

Ch. 21: Current, Resistance, Circuits

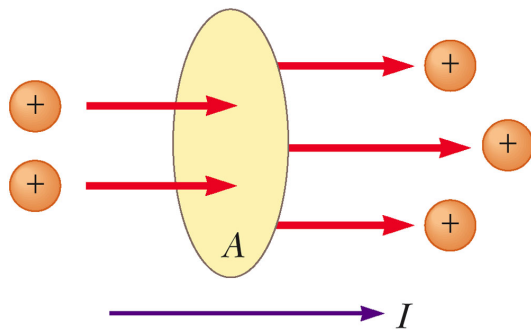
Current: How charges flow through circuits

Resistors: convert electrical energy into thermal/radiative energy

Electrical Energy & Power; Household
Circuits

Time-Dependent Circuits

Current : Rate at which charge flows through an area A (cross-section of a wire)



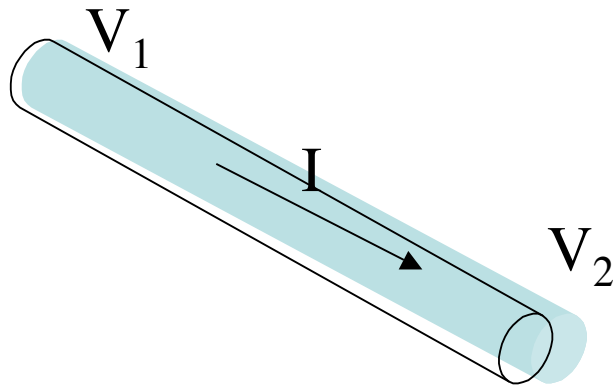
$$I \equiv \frac{\Delta Q}{\Delta t}$$

Flow is assumed to be perpendicular to area.

Units = Coul/sec = Amp.

Remember: I is defined as the direction in which positive charges will travel (in metal, the charge carriers are actually electrons)

Potential difference sets up E-field to drive Current



$$V_1 - V_2 = \Delta V$$

Example: Terminals of a battery

Example:

A flashlight bulb carries a current of 0.1 A.

Find the charge that passes through the bulb in 0.5 seconds:

$$I = \Delta Q / \Delta T \rightarrow \Delta Q = I \times \Delta T = 0.1 \text{ C/s} \times 0.5 \text{ s} = 0.05 \text{ C}$$

How many electrons does this correspond to?

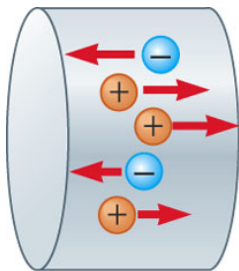
$$\Delta Q = N \times e$$

$$N = \Delta Q / e = 0.05 \text{ C} / (1.6 \times 10^{-19} \text{ C/e}^-) = 3.1 \times 10^{17} \text{ e}^- \text{'s}$$

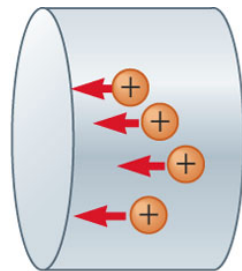
Remember: I is defined as the direction in which positive charges will travel (in metal, the charge carriers are actually electrons)

From quick quiz 21.1:

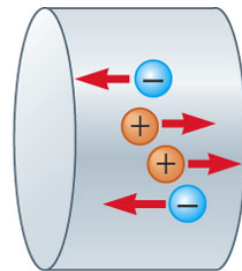
Rank the currents from lowest to highest:



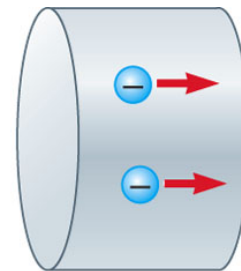
(a)



(b)



(c)

