21.8 Kirchhoff's Rules for Complex DC circuits

Used in analyzing relatively more complex DC circuits, e.g., when multiple circuit loops exist

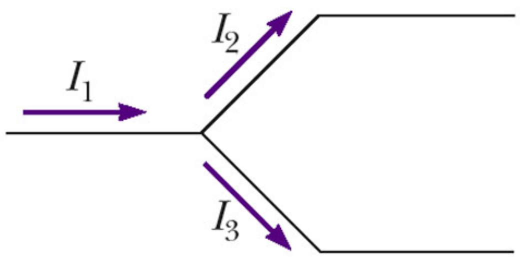
1. Junction rule
2. Loop rule

Junction Rule

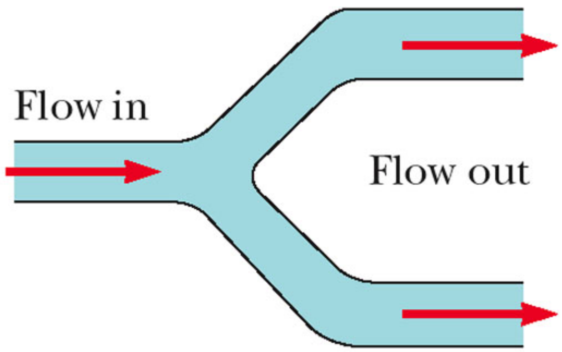
Sum of currents entering any junction must equal the sum of the currents leaving that junction:

$I_1 = I_2 + I_3$

A consequence of conservation of charge (charge can't disappear/appear at a point)

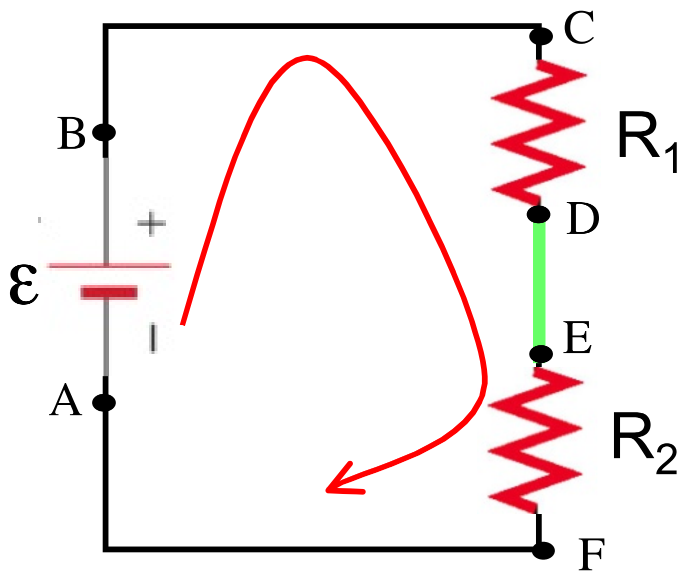


(a)



(b)

Loop Rule



"The sum of voltage differences in going around a closed current loop is equal to zero"

Stems from conservation of energy

$$+\varepsilon - IR_1 - IR_2 = 0$$

$$\varepsilon = IR_1 + IR_2$$

Application of Loop Rule

Choose a current direction (a to b)

When crossing a resistor: $\Delta V = -IR$ in traversal direction

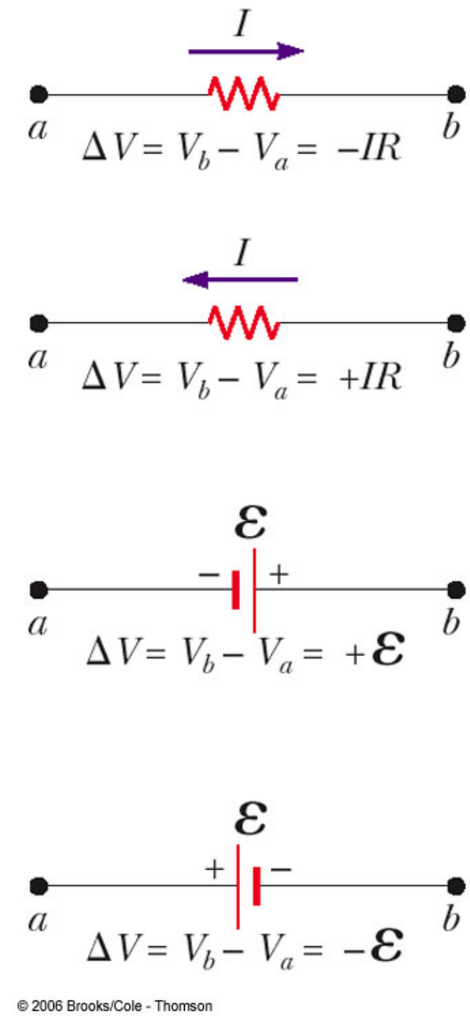
When crossing a resistor: $\Delta V = +IR$ in opposing direction

