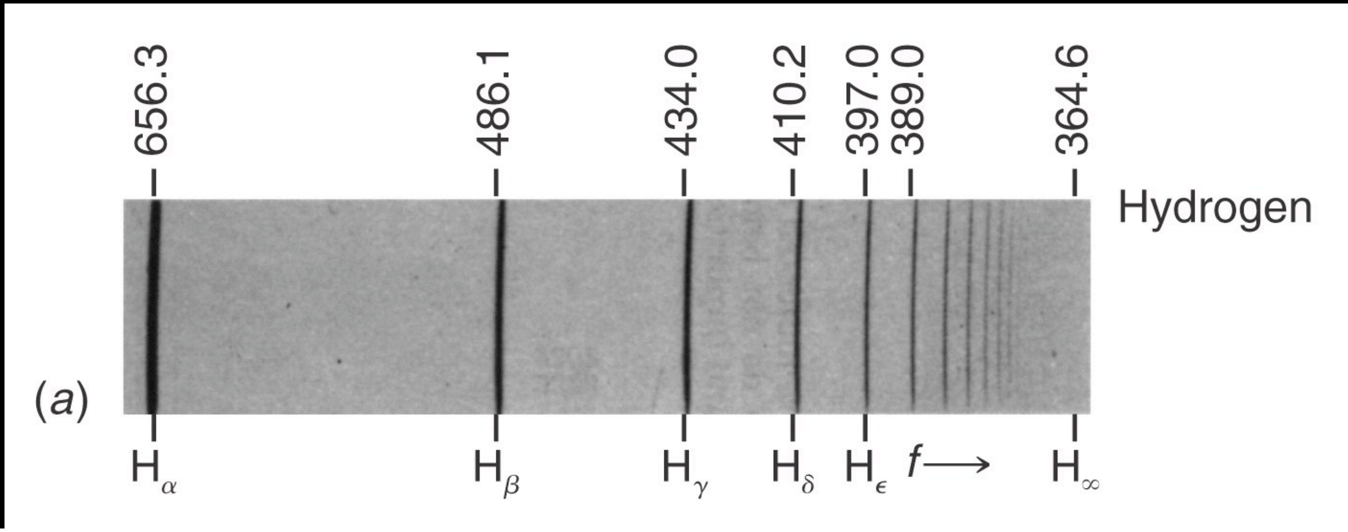


Visible Spectrum of Sun Through a Prism

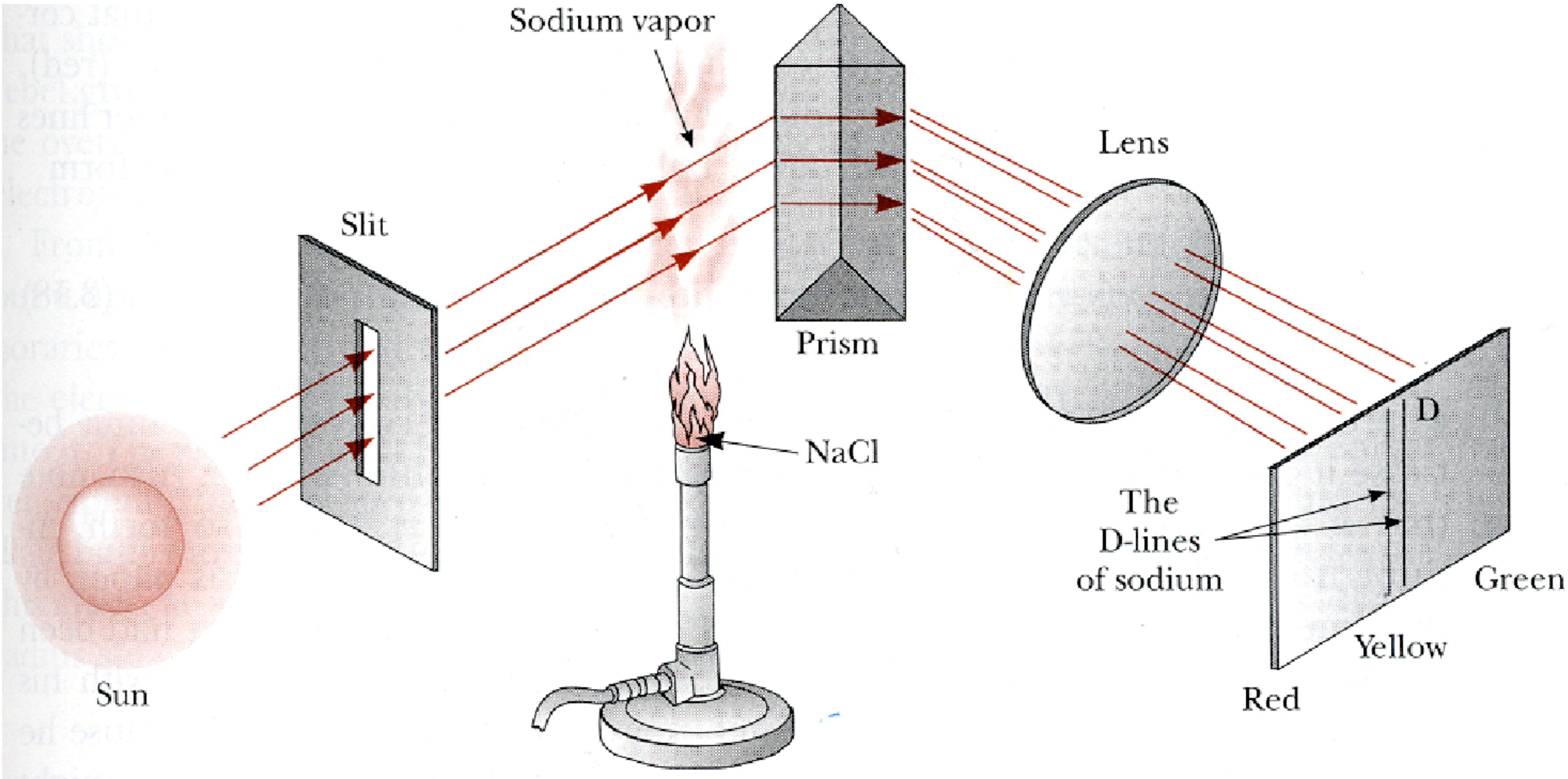


Emission & Absorption Line Spectra of Elements





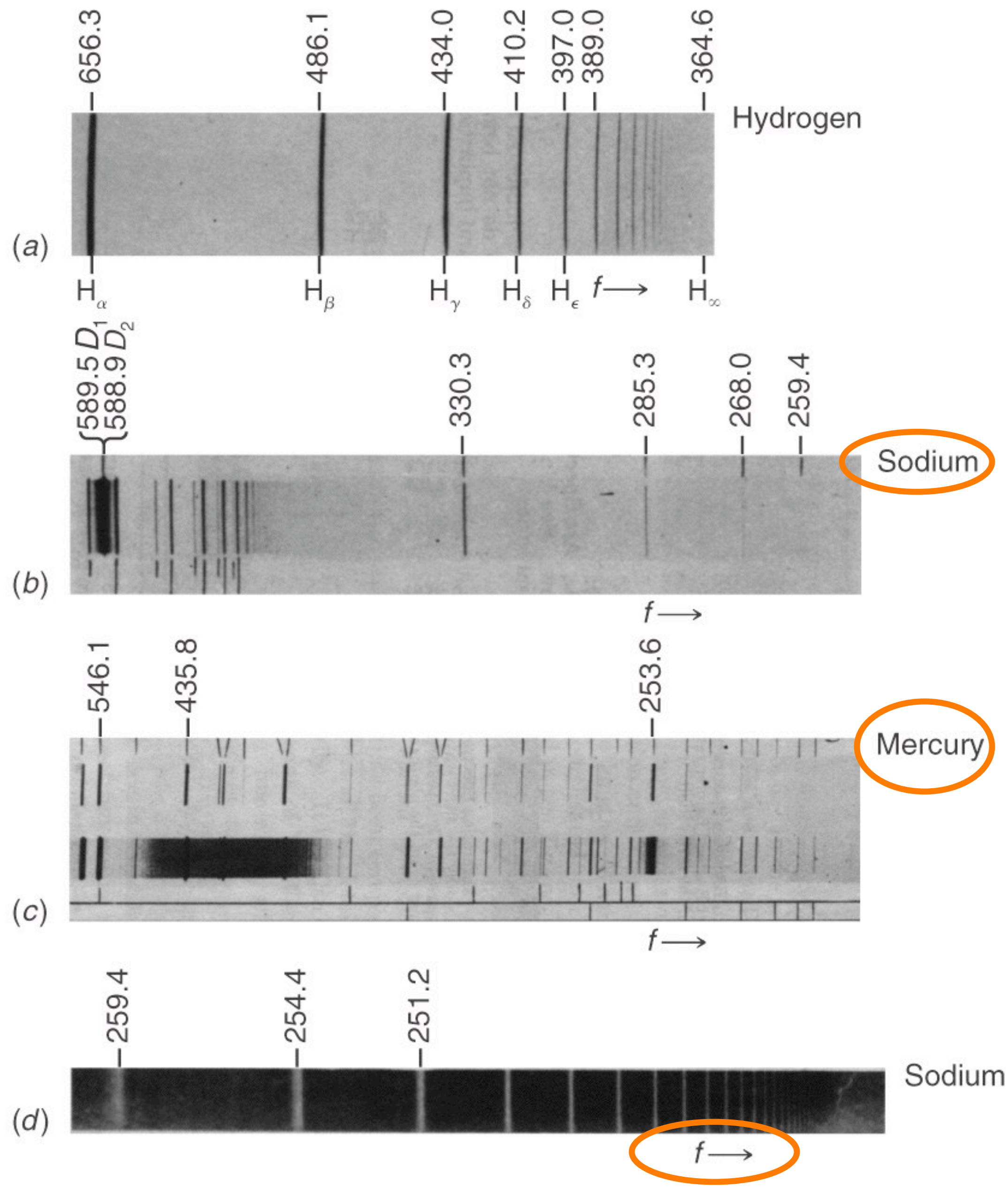
Kirchhoff’ Experiment : “D” Lines in Na



D lines darken noticeably when Sodium vapor introduced
Between slit and prism

Emission & Absorption Line Spectrum of Elements