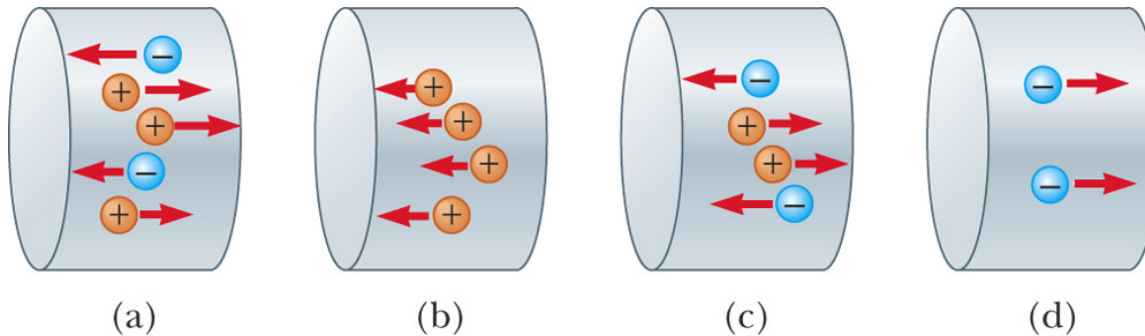


(d)

Remember: I is defined as the direction in which positive charges will travel (in metal, the charge carriers are actually electrons)

From quick quiz 21.1:

Rank the currents from lowest to highest:



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Negative charges moving left are equivalent to positive charges moving right.

- (a) Equivalent to 5 +’s moving right.
- (b) 4 +’s moving left
- (c) Equivalent to 4 +’s moving right
- (d) Equivalent to 2 +’s moving right

Amp-hour

Unit of charge

charge = current \times time

Ex.: Ni-metal hydride battery:
How much charge (in C) is
equal to 2100 mAh?

$$\begin{aligned}\text{Charge} &= (2100 \times 10^{-3} \text{ A}) (1 \text{ hour}) \\ &= (2100 \times 10^{-3} \text{ C/s})(3600\text{s}) \\ &= 7560 \text{ C.}\end{aligned}$$



Amp-hour

If one of these batteries is used to power a device which draws 0.15 Amps, how long will the battery last?

$$I = \Delta Q / \Delta T$$

$$\Delta T = \Delta Q / I = (2100 \times 10^{-3} \text{ Amp} \times \text{hr}) / 0.15 \text{ Amps} = 14 \text{ hours.}$$

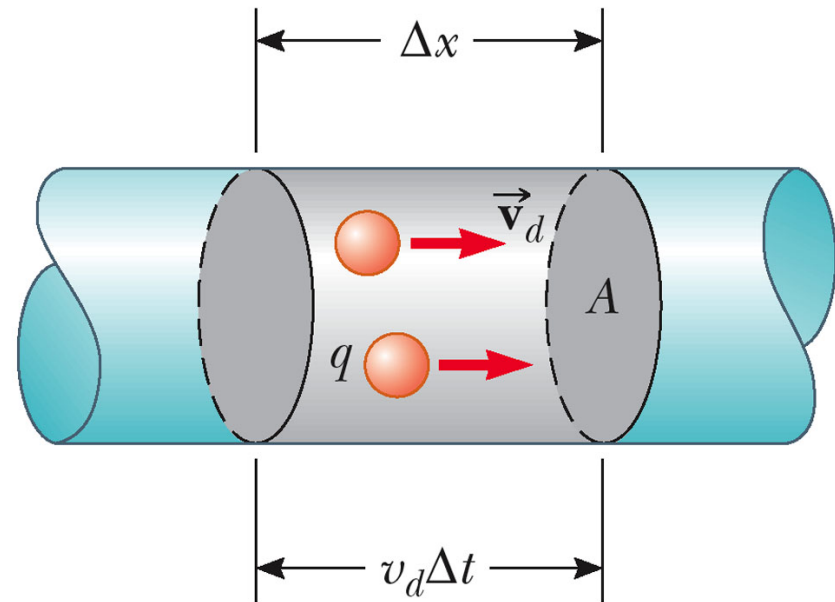


Drift Velocity, v_d

$$\text{Volume} = A \Delta x$$

n = density of charge carriers = # of charge carriers per unit vol.