When crossing a battery: – to + terminals:

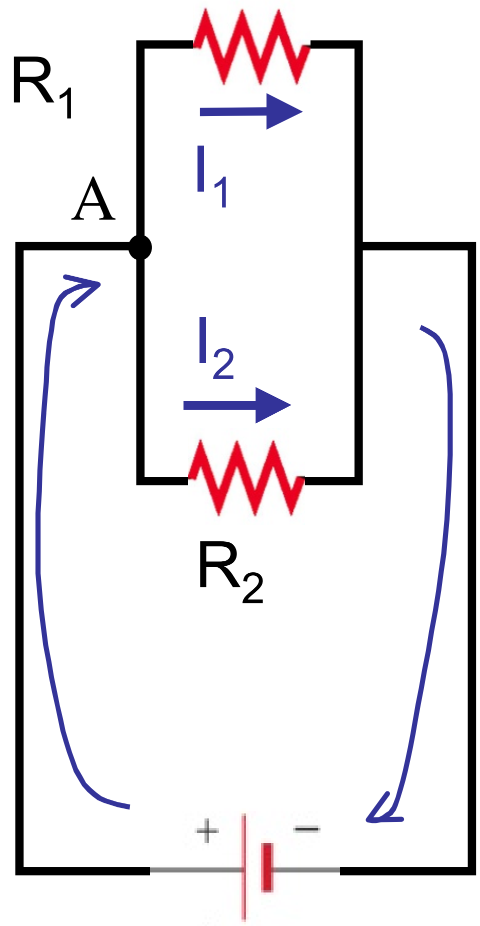
$\Delta V = +\mathcal{E}$

When crossing a battery: + to – terminals:

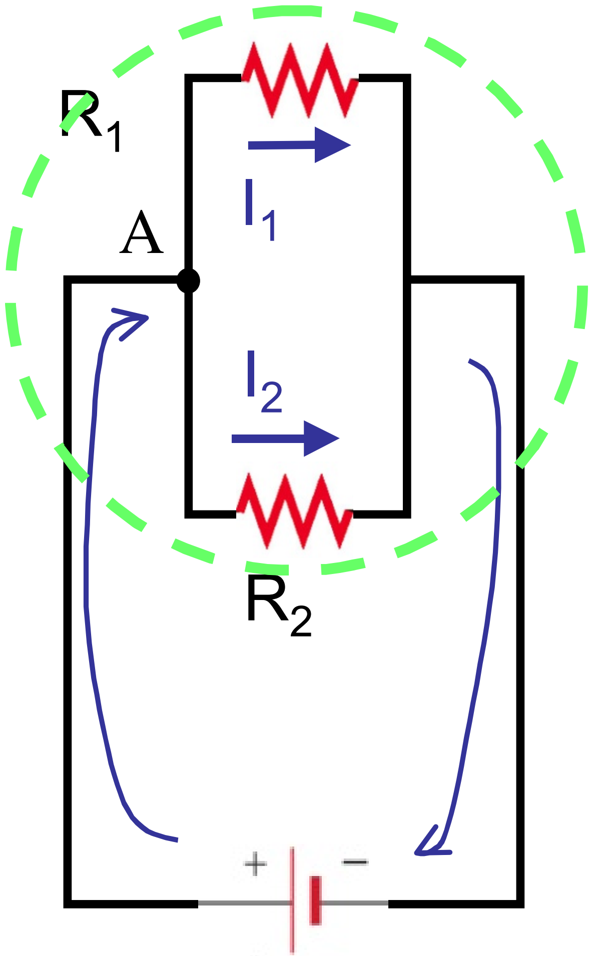
$\Delta V = -\mathcal{E}$



Example of loop/junction rules



Example of loop/junction rules



Loop rule:

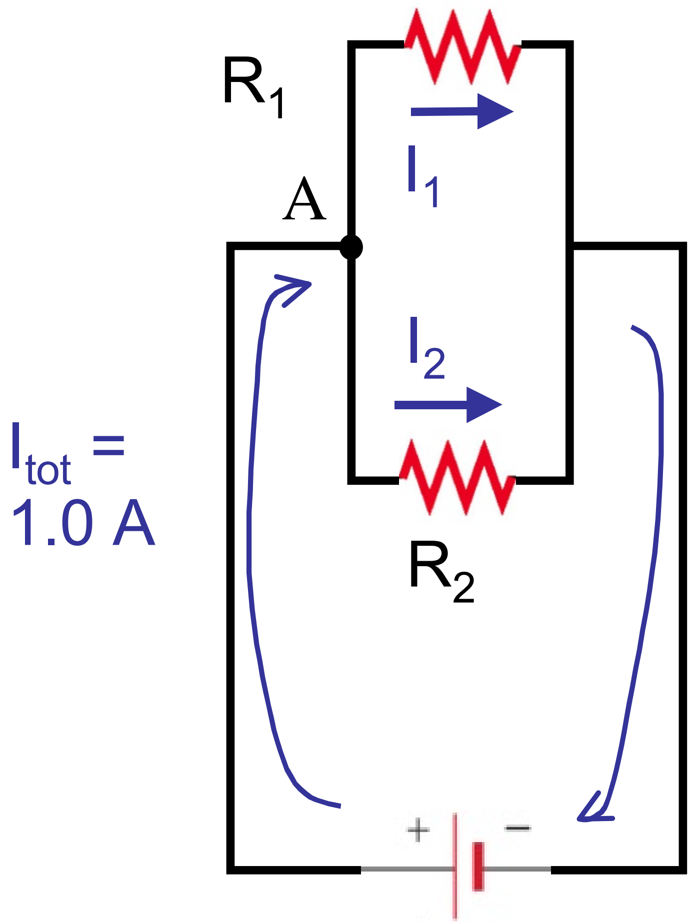
Start at point A, go in CW direction:

$$-I_1R_1 + I_2R_2 = 0$$

$$I_1R_1 = I_2R_2$$

$$I_1/I_2 = R_2/R_1$$

Example of loop/junction rules



Suppose I_{tot} = 1.0 A, R_1 = 3 Ω and R_2 = 6Ω.

Find I_1 & I_2.

$I_1/I_2 = R_2/R_1 = 2$

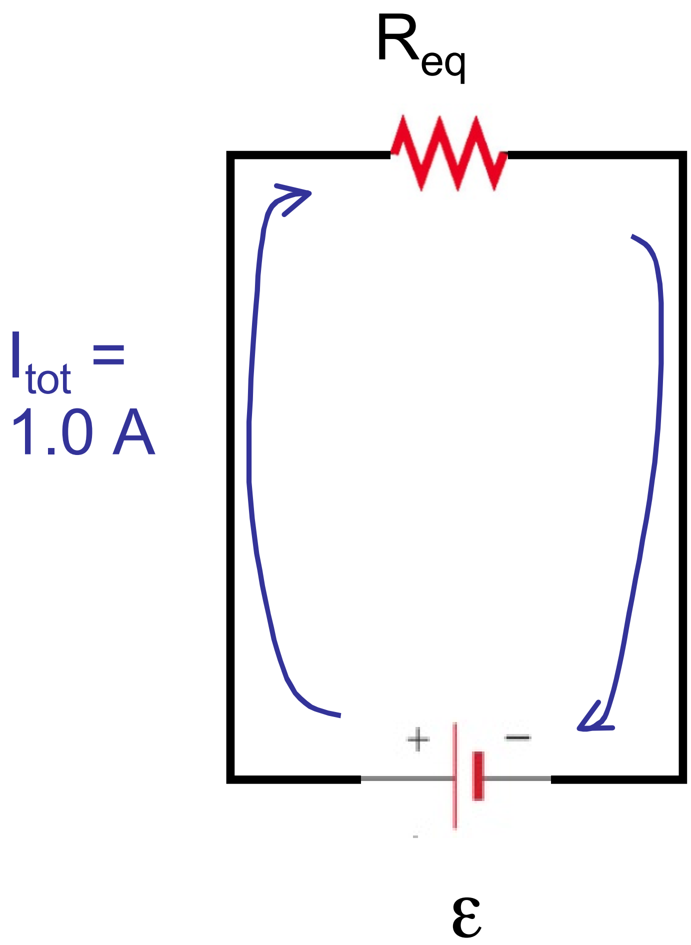
or, $I_1 = 2I_2$

But $I_1 + I_2 = I_{tot} = 1.0\text{A}$.

$2I_2 + I_2 = 1.0\text{ A}$

So I_2 = 0.33 A, and I_1 = 0.67 A.

Example of loop/junction rules



Now, calculate ε of the battery.

