

**Problem 1.34.** It's easiest to first compute the work done during each step, using  $W = -P\Delta V$ . For steps *A* and *C* there is no work done because the volume doesn't change; for steps *B* and *D* the pressure is constant so we don't need to set up an integral. So, for instance, the work done on the gas during step *D* is  $+P_1(V_2 - V_1)$ .

Since each molecule has five degrees of freedom, the thermal energy of the gas at any point is  $U = \frac{5}{2}NkT = \frac{5}{2}PV$ . Therefore  $\Delta U$  during any step is  $\frac{5}{2}(P_f V_f - P_i V_i)$ , where *f* stands for final and *i* stands for initial. For instance, during step *D*,  $\Delta U = -\frac{5}{2}P_1(V_2 - V_1)$ .

The heat added to the gas during any step is just  $Q = \Delta U - W$ . So again for step *D* we have  $Q = -\frac{5}{2}P_1(V_2 - V_1) - P_1(V_2 - V_1) = -\frac{7}{2}P_1(V_2 - V_1)$ .

Here's a table of values for all four steps, computed in this way:

|                 | $W$                       | $\Delta U$                   | $Q$                          |
|-----------------|---------------------------|------------------------------|------------------------------|
| step <i>A</i> : | 0                         | $\frac{5}{2}V_1(P_2 - P_1)$  | $\frac{5}{2}V_1(P_2 - P_1)$  |
| step <i>B</i> : | $-P_2(V_2 - V_1)$         | $\frac{5}{2}P_2(V_2 - V_1)$  | $\frac{7}{2}P_2(V_2 - V_1)$  |
| step <i>C</i> : | 0                         | $-\frac{5}{2}V_2(P_2 - P_1)$ | $-\frac{5}{2}V_2(P_2 - P_1)$ |
| step <i>D</i> : | $P_1(V_2 - V_1)$          | $-\frac{5}{2}P_1(V_2 - V_1)$ | $-\frac{7}{2}P_1(V_2 - V_1)$ |
| whole cycle:    | $-(P_2 - P_1)(V_2 - V_1)$ | 0                            | $(P_2 - P_1)(V_2 - V_1)$     |

I found the entries in the last row by adding up each of the columns and simplifying the result as much as possible.

What's actually happening must be something like the following: During step *A* we hold the piston fixed but put heat in (say from a flame); during step *B* we let the piston out and continue putting heat in at such a rate as to maintain constant pressure; during step *C* we hold the piston fixed but suck heat out, perhaps by immersing the whole thing in an ice bath; and during step *D* we push in the piston while still sucking heat out so the pressure again remains steady.

The net work done on the gas during the whole cycle is negative; in other words, the net work done *by* the gas is positive. This is as expected, because the pressure is higher when the gas is expanding than when it is being compressed. Notice that the net work is just minus the area enclosed by the rectangular cycle on the diagram. The net change in the energy of the gas is zero, as it must be: the state of the gas (as determined by its pressure and volume) is the same at the end of a cycle as at the beginning. Therefore the net heat put into the gas must be minus the net work done, as indeed it is. In summary, this procedure results in a net conversion of heat input into work output.