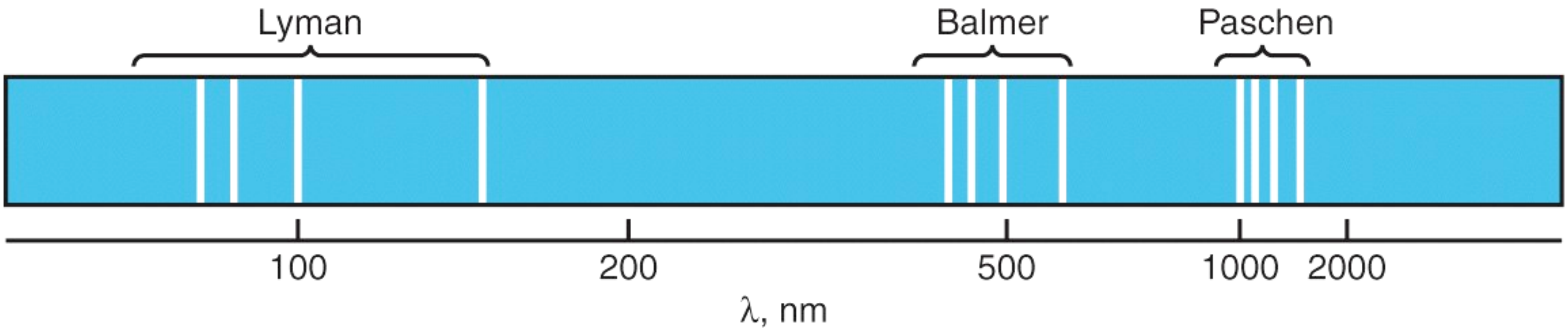
•Emission line appear dark because of photographic exposure



Absorption spectrum of Na

While light passed thru Na vapor is absorbed at specific λ

Spectral Observations : series of lines with a pattern



- Empirical observation (by trial & error)
- All these series can be summarized in a simple formula

$$\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right), n_i > n_f, n_i = 1,2,3,4..$$

