21.8 Kirchhoff's Rules for Complex DC circuits

Used in analyzing relatively more complex DC circuits, e.g., when multiple circuit loops exist

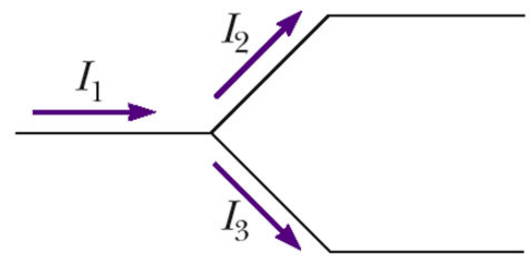
1. Junction rule
2. Loop rule

Junction Rule

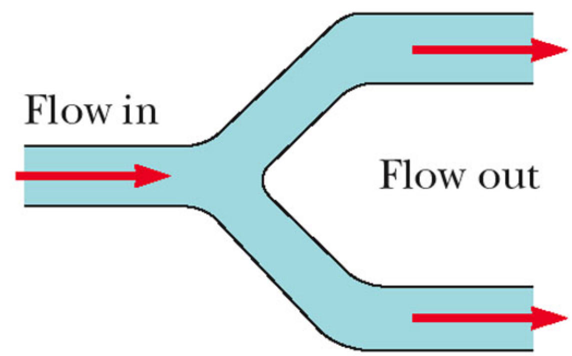
Sum of currents entering any junction must equal the sum of the currents leaving that junction:

$I_1 = I_2 + I_3$

A consequence of conservation of charge (charge can't disappear/appear at a point)

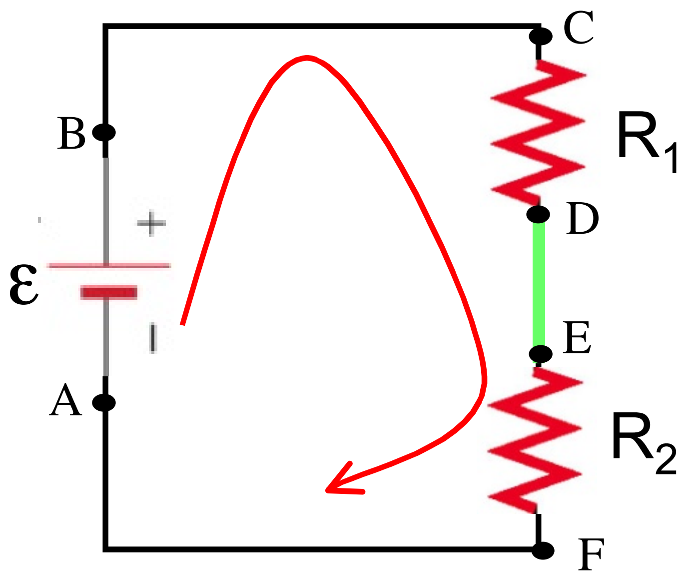


(a)



(b)

Loop Rule



"The sum of voltage differences in going around a closed current loop is equal to zero"

Stems from conservation of energy

$$+\varepsilon - IR_1 - IR_2 = 0$$

$$\varepsilon = IR_1 + IR_2$$

Application of Loop Rule

Choose a current direction (a to b)