$$N = \text{Total \# of charge carriers} = n A \Delta x$$



Total charge in this volume: $\Delta Q = N \times \text{charge/carrier} = n A \Delta x q$

$$\Delta x = v_d \Delta t$$

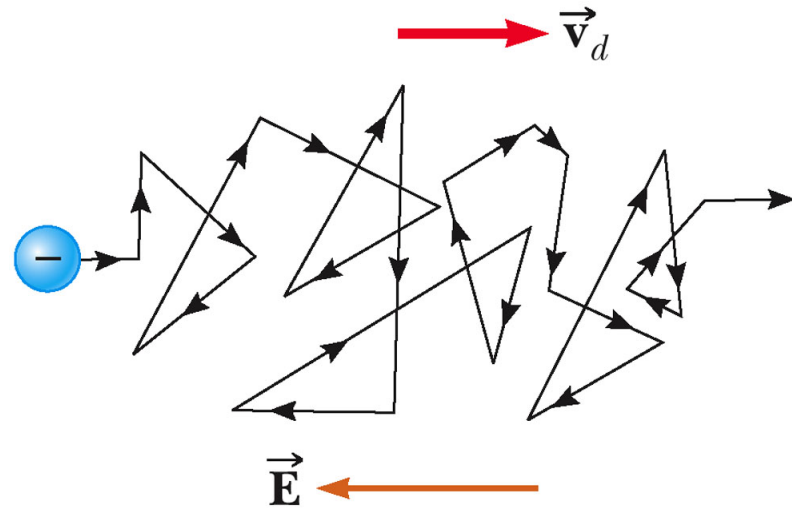
$$\Delta Q = n A v_d \Delta t q$$

$$I = \Delta Q / \Delta t = n A v_d q$$

Drift Velocity, v_d

Electrons undergo repeated collisions and move randomly. Typical velocity for Cu is 2×10^6 m/s

In the presence of an external field, the average motion is a slow drift



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Electric signal travels very fast -- almost at the speed of light: electrons interact and "push" other electrons in the conductor.

Example:

Find the drift velocity of electrons in a copper conductor whose diameter is 2 mm when the applied current is 0.5 A. The mass density of Cu is $\rho = 8.95\text{g/cm}^3$. Each Cu atom contributes 1 electron. One mole of Cu has a mass of 63.5 gm.

Soln: Need to calculate density of charge carriers (# of e^- 's/ m^3)

How many moles per cm^3 ? $(8.95\text{g/cm}^3)/(63.5\text{g/mol}) = 0.14 \text{ mol/cm}^3$

Every mol contain 6×10^{23} atoms.

Number of atoms per cm^3 : $(0.14 \text{ mol/cm}^3)(6 \times 10^{23} \text{ atoms/mol}) = 8.4 \times 10^{22} \text{ atoms/cm}^3$

Density of charge carriers (given that $1e^-/\text{atom}$) = $8.4 \times 10^{22} \text{ e}^-/\text{cm}^3$

$v_d = I/(nqA) = 0.5\text{A}/(8.4 \times 10^{22} \text{e}^-/\text{m}^3 \cdot 1.6 \times 10^{-19}\text{C} \cdot 3.14 \cdot (.001\text{m})^2)$
 $= 1.2 \times 10^{-5} \text{ m/s} = 0.012 \text{ mm/s}$

If $A = 50$ Amps: v_d would be 1.2 mm/s -- still a snail's pace!