Fitting to spectral line series data

$$R = 1.09737 \times 10^7 m^{-1}$$

How does one explain this ?

The Rapidly Vanishing Atom: A Classical Disaster !

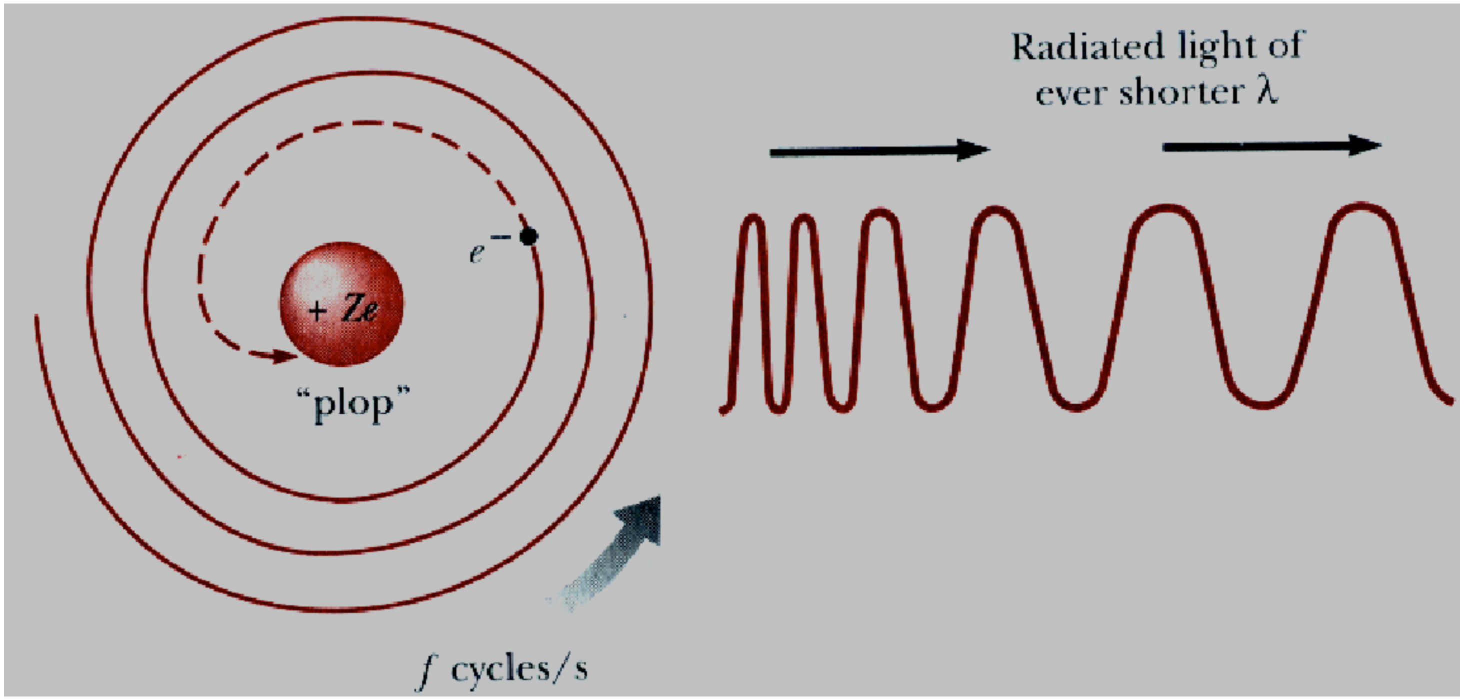
Not too hard to draw analogy with dynamics under another Central Force

Think of the Gravitational Force between two objects and their circular orbits.

