PHYSICS 4E QUIZ 3 SPRING QUARTER 2009
PROF. HIRSCH May 8

Formulas:

Relativistic energy - momentum relation $E=\sqrt{m^2c^4+p^2c^2}$; $c=3\times10^8 m/s$

Electron rest mass : $m_e = 0.511\ MeV/c^2$;Proton : $m_p = 938.26\ MeV/c^2$;Neutron : $m_n = 939.55\ MeV/c^2$

Planck's law : $u(\lambda)=n(\lambda)\bar{E}(\lambda)$; $n(\lambda)=\frac{8\pi}{\lambda^4}$; $\bar{E}(\lambda)=\frac{hc}{\lambda}\frac{1}{e^{hc/\lambda k_B T}-1}$

Energy in a mode/oscillator : $E_f = nhf$; probability $P(E)\propto e^{-E/k_BT}$

Stefan's law : $R=\sigma T^4$; $\sigma = 5.67\times10^{-8} W/m^2K^4$; $R=cU/4$, $U=\int_0^\infty u(\lambda)d\lambda$

Wien's displacement law : $\lambda_m T = \frac{hc}{4.96 k_B}$

Photons : $E=pc$; $E=hf$; $p = h/\lambda$; $f=c/\lambda$

Photoelectric effect : $eV_0 = (\frac{1}{2}mv^2)_{\max} = hf-\phi$, $\phi \equiv$ work function

Compton scattering : $\lambda' - \lambda = \frac{h}{m_e c}(1-\cos\theta)$

Rutherford scattering: $b=\frac{kq_\alpha Q}{m_\alpha v^2}\cot(\theta/2)$; $\Delta N \propto \frac{1}{\sin^4(\theta/2)}$

Constants : $hc = 12{,}400\ eV\,A$; $\hbar c = 1{,}973\ eV\,A$; $k_B = 1/11{,}600\ eV/K$; $ke^2 = 14.4 eVA$

Electrostatics : $F=\frac{kq_1q_2}{r^2}$ (force) ; $U=q_0V$ (potential energy) ; $V=\frac{kq}{r}$ (potential)

Hydrogen spectrum : $\frac{1}{\lambda}=R(\frac{1}{m^2}-\frac{1}{n^2})$; $R=1.097\times10^7\ m^{-1} = \frac{1}{911.3A}$

Bohr atom : $E_n = -\frac{ke^2Z}{2r_n} = -\frac{Z^2E_0}{n^2}$; $E_0 = \frac{ke^2}{2a_0} = \frac{mk^2e^4}{2\hbar^2} = 13.6eV$; $E_n = E_{kin}+E_{pot}, E_{kin} = -E_{pot}/2 = -E_n$

$hf = E_i - E_f$; $r_n = r_0 n^2$; $r_0 = \frac{a_0}{Z}$; $a_0 = \frac{\hbar^2}{mke^2} = 0.529A$; $L = mvr = n\hbar$ angular momentum

X - ray spectra : $f^{1/2} = A_n(Z-b)$; K : $b=1$, L : $b=7.4$

de Broglie : $\lambda = \frac{h}{p}$; $f=\frac{E}{h}$; $\omega = 2\pi f$; $k=\frac{2\pi}{\lambda}$; $E=\hbar\omega$; $p=\hbar k$; $E=\frac{p^2}{2m}$

group and phase velocity : $v_g = \frac{d\omega}{dk}$; $v_p = \frac{\omega}{k}$; Heisenberg : $\Delta x\Delta p \sim \hbar$; $\Delta t \Delta E \sim \hbar$

Wave function $\Psi(x,t) = |\Psi(x,t)|e^{i\theta(x,t)}$; $P(x,t)\,dx = |\Psi(x,t)|^2\,dx$ = probability

Schrodinger equation : $-\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + V(x)\Psi(x,t) = i\hbar\frac{\partial\Psi}{\partial t}$; $\Psi(x,t) = \psi(x)e^{-i\frac{E}{\hbar}t}$

Time – independent Schrodinger equation : $-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V(x)\psi(x) = E\psi(x)$; $\int_{-\infty}^{\infty} dx\ \psi^*\psi = 1$