Current Density

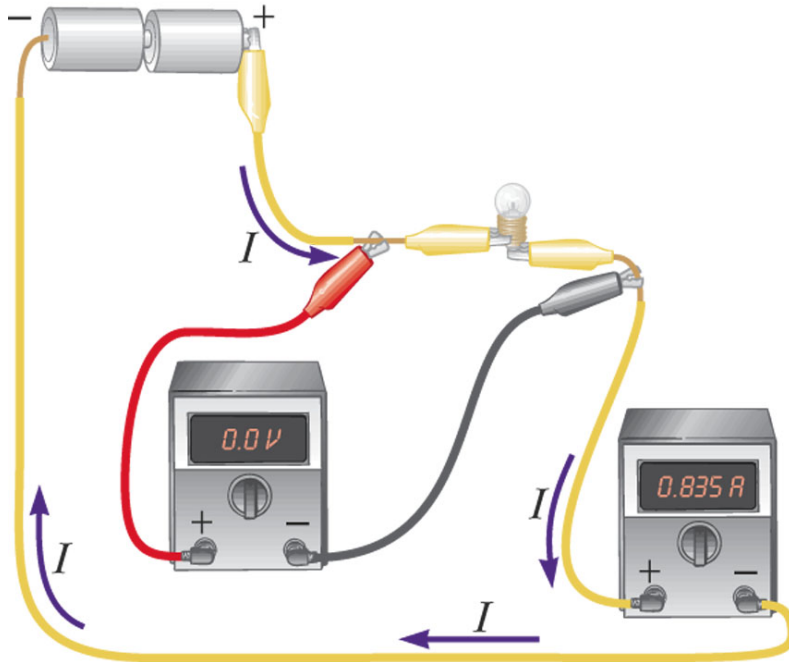
$$J = I/A = n q v_d$$

SI unit: Amps / m²

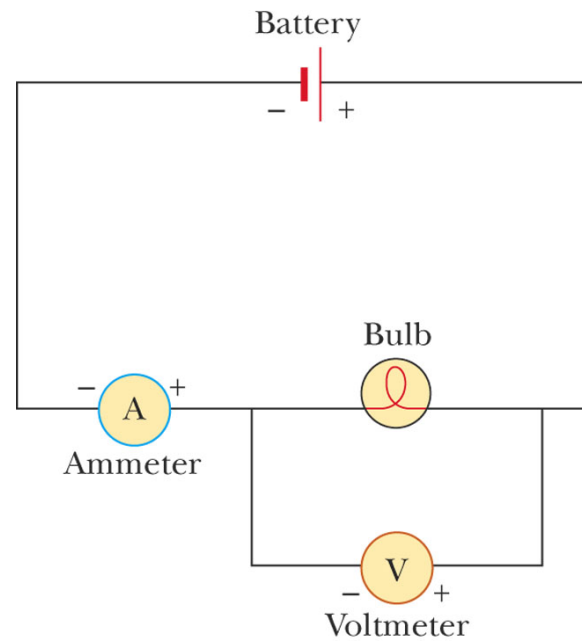
Ammeter

Device used to measure current

All charge must pass through ammeter



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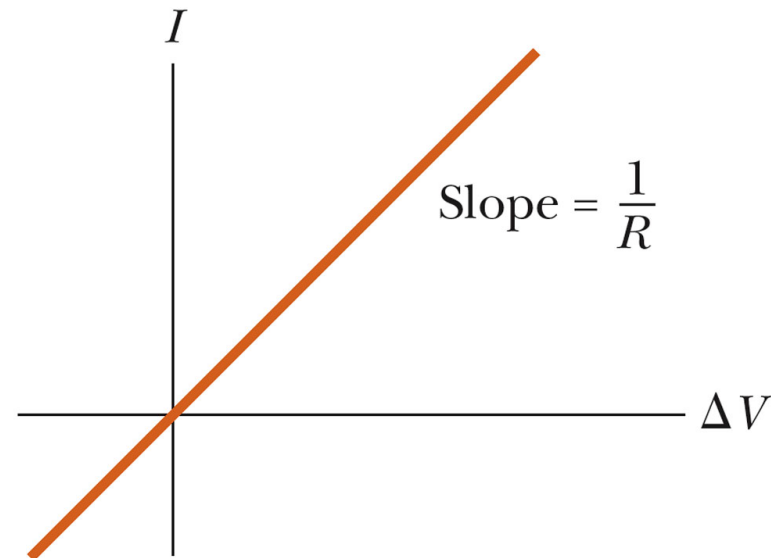
21.2: Resistance & Ohm's Law