When crossing a resistor: $\Delta V = - IR$ in traversal direction

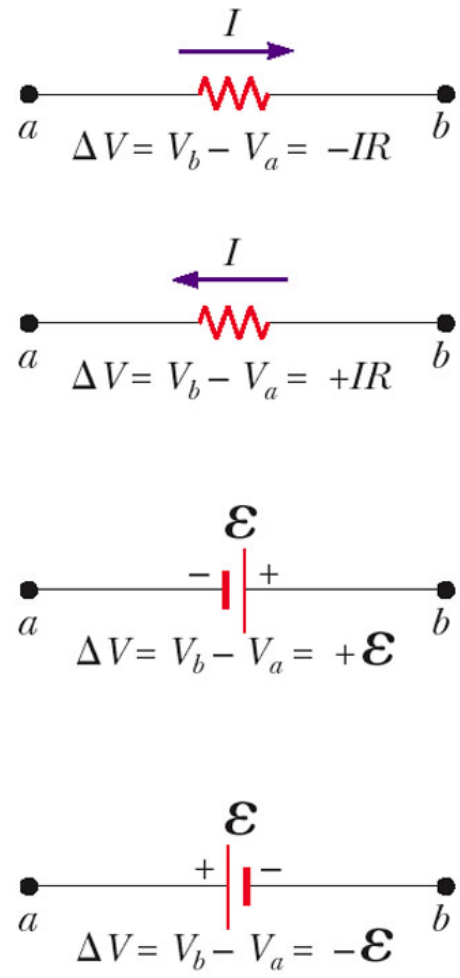
When crossing a resistor: $\Delta V = + IR$ in opposing direction

When crossing a battery: – to + terminals:

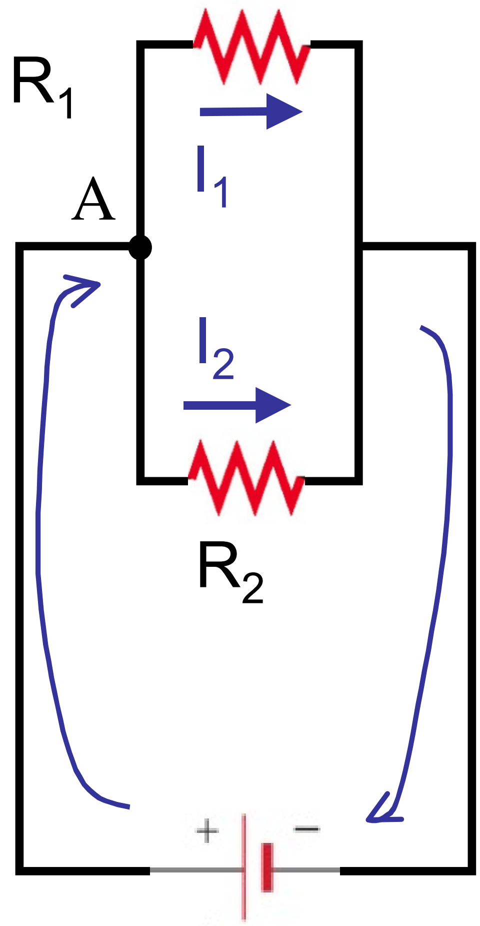
$\Delta V = +\mathcal{E}$

When crossing a battery: + to – terminals:

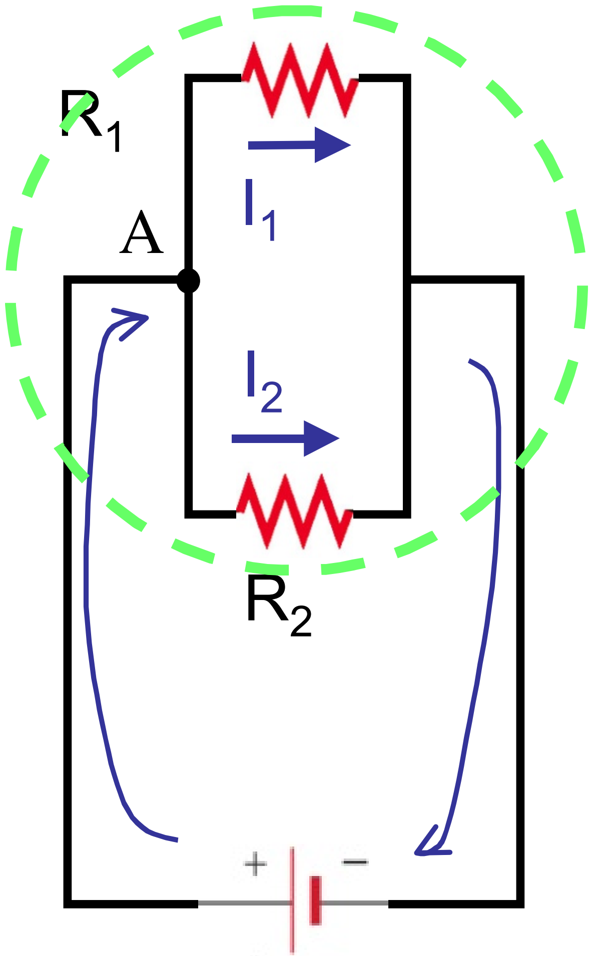
$\Delta V = -\mathcal{E}$



Example of loop/junction rules



Example of loop/junction rules



Loop rule:

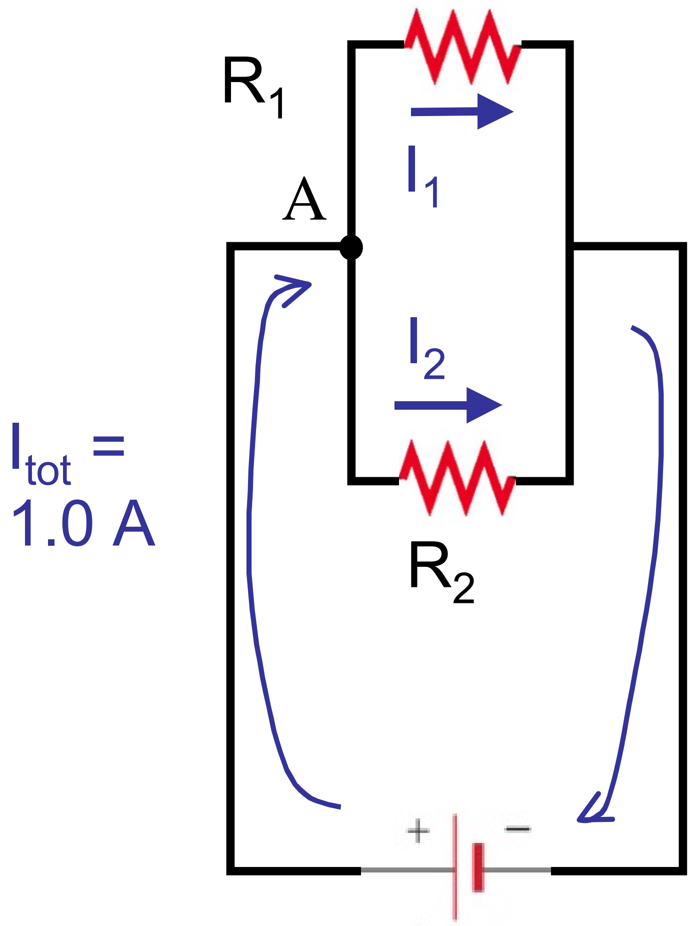
Start at point A, go in CW direction:

$$-I_1R_1 + I_2R_2 = 0$$

$$I_1R_1 = I_2R_2$$

$$I_1/I_2 = R_2/R_1$$

Example of loop/junction rules



Suppose I_{tot} = 1.0 A, R_1 = 3 Ω and R_2 = 6Ω.

Find I_1 & I_2.

