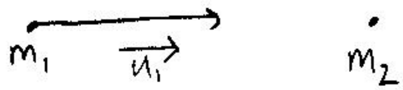
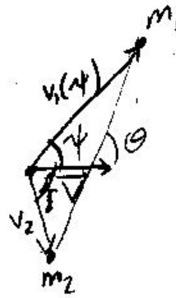
9.35





we need to find a surface a distance $\vec{r}$ from the scattering location where the time to travel $\vec{r}$ is constant.

ie: we have $r = v_1 t_0$, where t_0 is constant and v_1 is the final velocity of m_1.

Using conservation of energy;

$T_0 = \frac{1}{2} m_1 u_1^2$: intial kinetic energy of system

-following the derivation in M. & T.,

$$\frac{T_1}{T_0} = \frac{V_1^2}{u_1^2} \quad \text{and} \quad \frac{T_1}{T_0} = \frac{m_1^2}{(m_1+m_2)^2}\left[\cos\psi \pm \sqrt{\left(\frac{m_2}{m_1}\right)^2 - \sin^2\psi}\right]^2$$

$$\therefore \quad V_1 = \frac{m_1 u_1}{m_1+m_2}\left[\cos\psi \pm \sqrt{\left(\frac{m_2}{m_1}\right)^2 - \sin^2\psi}\right]$$

and

$$r = \frac{m_1 u_1 t_0}{m_1+m_2}\left[\cos\psi \pm \sqrt{\left(\frac{m_2}{m_1}\right)^2 - \sin^2\psi}\right]$$

a) $m_2 = m_1$:

$$r = \frac{u_1 t_0}{2}\left[\cos\psi \pm \sqrt{1-\sin^2\psi}\,\right] = u_1 t_0 \cos\psi \text{ or } 0$$

b) $m_2 = 2m_1$:

$$r = \frac{u_1 t_0}{3}\left[\cos\psi \pm \sqrt{4-\sin^2\psi}\,\right]$$

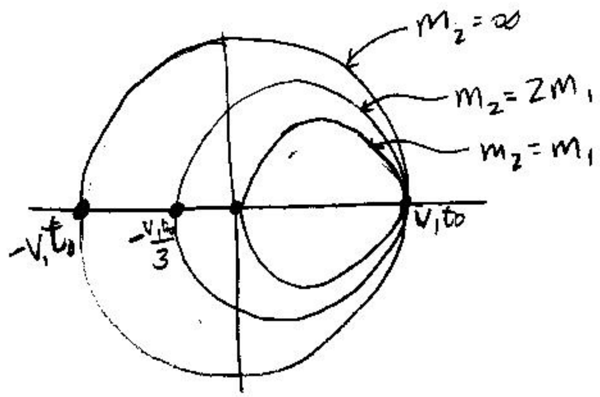
c) $m_2 = \infty$:

rewrite: $$r = \frac{m_1 u_1 t_0}{1+\frac{m_1}{m_2}}\left[\frac{\cos\psi}{m_2} \pm \sqrt{\left(\frac{1}{m_1}\right)^2 - \frac{\sin^2\psi}{m_2^2}}\,\right]$$

in the limit $m_2 \to \infty$, we have simply

$$r = u_1 t_0$$

• in each case, the surface is spherical but with the center displaced:



9.50 $\quad F = \frac{k}{r^3} = -\frac{du}{dr} \quad \therefore \; u = \frac{k}{2r^2}$

$r_{min}: \quad \sqrt{1 - \frac{b^2}{r^2} - \frac{u}{T_0}} = 0$

$$1 - \frac{b^2}{r^2} - \frac{k}{r^2 m u_0^2} = 0$$

$$r_{min} = \sqrt{b^2 + \frac{k}{m u_0^2}}$$

$$\Theta_1 = \int_{r_{min}}^{\infty} \frac{\frac{b}{r^2}\,dr}{\sqrt{1 - \frac{b^2}{r^2} - \frac{u}{T_0}}} = \int_{r_{min}}^{\infty} \frac{b\,dr}{r\sqrt{r^2 - \left(b^2 + \frac{k}{2T_0}\right)}}$$

let $\quad a = \sqrt{b^2 + \frac{k}{2T_0}}$

$$\Theta_1 = \frac{b}{a}\sec^{-1}\left(\frac{r}{a}\right)\Big|_{r_{min}}^{\infty} = \frac{b}{a}\left(\frac{\pi}{2} - \sec^{-1}\left(\frac{r_{min}}{a}\right)\right)$$

$$= \frac{b}{a}\frac{\pi}{2} - \cancelto{0}{\sec^{-1}(1)} = \frac{b}{a}\frac{\pi}{2} = \frac{b}{\sqrt{b^2 + \frac{k}{2T_0}}}\frac{\pi}{2}$$

$$\theta = \pi - 2\Theta_1 = \pi - \pi\frac{b}{\sqrt{b^2 + \frac{k}{2T_0}}}$$

$$\theta - \pi = -\pi\frac{b}{\sqrt{b^2 + \frac{k}{m u_0^2}}} \rightarrow (\pi - \theta)^2 = \pi^2\frac{b^2}{b^2 + \frac{k}{m u_0^2}}$$

$$\Rightarrow b = \sqrt{\frac{\frac{k}{m u_0^2}(\pi - \theta)^2}{\pi^2 - (\pi - \theta)^2}}$$

$$\frac{db}{d\theta} = \frac{k\pi^2(\pi - \theta)}{m u_0^2 (2\pi - \theta)^2 \sqrt{k(\pi-\theta)^2 / m u_0^2 (2\pi - \theta)\theta}}$$

$$\sigma(\theta) = \frac{b}{\sin\theta}\left|\frac{db}{d\theta}\right|$$

after much simplification:

$$\sigma(\theta) = \frac{k\pi^2(\pi-\theta)}{mu_0^2\,\theta^2(2\pi-\theta)^2\sin\theta}$$