Resistance of a conductor is defined as ratio of potential difference across it to the current that results:
Ohm's Law: For many materials, R remains constant over a wide range of applied ΔV or I .

$$R \equiv \frac{\Delta V}{I}$$

$$\Delta V = IR$$

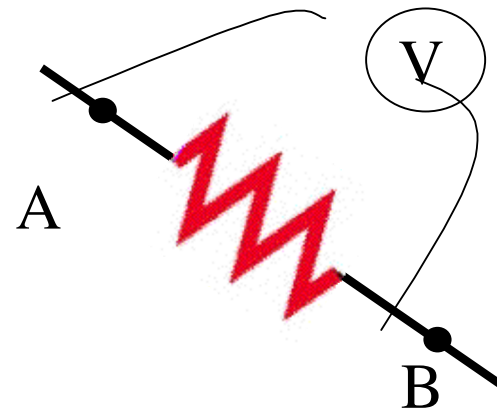
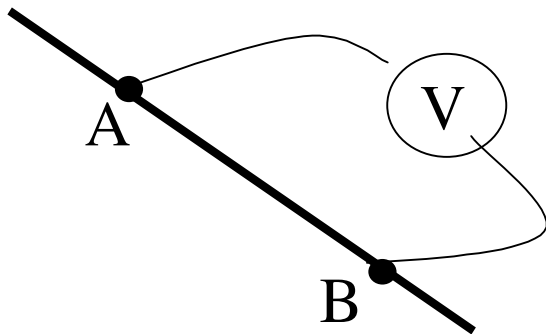
Units of R : Ohms (Ω)



Resistors

In a circuit: the resistance of the conducting wires is negligible, so $\Delta V = 0$ (no extra loss in potential) between points A & B.

But a resistor can cause a significant drop in ΔV (comparing V before/after the resistor):