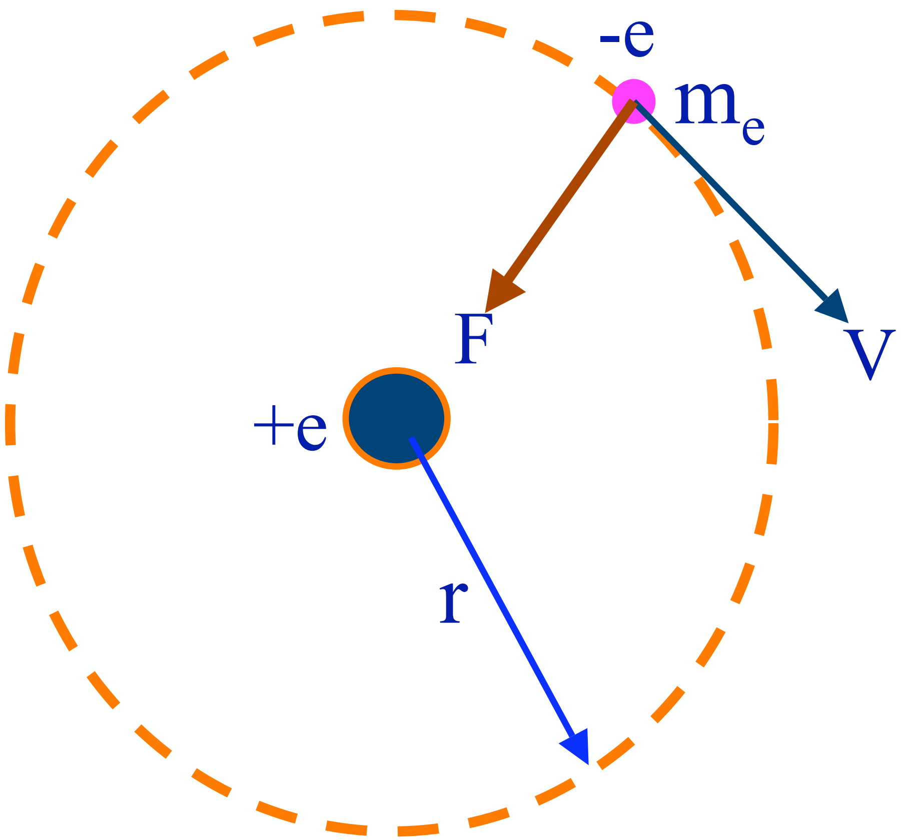
Perhaps the electron rotates around the Nucleus and is bound by their electrical charge

$$F = G\frac{M_1 M_2}{r^2} \Rightarrow \boxed{k\,\frac{Q_1 Q_2}{r^2}}$$

Laws of E&M destroy this equivalent picture : Why ?