∞ square well : $\psi_n(x) = \sqrt{\frac{2}{L}}\sin(\frac{n\pi x}{L})$; $E_n = \frac{\pi^2\hbar^2n^2}{2mL^2}$; $x_{op} = x$, $p_{op} = \frac{\hbar}{i}\frac{\partial}{\partial x}$; $\langle A\rangle = \int_{-\infty}^{\infty} dx\psi^* A_{op}\psi$

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Eigenvalues and eigenfunctions: $A_{op}\,\Psi = a\,\Psi$ (a is a constant) ; uncertainty: $\Delta A = \sqrt{<A^2> - <A>^2}$

Harmonic oscillator: $\Psi_n(x) = C_n H_n(x) e^{-\frac{m\omega}{2\hbar}x^2}$; $E_n = (n+\frac{1}{2})\hbar\omega$; $E = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2 = \frac{1}{2}m\omega^2 A^2$; $\Delta n = \pm 1$

Step potential: $R = \frac{(k_1 - k_2)^2}{(k_1 + k_2)^2}$, $T = 1 - R$; $k = \sqrt{\frac{2m}{\hbar^2}(E - V)}$

Tunneling: $\psi(x) \sim e^{-\alpha x}$; $T \sim e^{-2\alpha\Delta x}$; $T \sim e^{-2\int_a^b \alpha(x)dx}$; $\alpha(x) = \sqrt{\frac{2m[V(x) - E]}{\hbar^2}}$

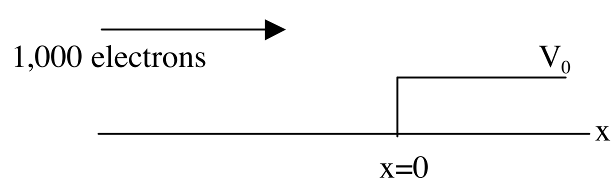
3D square well: $\Psi(x,y,z) = \Psi_1(x)\Psi_2(y)\Psi_3(z)$; $E = \frac{\pi^2\hbar^2}{2m}(\frac{n_1^2}{L_1^2} + \frac{n_2^2}{L_2^2} + \frac{n_3^2}{L_3^2})$

Justify all your answers to all problems

Problem 1 (10 pts)
An electron is in a harmonic oscillator potential where it can absorb and emit photons of wavelength λ=3000A. Assuming the electron is in the ground state.
(a) Find the classical amplitude of oscillation, in Angstrom (classical amplitude=amplitude of oscillation of a classical electron with the same total energy).
(b) Calculate the uncertainty in the position of this electron, Δx, in Angstrom. Show all steps. How does it compare to the classical amplitude of oscillation?
(c) Find the uncertainty in the momentum of this electron, Δp, in units eV/c.
Use:

$$\psi_0(x) = (\frac{m\omega}{\pi\hbar})^{1/4} e^{-\frac{m\omega}{2\hbar}x^2} \quad ; \quad \int_{-\infty}^{\infty} dx\, e^{-\lambda x^2} = \sqrt{\frac{\pi}{\lambda}} \quad ; \quad \int_{-\infty}^{\infty} dx\, x^2 e^{-\lambda x^2} = \frac{1}{2}\sqrt{\frac{\pi}{\lambda^3}}$$

Problem 2 (10 pts)



A beam of 1,000 electrons of kinetic energy E=14eV is incident from the left on the potential step of unknown height V_0 for x>0, shown in the figure. The wavefunction for each electron is

$\psi_I(x) = e^{ikx} + Be^{-ikx}$ for x < 0

$\psi_{II}(x) = 1.3e^{ik_2x}$ for x > 0

(except for an overall normalization constant that is unimportant).
(a) Find the wavenumber of the transmitted electrons, k_2, in A^{-1}.
(b) How many electrons are reflected, how many are transmitted?
(c) Find the value of V_0 in eV.
Use $\hbar^2/2m_e = 3.81\ eV\,A^2$.

Problem 3 (10 pts)
A finite one-dimensional square well has height V_0=10eV. It has only 2 bound states for an electron.
(a) What can you say about the length of this well, L? Give an exact upper bound for L, in Angstrom.
(b) If the particle in this well is a proton instead of an electron, estimate roughly the number of bound states.
(c) How do the uncertainties in the position (Δx) of an electron and a proton in their lowest energy state in this well compare? Are they equal, and if not, which one is larger? Explain.

Justify all your answers to all problems