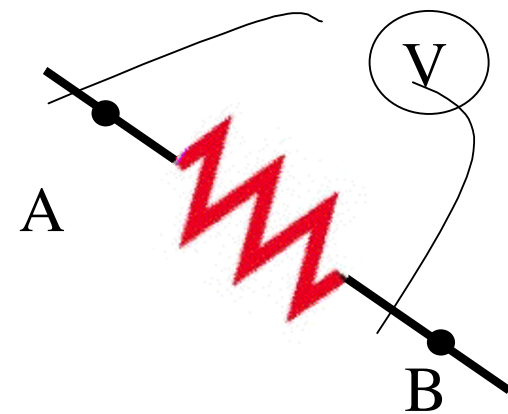
Resistors

Analogy: Waterfalls: sudden drop
in gravitational potential energy

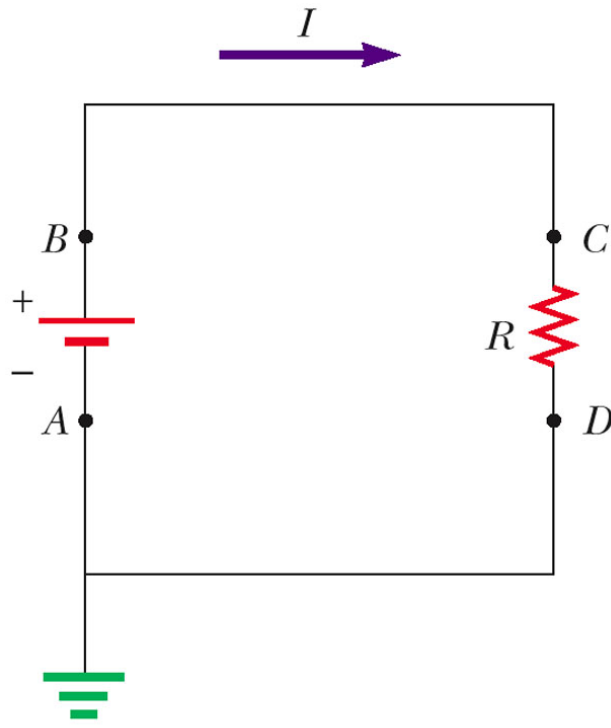
ΔPE converted to kinetic energy
of water



$$q\Delta V = \Delta PE$$



electrical potential energy
converted to thermal
energy in resistor



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Change in PE is
 $+q\Delta V$ (battery)
or $-q\Delta V$ (resistor)

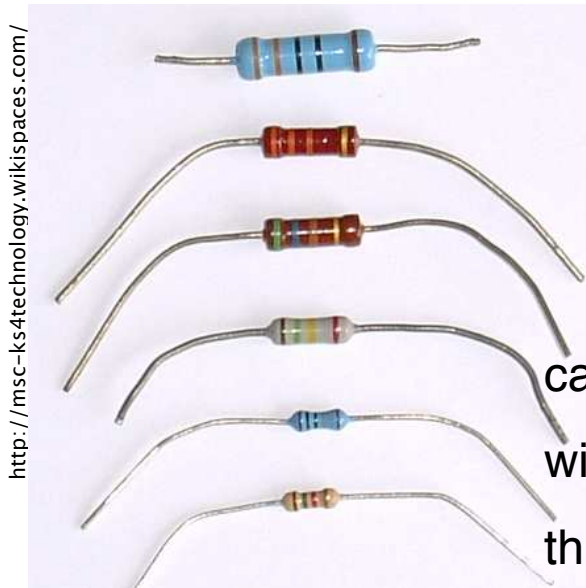
Points A and D are “grounded” -- Potential $V = 0$.

Points B and C are both at higher potential

Resistors

RESISTANCE regulates current and causes conversion of electrical potential energy to heat.

Common examples: heating elements in toasters, hair dryers, space heaters; light bulb filaments



<http://msc-ks4technology.wikispaces.com/>



kidbots.com

carbon resistors

wire wound resistors

thin metal film resistors



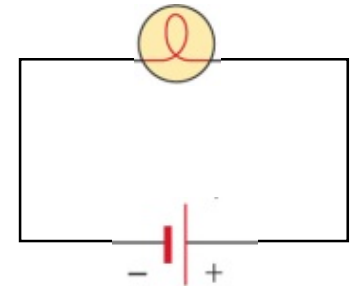
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Examples:

Consider a simple V-R circuit comprising a light bulb. Assume there is a 1.5-volt battery and the light bulb draws a current of 0.2 Amps. Find the R of the light bulb filament:

$$R = \Delta V / I = 1.5V / 0.2 A = 7.5 \Omega$$



A 120-Volt (rel. to ground) household circuit is connected to a lamp; the light bulb filament has $R = 240 \Omega$. Find I.