Bohr's Bold Model of Atom: Semi Quantum/Classical



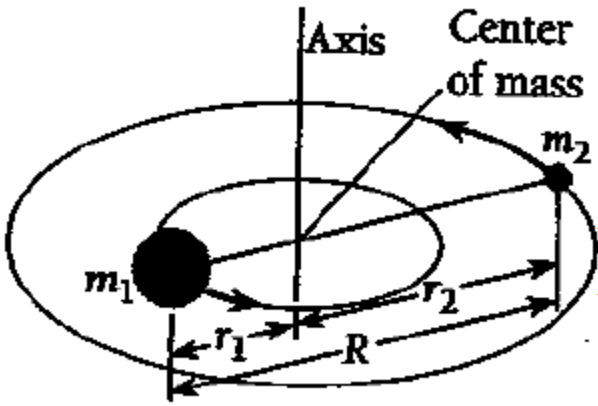
$$U(r) = -k\frac{e^2}{r}$$

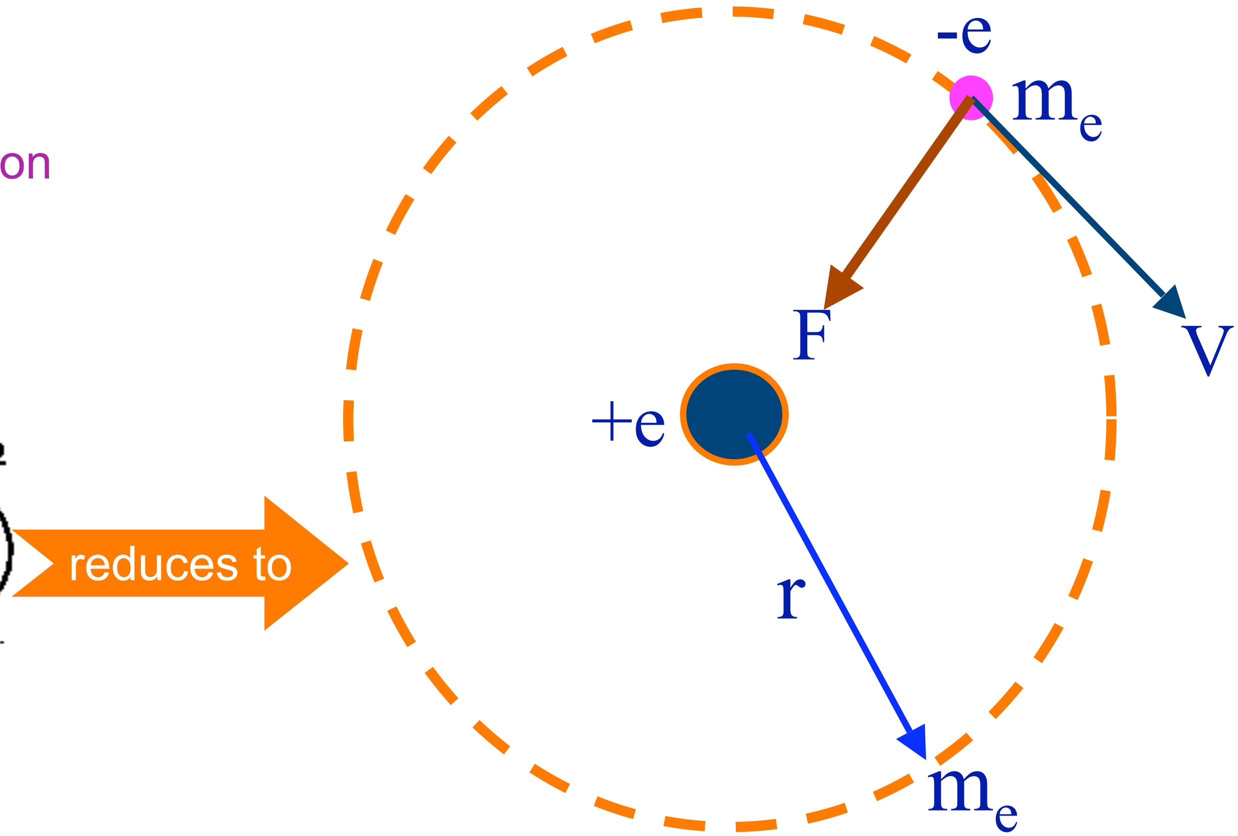
$$KE = \frac{1}{2}m_e v^2$$

1. Electron in circular orbit around proton with vel=v
2. Only stationary orbits allowed . Electron does not radiate when in these stable (stationary) orbits
3. Orbits quantized:
   - $M_e v\, r = n\, h/2\pi$ (n=1,2,3…)
4. Radiation emitted when electron "jumps" from a stable orbit of higher energy → stable orbit of lower energy $E_f - E_i = hf = hc/\lambda$
5. Energy change quantized
   - f = frequency of radiation

Reduced Mass of 2-body system

General Two body Motion under a central force





- Both Nucleus & e^- revolve around their common center of mass (CM)
- Such a system is equivalent to single particle of "reduced mass" μ that revolves around position of Nucleus at a distance of (e^- -N) separation
  - $\mu=(m_eM)/(m_e+M)$, when M>>m, μ=m (Hydrogen atom)
  - Not so when calculating Muon ($m_\mu=207\,m_e$) or equal mass charges rotating around each other (similar to what you saw in gravitation)

Allowed Energy Levels & Orbit Radii in Bohr Model

$$E=KE+U=\frac{1}{2}m_e v^2 - k\frac{e^2}{r}$$

Force Equality for Stable Orbit

$\Rightarrow$ Coulomb attraction = CP Force

$$k\frac{e^2}{r^2}=\frac{m_e v^2}{r}$$

$$\Rightarrow KE=\frac{m_e v^2}{2}=k\frac{e^2}{2r}$$

Total Energy $\boxed{E = KE+U = -k\frac{e^2}{2r}}$

Negative E $\Rightarrow$ Bound system

This much energy must be added to the system to break up the bound atom

Radius of Electron Orbit :