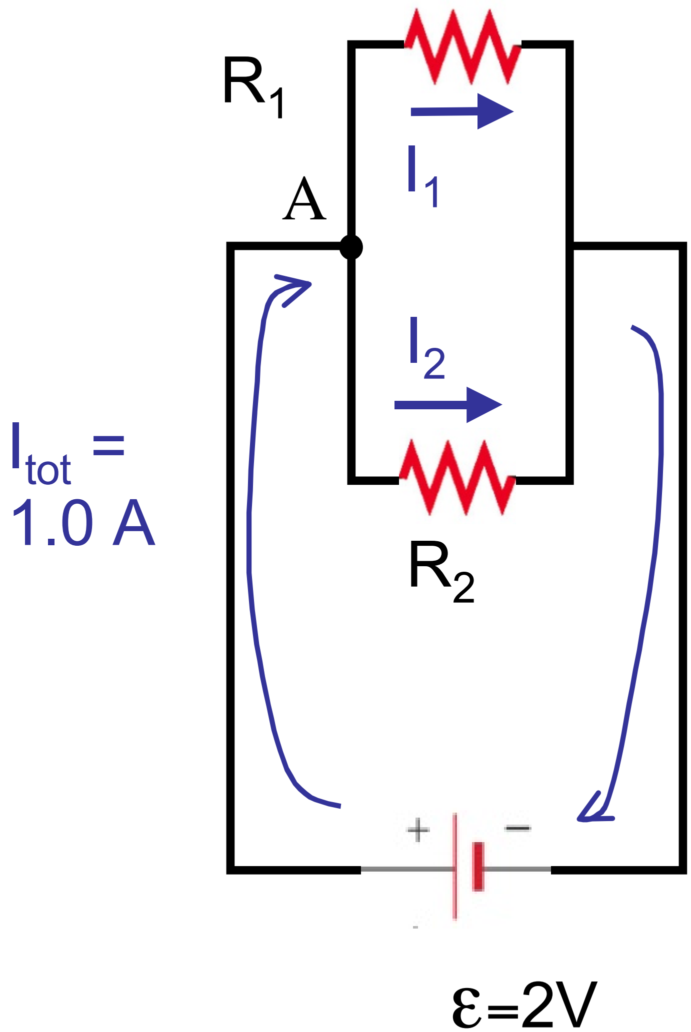
$1/R_{eq} = 1/(3\Omega) + 1/(6\Omega) = 1/(2\Omega)$

$R_{eq} = 2\Omega$

Loop rule for simplified circuit:

$\varepsilon = I_{tot} R_{eq} = 1.0\text{ A } 2\Omega = 2.0\text{ V}$

Example of loop/junction rules

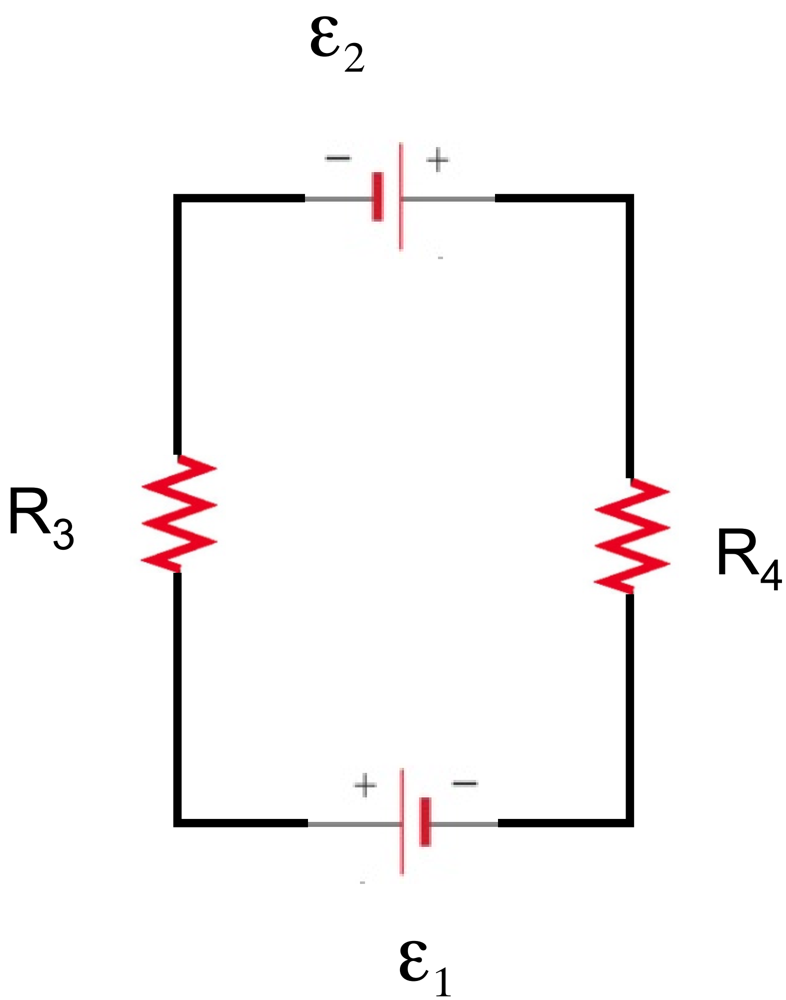


Confirm that the amount of the voltage drop across each resistor is 2V:

$\Delta V_1 = I_1 R_1 = (0.67A)(3\Omega) = 2V$

$\Delta V_2 = I_2 R_2 = (0.33A)(6\Omega) = 2V.$

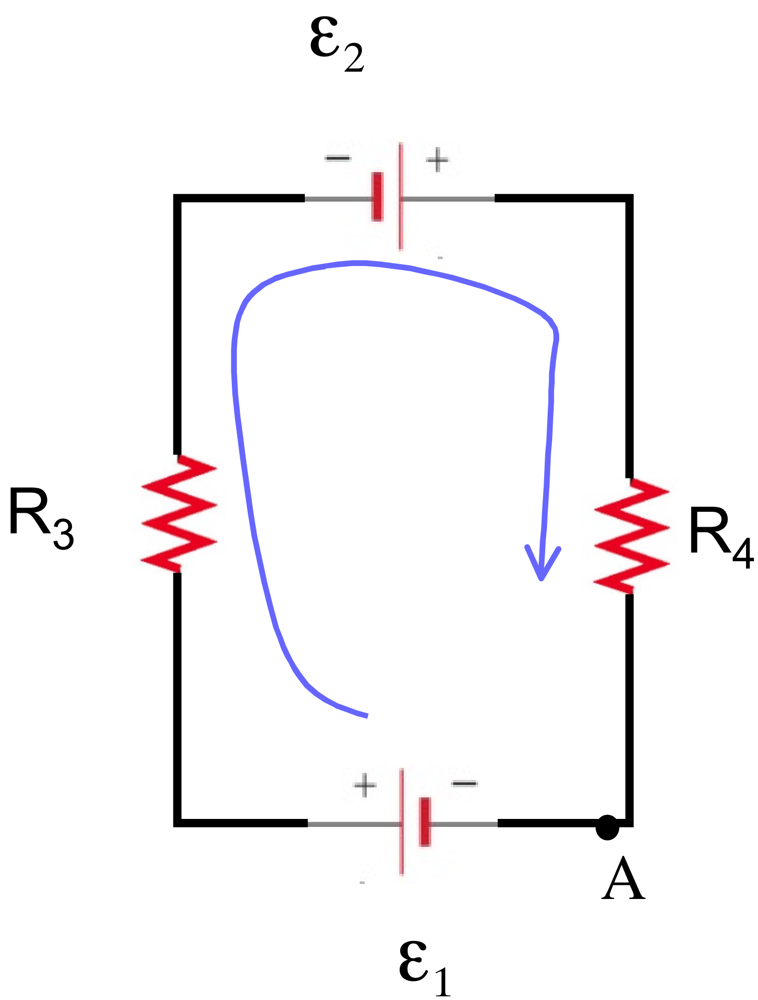
Loop rule, cont'd



Another example:

In which direction will the current flow?

Loop rule, cont'd



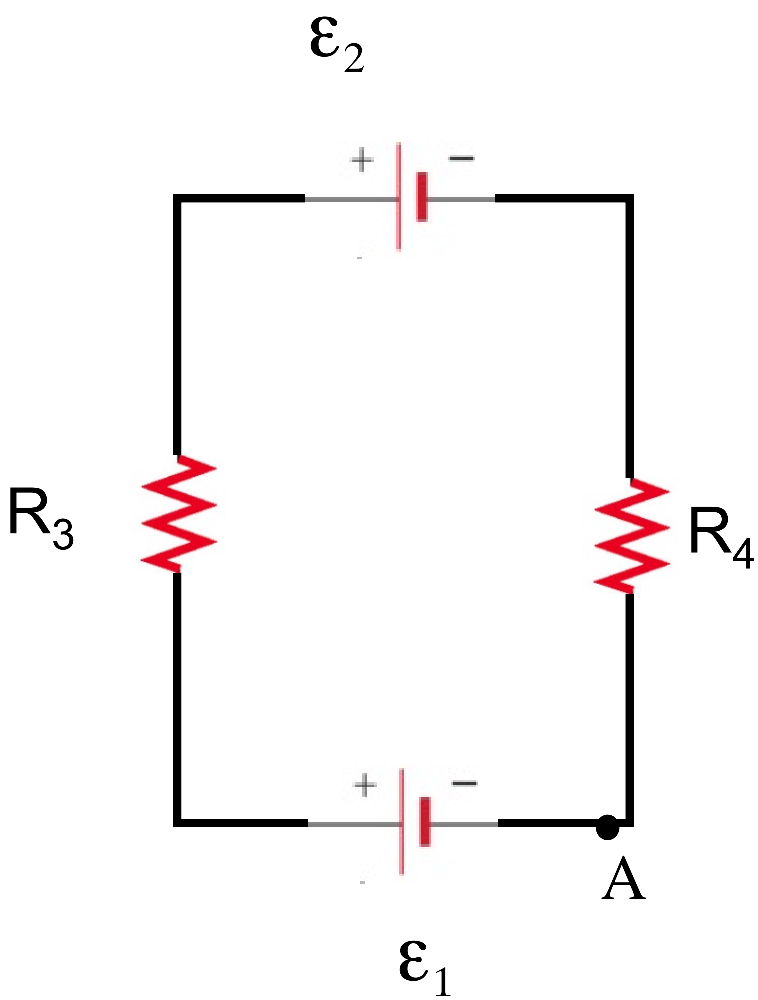
Starting at point A, and going with the current:

$+\varepsilon_1 - IR_3 + \varepsilon_2 - IR_4 = 0$

$+\varepsilon_1 + \varepsilon_2 - IR_4 - IR_3 = 0$

$+\varepsilon_1 + \varepsilon_2 = IR_4 + IR_3$

Another example:



But watch the direction of EMF in batteries:

Starting at point A, and going with the current:

$$+\varepsilon_1 - IR_3 - \varepsilon_2 - IR_4 = 0$$

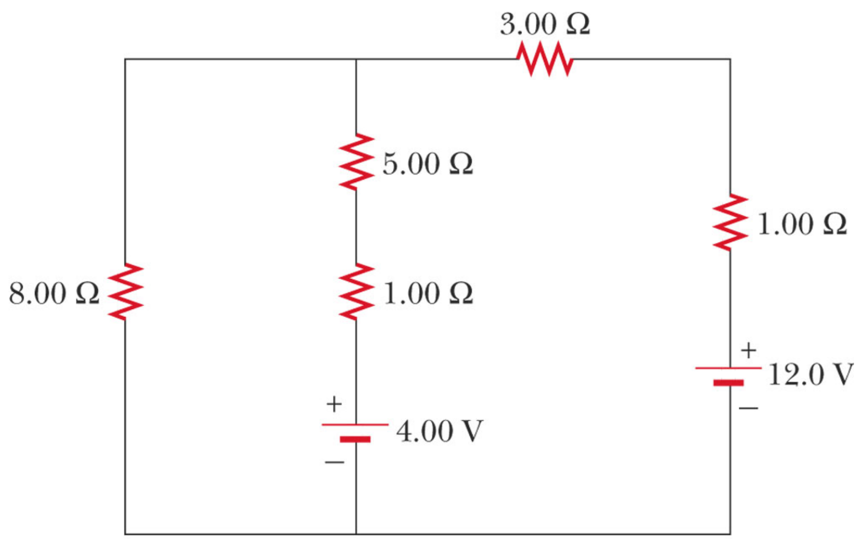
$$+\varepsilon_1 - \varepsilon_2 - IR_4 - IR_3 = 0$$

$$+\varepsilon_1 - \varepsilon_2 = IR_4 + IR_3$$

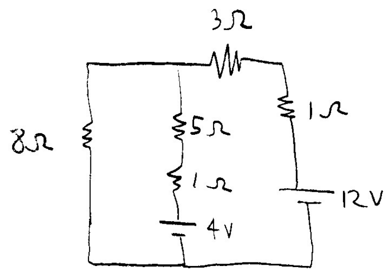
How to use Kirchhoff's Rules

- Draw the circuit diagram and assign labels and symbols to all known and unknown quantities
- Assign directions to currents.
- Apply the junction rule to any junction in the circuit
- Apply the loop rule to as many loops as are needed to solve for the unknowns
- Solve the equations simultaneously for the unknown quantities
- <u>Check your answers -- substitute them back into the original equations!</u>

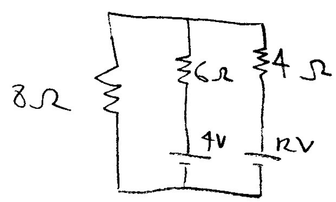
Example for Kirchhoff's Rules: #21.35



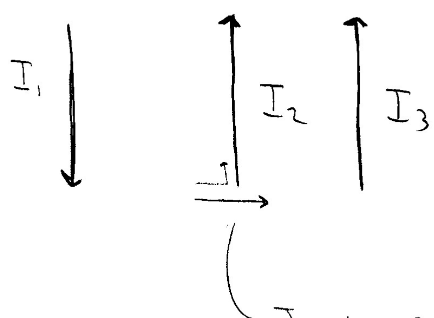
21.35



First, we can use the Series Law and redraw the circuit:



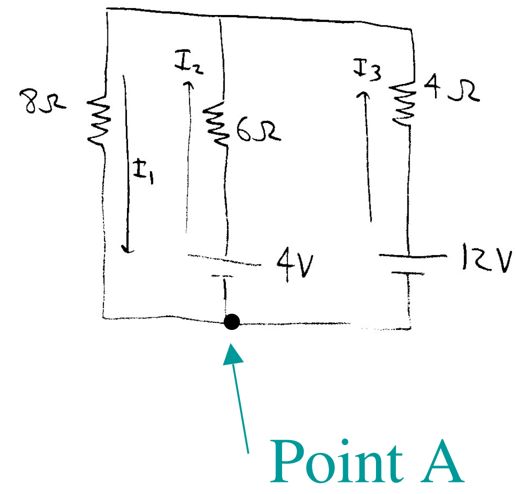
Next, we assign directions to currents. Let's guess that the currents flow as follows:



Junction Rule $I_1 = I_2 + I_3$ (#1)

Apply Loop rule, left-hand loop, going counter-clockwise from point A:

$$+4V - (6\Omega)I_2 - (8\Omega)I_1 = 0 \quad \#2$$



Loop Rule, biggest loop, going counter-clockwise from point A

$$+12V - (4\Omega)I_3 - (8\Omega)I_1 = 0 \quad \#3$$

Now we have three equations and three unknowns (I_1, I_2, and I_3)

Alternate loop rule: Right-hand loop, going counter-clockwise from point A:

$$+12V - (4\Omega)I_3 + (6\Omega)I_2 - 4V = 0$$

Substitute ⟨#1⟩ into ⟨#2⟩ and ⟨#3⟩

$$+4V - (6\Omega)I_2 - 8\Omega(I_2 + I_3) = 0$$ ⟨#5⟩

$$+12V - (4\Omega)I_3 - 8\Omega(I_2 + I_3) = 0$$ ⟨#6⟩

Now we have two equations and two unknowns (I_2 and I_3)

Solve for I_3 in terms of I_2, using ⟨#5⟩:

$$+4V - (6\Omega)I_2 - (8\Omega)I_2 - (8\Omega)I_3 = 0$$

$$4V - (14\Omega)I_2 = (8\Omega)I_3$$] Units are volts

$$\frac{4V}{8\Omega} - \left(\frac{14\Omega}{8\Omega}\right)I_2 = I_3$$] Units are now Amps

$$0.5A - 1.75 I_2 = I_3$$ ⟨#5A⟩

Rearrange #6: $12V - (4\Omega)I_3 - (8\Omega)I_2 - (8\Omega)I_3 = 0$

$$12V - (8\Omega)I_2 - (12\Omega)I_3 = 0$$

$$3V - (2\Omega)I_2 - (3\Omega)I_3 = 0 \quad \#6A$$

Substitute expression for I_3 (#5A) into #6A:

$$3V - (2\Omega)I_2 - 3\Omega(0.5A - 1.75 I_2) = 0$$

$$3V - (2\Omega)I_2 - 1.5V + (5.25\Omega)I_2 = 0$$

$$1.5V + (3.25\Omega)I_2 = 0$$

$$I_2 = -0.462 A$$

Our initial guess for the direction of I_2 was incorrect!

Substitute value for I_2 back into #5A.

$$0.5A - 1.75\, I_2 = I_3$$

$$0.5A - 1.75\,(-0.462A) = I_3$$

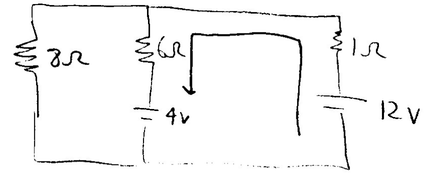
$$0.5A + 0.81A = I_3$$

$$I_3 = +1.31\ A$$

Finally, substitute back into #1 (Junction rule): $I_1 = I_2 + I_3 = -0.462A + 1.31\ A = 0.85A$

The initial guess of direction of currents was motivated by the (reasonable) assumption that both batteries would force (positive-)current to travel up in both the middle and right branches, forcing current to travel down in the left branch

But in the left branch, there is the 8Ω resistor (the largest-resistance resistor). Because of the strong opposition to current in the left branch, current is forced downward in the middle branch — "over-riding" the upward-directed EMF supplied by the 4-V battery.