$$I = \Delta V / R = 120V / 240\Omega = 0.5 A$$

Resistance is determined by geometry & resistivity

$$R = \rho \frac{L}{A}$$

ρ = resistivity.
units are Ωm

semi-conductors {

insulators {

Material	Resistivity ^a ($\Omega \cdot \text{m}$)	Temperature Coefficient α [$(^\circ\text{C})^{-1}$]
Silver	1.59×10^{-8}	3.8×10^{-3}
Copper	1.7×10^{-8}	3.9×10^{-3}
Gold	2.44×10^{-8}	3.4×10^{-3}
Aluminum	2.82×10^{-8}	3.9×10^{-3}
Tungsten	5.6×10^{-8}	4.5×10^{-3}
Iron	10×10^{-8}	5.0×10^{-3}
Platinum	11×10^{-8}	3.92×10^{-3}
Lead	22×10^{-8}	3.9×10^{-3}
Nichrome ^b	1.50×10^{-6}	0.4×10^{-3}
Carbon	3.5×10^{-5}	-0.5×10^{-3}
Germanium	0.46	-48×10^{-3}
Silicon	640	-75×10^{-3}
Glass	10^{10} to 10^{14}	
Hard rubber	$\sim 10^{13}$	
Sulfur	10^{15}	
Quartz (fused)	75×10^{16}	

^aAll values are at 20°C.

^bNichrome is a nickel–chromium alloy commonly used in heating elements.

$$R = \rho \frac{L}{A}$$

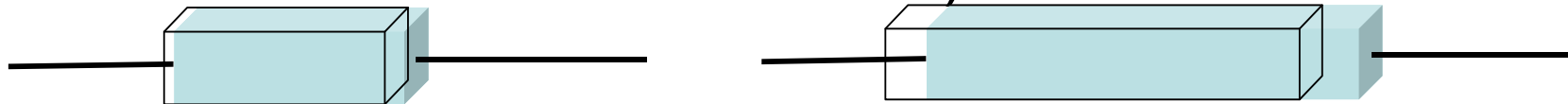
Resistance caused by charge carriers colliding with the lattice of the conductor.

More collisions = more resistance

L = length

Double the length → double the resistance

(electrons must undergo twice as many collisions across the resistor)



$$R = \rho \frac{L}{A}$$

A = cross-section area

Decrease Area: Resistance is raised since flow of charge carriers is constricted



Conductivity σ

$$\sigma = 1/\rho$$

$$R = \rho L/A$$

$$R = L/(\sigma A)$$

$$V/I = L/(\sigma A)$$

$$V/L = I/(\sigma A)$$

$$E = J/\sigma$$

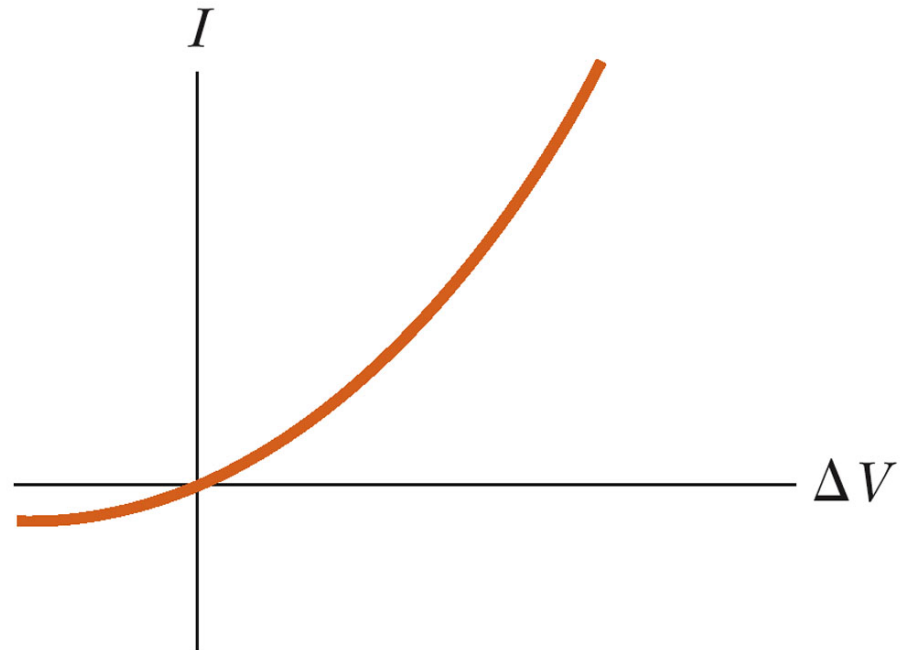
$$J = \sigma E$$

Resistivity and conductivity are “microscopic” properties of the material

Resistance is a macroscopic property of an object, and is a function of geometry and resistivity

