

**Formulas:**

$$\text{Relativistic energy - momentum relation } E = \sqrt{m^2 c^4 + p^2 c^2} ; \quad c = 3 \times 10^8 \text{ m/s}$$

Electron rest mass :  $m_e = 0.511 \text{ MeV}/c^2$ ; Proton :  $m_p = 938.26 \text{ MeV}/c^2$ ; Neutron :  $m_n = 939.55 \text{ MeV}/c^2$

$$\text{Planck's law : } u(\lambda) = n(\lambda) \bar{E}(\lambda) ; \quad n(\lambda) = \frac{8\pi}{\lambda^4} ; \quad \bar{E}(\lambda) = \frac{hc}{\lambda} \frac{1}{e^{hc/\lambda k_B T} - 1}$$

Energy in a mode/oscillator :  $E_f = nhf$  ; probability  $P(E) \propto e^{-E/k_B T}$

$$\text{Stefan's law : } R = \sigma T^4 ; \quad \sigma = 5.67 \times 10^{-8} \text{ W/m}^2 \text{ K}^4 ; \quad R = c U / 4 , \quad U = \int_0^\infty u(\lambda) d\lambda$$

$$\text{Wien's displacement law : } \lambda_m T = \frac{hc}{4.96 k_B}$$

Photons :  $E = pc$  ;  $E = hf$  ;  $p = h/\lambda$  ;  $f = c/\lambda$

$$\text{Photoelectric effect : } eV_0 = \left( \frac{1}{2} mv^2 \right)_{\max} = hf - \phi , \quad \phi = \text{work function}$$

$$\text{Compton scattering : } \lambda' - \lambda = \frac{h}{m_e c} (1 - \cos \theta)$$

$$\text{Rutherford scattering: } b = \frac{kq_a Q}{m_a v^2} \cot(\theta/2) ; \quad \Delta N \propto \frac{1}{\sin^4(\theta/2)}$$

Constants :  $hc = 12,400 \text{ eV A}$  ;  $\hbar c = 1,973 \text{ eV A}$  ;  $k_B = 1/11,600 \text{ eV/K}$  ;  $ke^2 = 14.4 \text{ eV A}$

$$\text{Electrostatics : } F = \frac{kq_1 q_2}{r^2} \text{ (force)} ; \quad U = q_0 V \text{ (potential energy)} ; \quad V = \frac{kq}{r} \text{ (potential)}$$

$$\text{Hydrogen spectrum : } \frac{1}{\lambda} = R \left( \frac{1}{m^2} - \frac{1}{n^2} \right) ; \quad R = 1.097 \times 10^7 \text{ m}^{-1} = \frac{1}{911.3 \text{ A}}$$

$$\text{Bohr atom : } E_n = -\frac{ke^2 Z}{2r_n} = -\frac{Z^2 E_0}{n^2} ; \quad E_0 = \frac{ke^2}{2a_0} = \frac{mk^2 e^4}{2\hbar^2} = 13.6 \text{ eV} ; \quad E_n = E_{kin} + E_{pot}, E_{kin} = -E_{pot}/2 = -E_n$$

$$hf = E_i - E_f ; \quad r_n = r_0 n^2 ; \quad r_0 = \frac{a_0}{Z} ; \quad a_0 = \frac{\hbar^2}{mke^2} = 0.529 \text{ A} ; \quad L = mvr = \hbar \quad \text{angular momentum}$$

X-ray spectra :  $f^{1/2} = A_n(Z-b)$  ; K :  $b=1$ , L :  $b=7.4$

$$\text{de Broglie : } \lambda = \frac{h}{p} ; \quad f = \frac{E}{h} ; \quad \omega = 2\pi f ; \quad k = \frac{2\pi}{\lambda} ; \quad E = \hbar\omega ; \quad p = \hbar k ; \quad E = \frac{p^2}{2m}$$

group and phase velocity :  $v_g = \frac{d\omega}{dk} ; \quad v_p = \frac{\omega}{k} ; \quad \text{Heisenberg : } \Delta x \Delta p \sim \hbar ; \quad \Delta t \Delta E \sim \hbar$

Wave function  $\Psi(x,t) = |\Psi(x,t)| e^{i\theta(x,t)}$  ;  $P(x,t) dx = |\Psi(x,t)|^2 dx = \text{probability}$

$$\text{Schrodinger equation : } -\frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x^2} + V(x) \Psi(x,t) = i\hbar \frac{\partial \Psi}{\partial t} ; \quad \Psi(x,t) = \psi(x) e^{-\frac{iE}{\hbar}t}$$

$$\text{Time-independent Schrodinger equation : } -\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + V(x) \psi(x) = E \psi(x) ; \quad \int_{-\infty}^{\infty} dx \psi^* \psi = 1$$

$$\infty \text{ square well: } \psi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right) ; \quad E_n = \frac{\pi^2 \hbar^2 n^2}{2m L^2} ; \quad x_{op} = x , \quad p_{op} = \frac{\hbar}{i} \frac{\partial}{\partial x} ; \quad \langle A \rangle = \int_{-\infty}^{\infty} dx \psi^* A_{op} \psi$$

Eigenvalues and eigenfunctions:  $A_{\text{op}} \Psi = a \Psi$  ( $a$  is a constant) ; uncertainty:  $\Delta A = \sqrt{\langle A^2 \rangle - \langle A \rangle^2}$

Harmonic oscillator:  $\Psi_n(x) = C_n H_n(x) e^{-\frac{m\omega}{2\hbar}x^2}$ ;  $E_n = (n + \frac{1}{2})\hbar\omega$ ;  $E = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2 = \frac{1}{2}m\omega^2 A^2$ ;  $\Delta n = \pm 1$

Step potential:  $R = \frac{(k_1 - k_2)^2}{(k_1 + k_2)^2}$ ,  $T = 1 - R$ ;  $k = \sqrt{\frac{2m}{\hbar^2}(E - V)}$

Tunneling:  $\psi(x) \sim e^{-\alpha x}$ ;  $T \sim e^{-2\alpha \Delta x}$ ;  $T \sim e^{-2 \int_a^b \alpha(x) dx}$ ;  $\alpha(x) = \sqrt{\frac{2m[V(x) - E]}{\hbar^2}}$

3D square well:  $\Psi(x,y,z) = \Psi_1(x)\Psi_2(y)\Psi_3(z)$ ;  $E = \frac{\pi^2 \hbar^2}{2m} \left( \frac{n_1^2}{L_1^2} + \frac{n_2^2}{L_2^2} + \frac{n_3^2}{L_3^2} \right)$