$$mvr = n\hbar$$

$$\Rightarrow v=\frac{n\hbar}{mr},$$

substitute in $KE=\frac{1}{2}m_e v^2=\frac{ke^2}{2r}$

$$\Rightarrow r_n=\frac{n^2\hbar^2}{mke^2},\ n=1,2,....\infty$$

$n=1 \Rightarrow$ Bohr Radius a_0

$$a_0=\frac{1^2\hbar^2}{mke^2}=0.529\times10^{-10}\,m$$

In general $r_n=n^2 a_0; n=1,2,....\infty$

Quantized orbits of rotation

Energy Level Diagram and Atomic Transitions

$$E_n = K + U = \frac{-ke^2}{2r}$$

since $r_n = a_0 n^2$, n =quantum number

$$E_n = \frac{-ke^2}{2a_0 n^2} = -\frac{13.6}{n^2} eV,\ n = 1, 2, 3..\infty$$

Interstate transition: $n_i \rightarrow n_f$

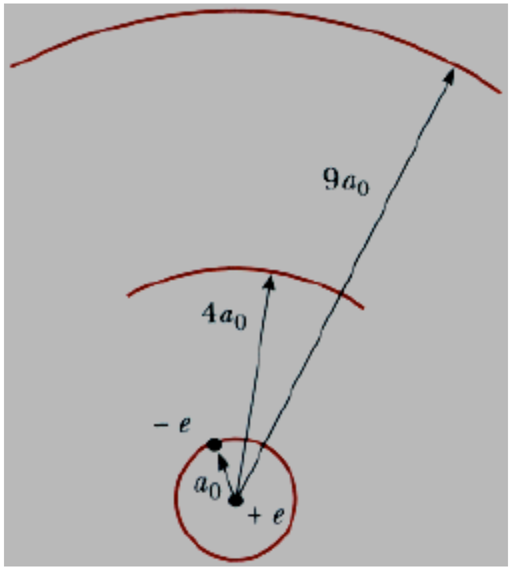
$$\Delta E = hf = E_i - E_f$$

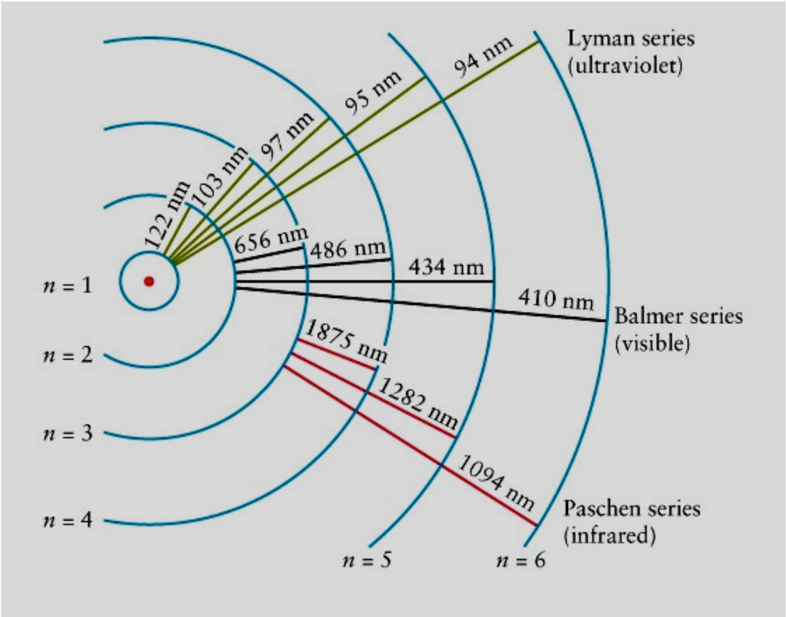
$$= \frac{-ke^2}{2a_0}\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right)$$

$$f = \frac{ke^2}{2ha_0}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

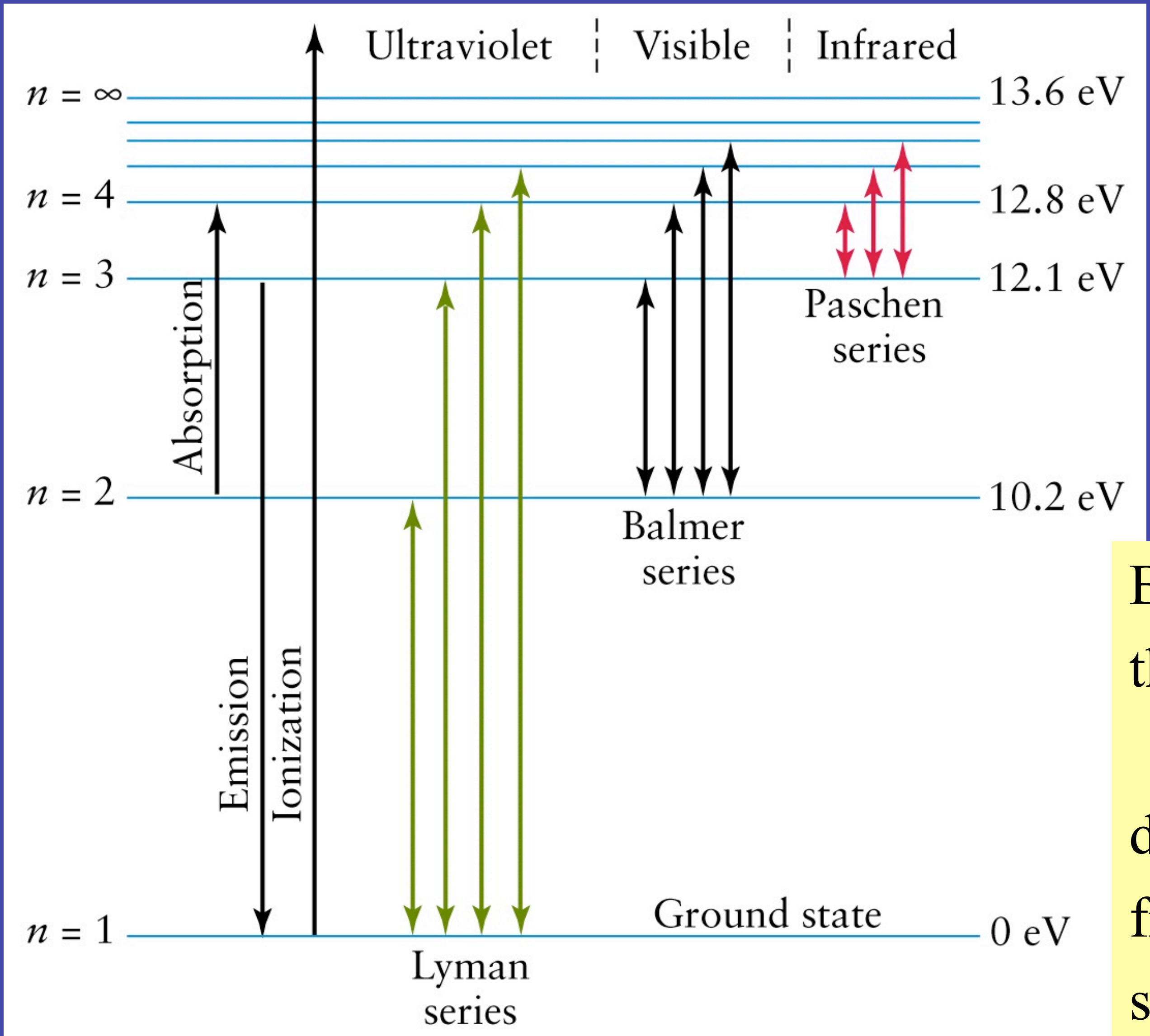
$$\frac{1}{\lambda} = \frac{f}{c} = \frac{ke^2}{2hca_0}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