Some materials exhibit non-Ohmic resistance

Current-voltage curve for a semi-conducting diode: it's non-linear



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In this course, assume Ohmic resistance unless otherwise stated

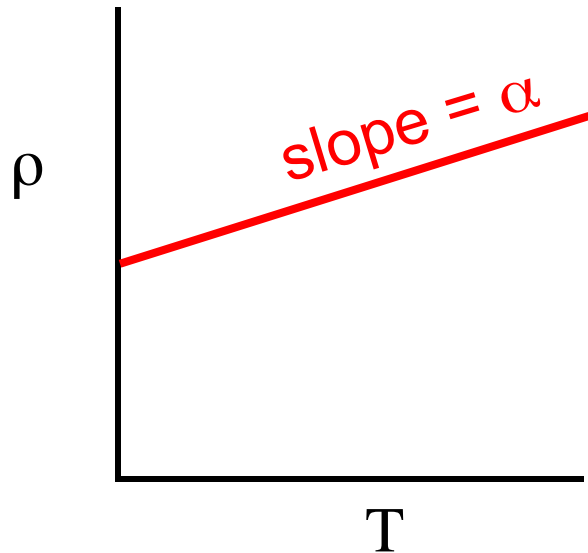
Temperature dependence of resistance

At higher T , the charge carriers' collisions with the lattice are more frequent.

v_d becomes lower. So I becomes lower.

And R becomes larger for a given potential.

Temperature coefficient of resistivity



$$\rho = \rho_0 [1 + \alpha(T - T_0)]$$

$$R = R_0 [1 + \alpha(T - T_0)]$$

T_0 = reference temperature

α = temperature coefficient of resistivity, units of $(^\circ\text{C})^{-1}$

For Ag, Cu, Au, Al, W, Fe, Pt, Pb: values of α are $\sim 3-5 \times 10^{-3} (^\circ\text{C})^{-1}$

Example: A platinum resistance thermometer uses the change in R to measure temperature. Suppose $R_0 = 50 \Omega$ at $T_0 = 20 \text{ }^\circ\text{C}$.

α for Pt is $3.92 \times 10^{-3} \text{ (}^\circ\text{C)}^{-1}$ in this temperature range.

What is R when $T = 50.0 \text{ }^\circ\text{C}$?

$$R = R_0 [1 + \alpha(T - T_0)]$$

$$R = 50 \Omega [1 + 3.92 \times 10^{-3} \text{ (}^\circ\text{C)}^{-1} (30.0 \text{ }^\circ\text{C})] = 55.88 \Omega$$

Temperature coefficient of resistivity

Example: A platinum resistance thermometer has a resistance $R_0 = 50.0 \Omega$ at $T_0 = 20^\circ\text{C}$. α for Pt is $3.92 \times 10^{-3} (\text{°C})^{-1}$. The thermometer is immersed in a vessel containing melting tin, at which point R increases to 91.6Ω . What is the melting point of tin?

$$R = R_0 [1 + \alpha(T - T_0)]$$

$$91.6 \Omega = 50 \Omega [1 + 3.92 \times 10^{-3} (\text{°C})^{-1} (T - 20^\circ\text{C})]$$

$$1.83 = [1 + 3.92 \times 10^{-3} (\text{°C})^{-1} (T - 20^\circ\text{C})]$$

$$0.83 = 3.92 \times 10^{-3} (\text{°C})^{-1} (T - 20^\circ\text{C})$$

$$212^\circ\text{C} = T - 20^\circ\text{C}$$

$$T = 232^\circ\text{C}$$