$I_1/I_2 = R_2/R_1 = 2$

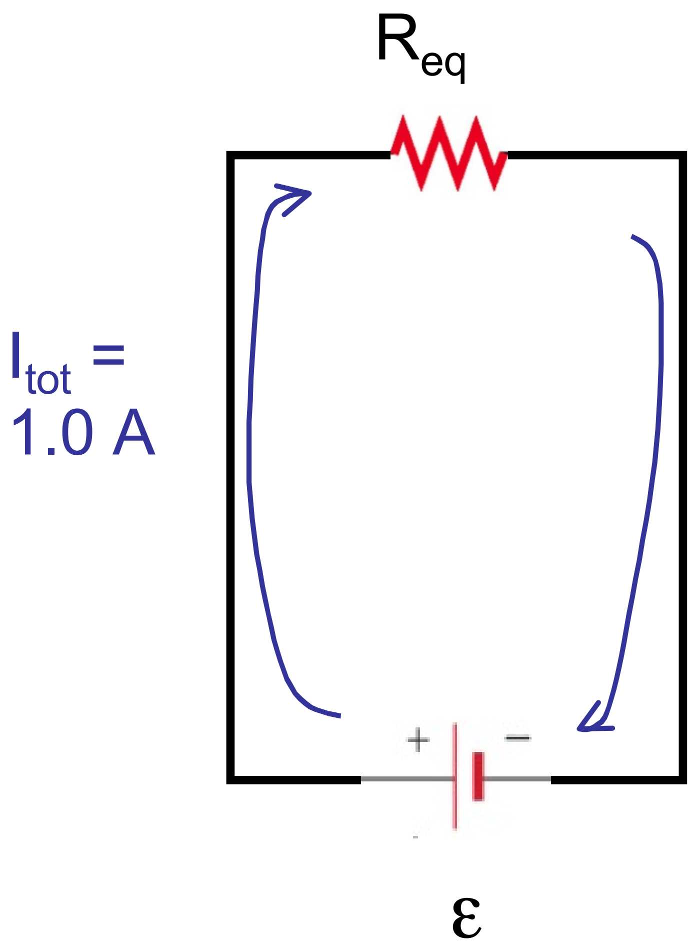
or, $I_1 = 2I_2$

But $I_1 + I_2 = I_{tot} = 1.0A$.

$2I_2 + I_2 = 1.0$ A

So $I_2 = 0.33$ A, and $I_1 = 0.67$ A.

Example of loop/junction rules



Now, calculate ε of the battery.

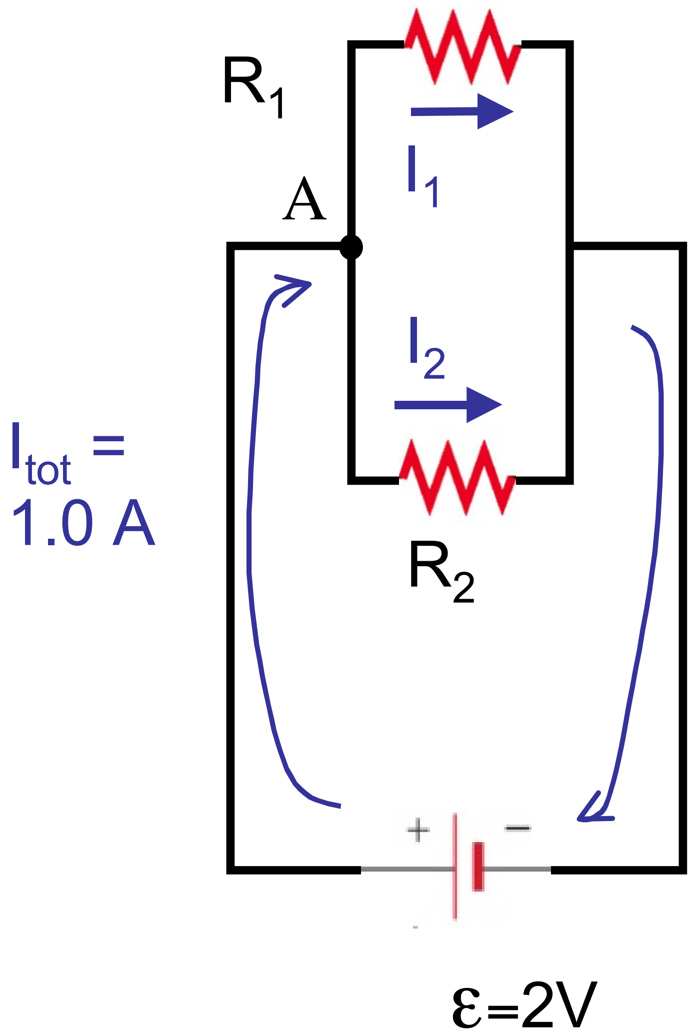
$1/R_{eq} = 1/(3\Omega) + 1/(6\Omega) = 1/(2\Omega)$