$$= R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$





Hydrogen Spectrum: as explained by Bohr



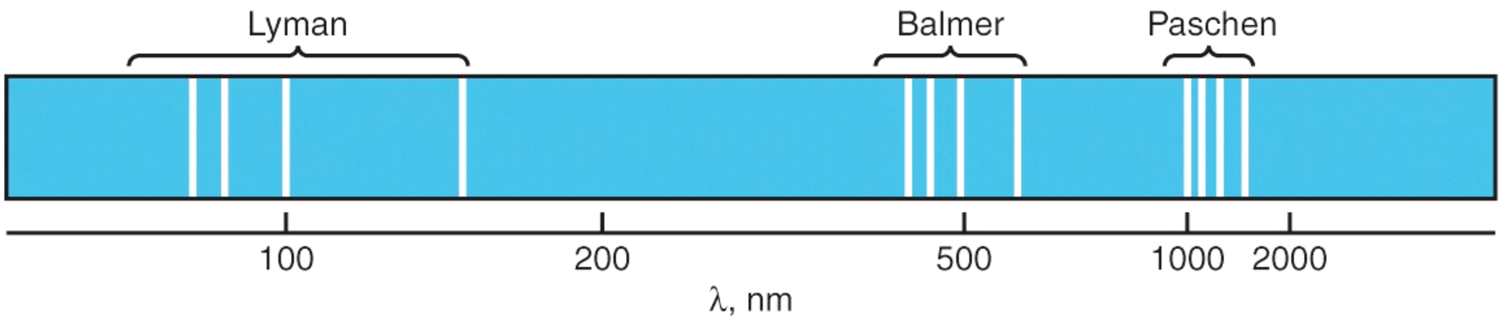
$$E_n = -\left(\frac{ke^2}{2a_0}\right)\frac{Z^2}{n^2}$$

Bohr's "R" same as the Rydberg Constant

R

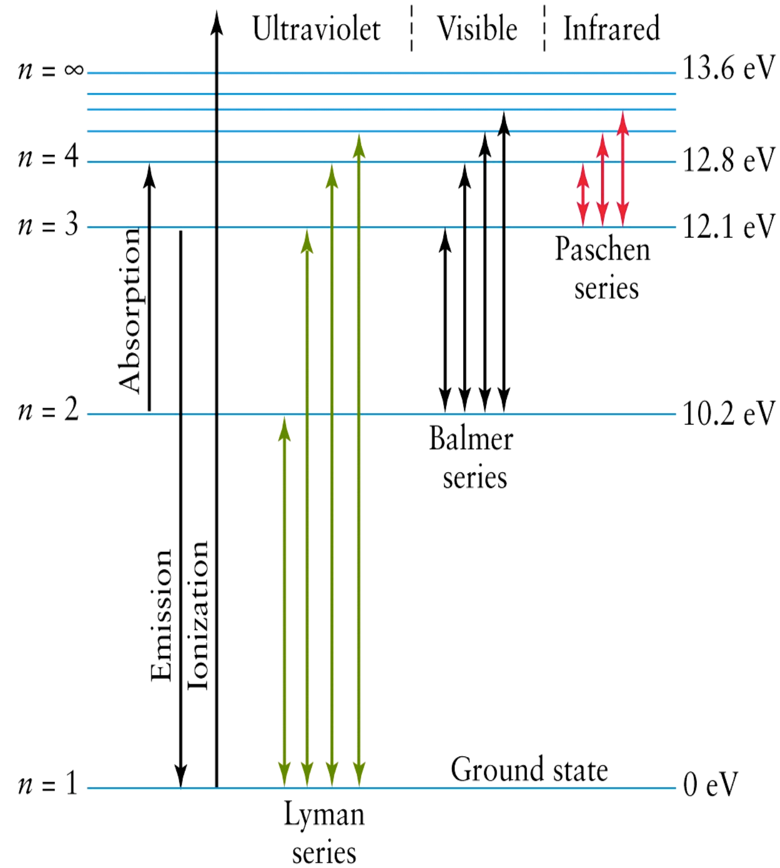
derived empirically from photographs of the spectral series