$R_{eq} = 2\Omega$

Loop rule for simplified circuit:

$\varepsilon = I_{tot} R_{eq} = 1.0\text{ A } 2\Omega = 2.0\text{ V}$

Example of loop/junction rules

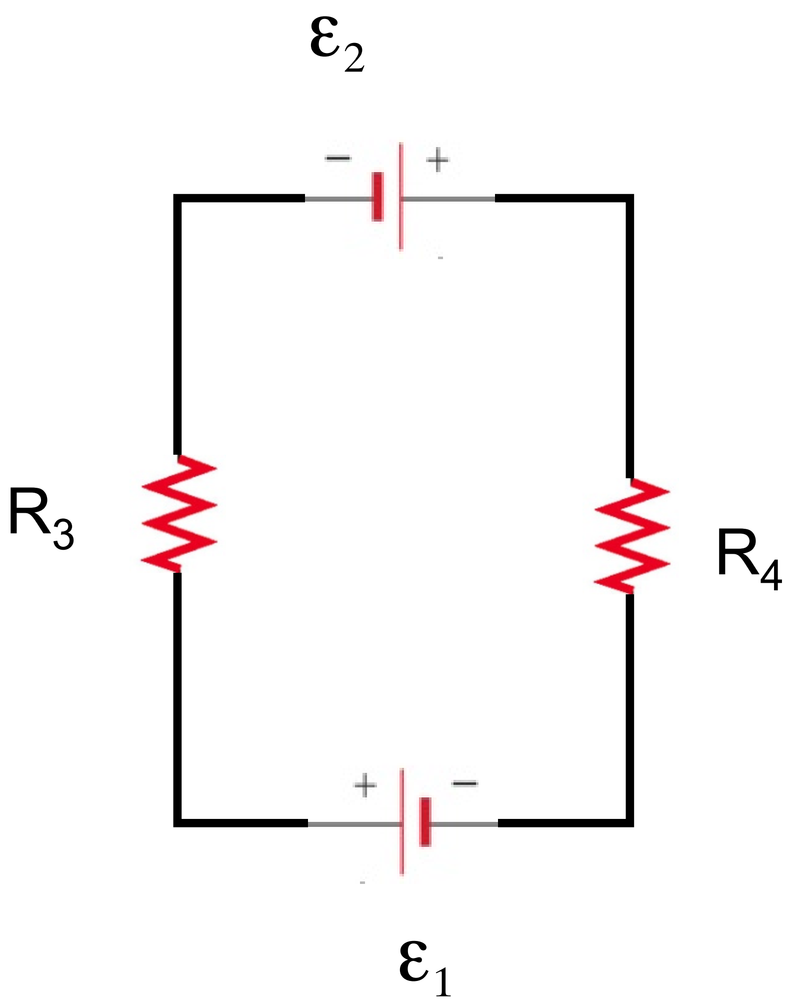


Confirm that the amount of the voltage drop across each resistor is 2V:

$\Delta V_1 = I_1 R_1 = (0.67A)(3\Omega) = 2V$

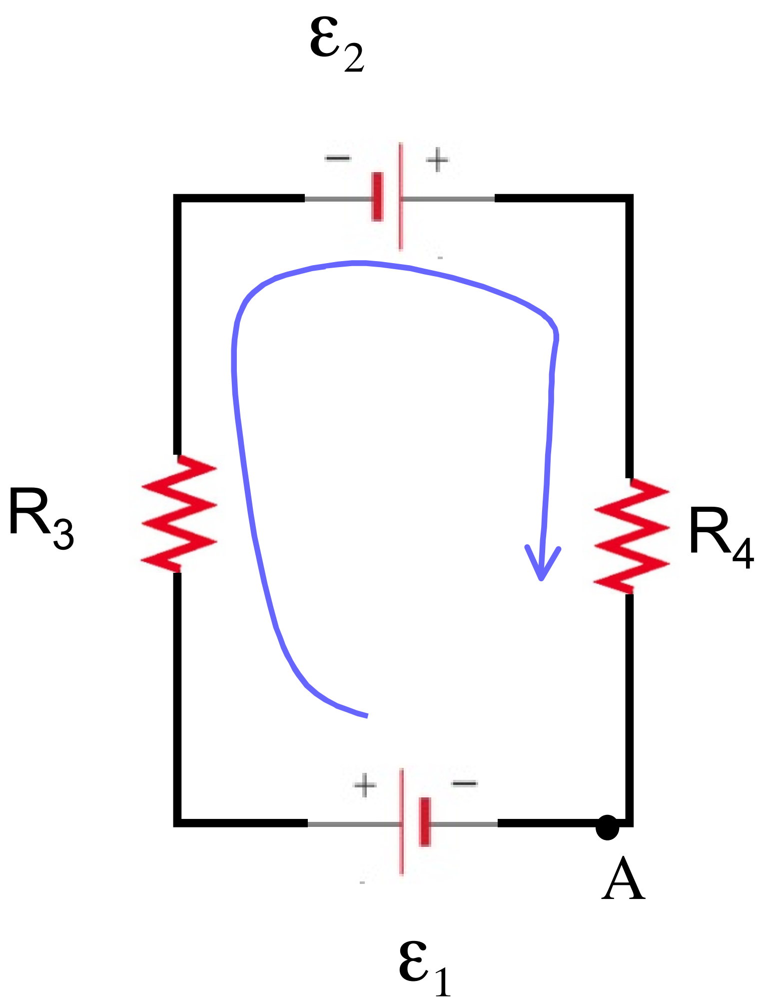
$\Delta V_2 = I_2 R_2 = (0.33A)(6\Omega) = 2V.$

more loop rule



which way will current flow?

more loop rule



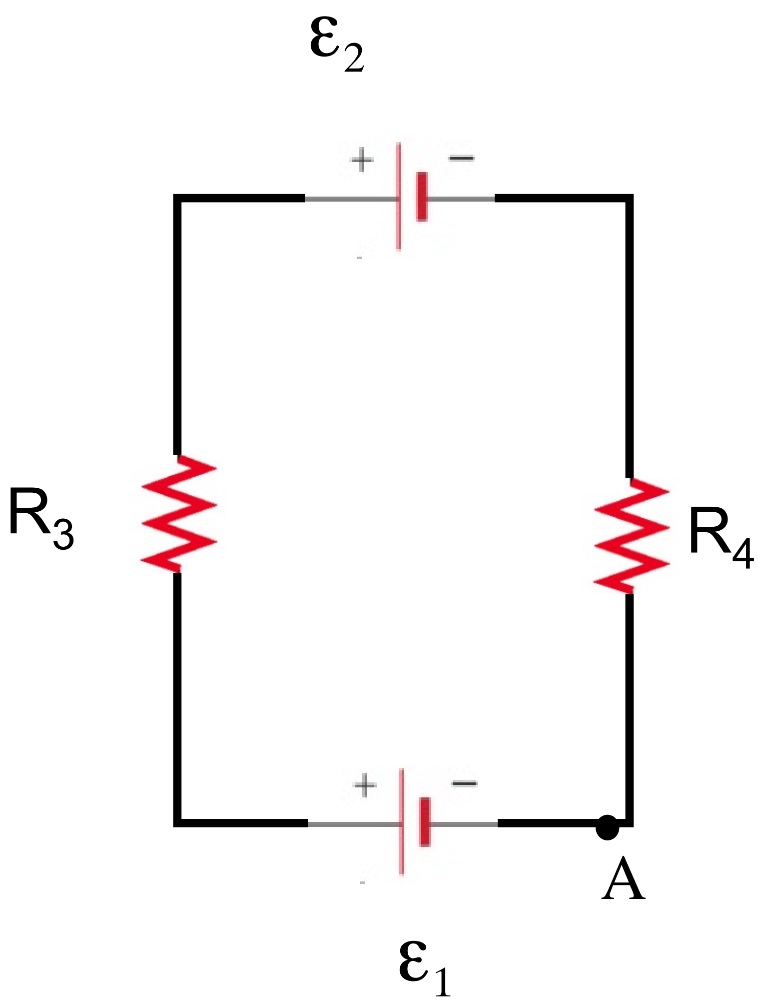
Starting at point A, and going with the current:

$$+\varepsilon_1 - IR_3 + \varepsilon_2 - IR_4 = 0$$

$$+\varepsilon_1 + \varepsilon_2 - IR_4 - IR_3 = 0$$

$$+\varepsilon_1 + \varepsilon_2 = IR_4 + IR_3$$

more loop rule



But watch the direction of EMF in batteries:

Starting at point A, and going with the current:

$$+\varepsilon_1 - IR_3 - \varepsilon_2 - IR_4 = 0$$

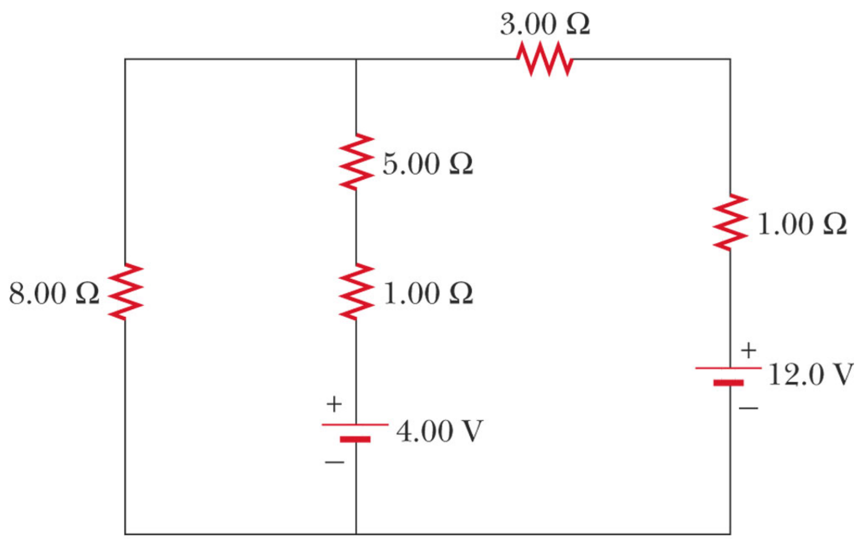
$$+\varepsilon_1 - \varepsilon_2 - IR_4 - IR_3 = 0$$

$$+\varepsilon_1 - \varepsilon_2 = IR_4 + IR_3$$

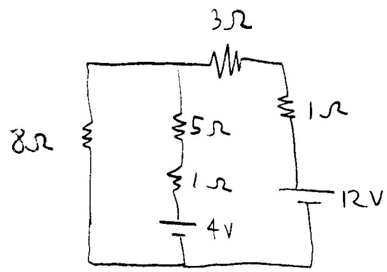
How to use Kirchhoff's Rules

- Draw the circuit diagram and assign labels and symbols to all known and unknown quantities
- Assign directions to currents.
- Apply the junction rule to any junction in the circuit
- Apply the loop rule to as many loops as are needed to solve for the unknowns
- Solve the equations simultaneously for the unknown quantities
- <u>Check your answers -- substitute them back into the original equations!</u>

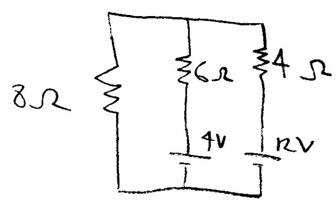
Example for Kirchoff's Rules: #21.35



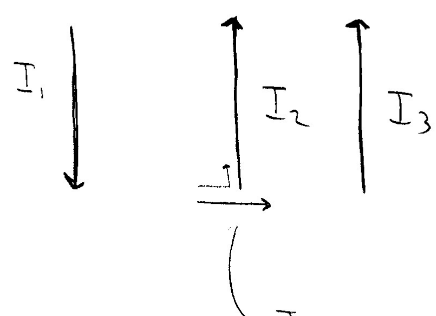
21.35



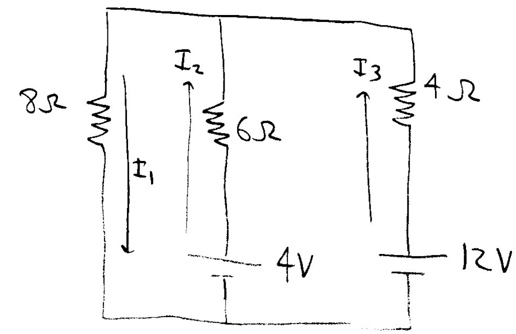
First, we can use the Series Law and redraw the circuit:



Next, we assign directions to currents. Let's <u>guess</u> that the currents flow as follows:



Junction Rule $I_1 = I_2 + I_3$ #1



Apply Loop rule, left-hand loop, going counter-clockwise from point A:

$$+4V - (6\Omega)I_2 - (8\Omega)I_1 = 0 \quad \#2$$

Loop Rule, biggest loop, going counter-clockwise from point A

$$+12V - (4\Omega)I_3 - (8\Omega)I_1 = 0 \quad \#3$$

Now we have three equations and three unknowns (I_1, I_2, and I_3)

Alternate loop rule: Right-hand loop, going counter-clockwise from point A:

$$+12V - (4\Omega)I_3 + (6\Omega)I_2 - 4V = 0$$

Substitute #1 into #2 and #3

$$+4V - (6\Omega)I_2 - 8\Omega\,(I_2 + I_3) = 0 \qquad \#5$$

$$+12V - (4\Omega)I_3 - 8\Omega\,(I_2 + I_3) = 0 \qquad \#6$$

Now we have two equations and two unknowns (I_2 and I_3)

Solve for I_3 in terms of I_2, using #5:

$$+4V - (6\Omega)I_2 - (8\Omega)I_2 - (8\Omega)\,I_3 = 0$$

$$4V - (14\Omega)\,I_2 = (8\Omega)\,I_3$$] Units are volts

$$\frac{4V}{8\Omega} - \left(\frac{14\Omega}{8\Omega}\right) I_2 = I_3$$] Units are now Amps

$$0.5\,A - 1.75\,I_2 = I_2 \qquad \#5A$$

Rearrange #6 $12V - (4\Omega)I_3 - (8\Omega)I_2 - (8\Omega)I_3 = 0$

$$12V - (8\Omega)I_2 - (12\Omega)I_3 = 0$$

$$3V - (2\Omega)I_2 - (3\Omega)I_3 = 0 \quad \#6A$$

Substitute expression for I_3 (#5A) into #6A:

$$3V - (2\Omega)I_2 - 3\Omega(0.5A - 1.75 I_2) = 0$$

$$3V - (2\Omega)I_2 - 1.5V + (5.25\Omega)I_2 = 0$$

$$1.5V + (3.25\Omega)I_2 = 0$$

$$I_2 = -0.462\,A$$

Our initial guess for the direction of I_2 was incorrect!

Substitute value for I_2 back into #5A.

$$0.5A - 1.75 I_2 = I_3$$
$$0.5A - 1.75(-0.462A) = I_3$$
$$0.5A + 0.81A = I_3$$
$$\boxed{I_3 = +1.31\,A}$$

Finally, substitute back into #1 (Junction rule): $\boxed{I_1 = I_2 + I_3 = -0.462A + 1.31\,A = \boxed{0.85A}}$

The initial guess of direction of currents was motivated by the (reasonable) assumption that both batteries would force (positive-)current to travel up in both the middle and right branches, forcing current to travel down in the left branch

But in the left branch, there is the 8 Ω resistor (the largest-resistance resistor). Because of the strong opposition to current in the left branch, current is forced downward in the middle branch – "over-riding" the upward-directed EMF supplied by the 4-V battery.