Another Look at the Energy levels

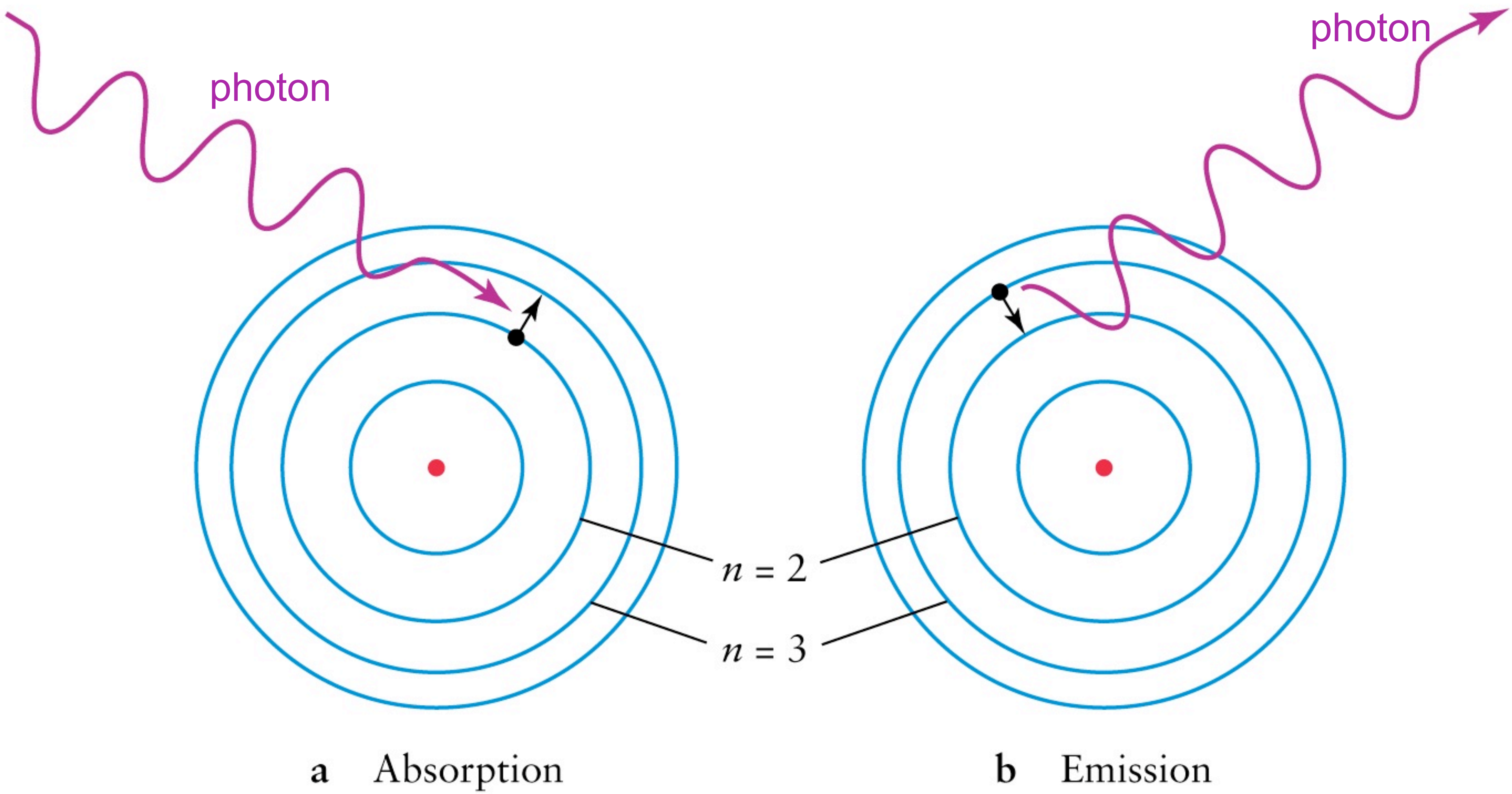


$$E_n = -\left(\frac{ke^2}{2a_0}\right)\frac{Z^2}{n^2}$$

Rydberg Constant



Bohr's Atom: Emission & Absorption Spectra



Some Notes About Bohr Like Atoms

- Ground state of Hydrogen atom (n=1) E_0= -13.6 eV
- Method for calculating energy levels etc applies to all Hydrogen-like atoms → -1e around +Ze
  - Examples : He^{+}, Li^{++}
- Energy levels would be different if replace electron with Muons
- Bohr's method can be applied in general to all systems under a central force (e.g. gravitational instead of Coulombic)