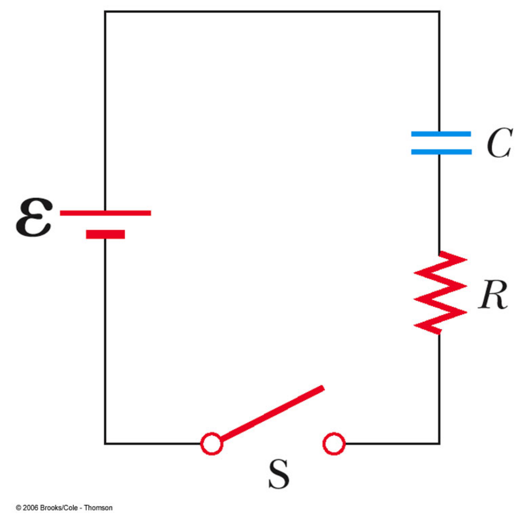


21.9 RC Circuits

Introduction to time-dependent currents and voltages.

Applications: timing circuits, clocks, computers, charging + discharging capacitors

An RC circuit

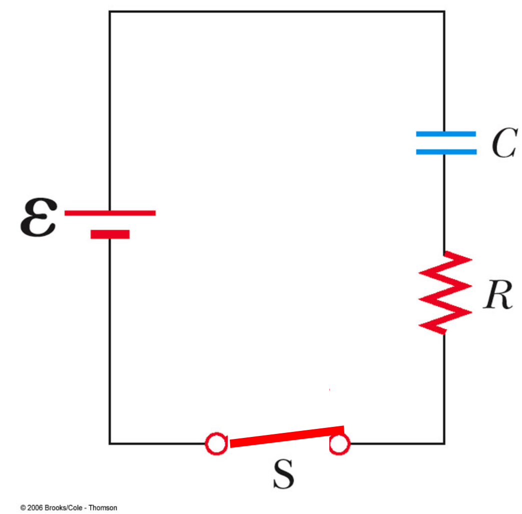


Battery, Capacitor, Resistor and switch connected in series.

Initially, the capacitor is uncharged. We're about to close the switch and allow the capacitor to start charging…

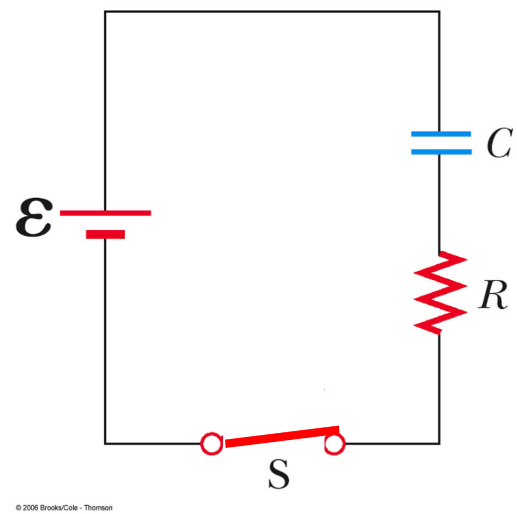
RC circuit: charging

At time t=0, close Switch



An RC circuit

At time t=0, close switch



Initially (at t=0), $Q = 0$.
ΔV_C (at t=0) = $Q(t=0) / C = 0$

Loop Rule: $\Sigma\Delta V=0 = +\varepsilon - \Delta V_R - \Delta V_C$

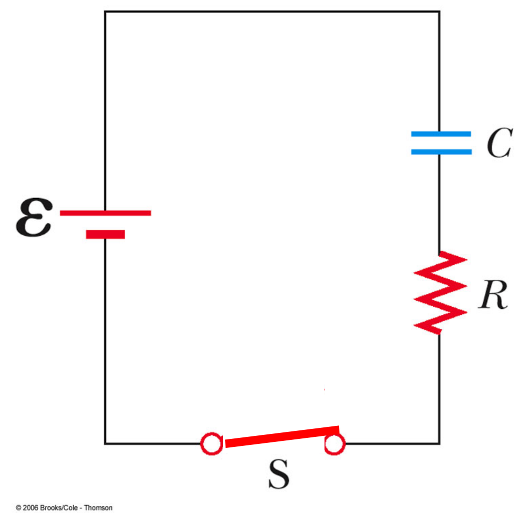
$+\varepsilon - IR - \Delta V_C = 0$

So when charge = 0 (which occurs at t=0), $\varepsilon = IR$ and so the current I jumps immediately up to ε/R

(at this instant, capacitor has no effect)

RC circuit: charging

Immediately after time t=0: Current starts to flow. Charge starts to accumulate on Capacitor (at a rate $I=dQ/dt$).



As Q increases over time, $\Delta V_C = Q/C$ also increases.

But remember $\Delta V_C + \Delta V_R = \varepsilon$.

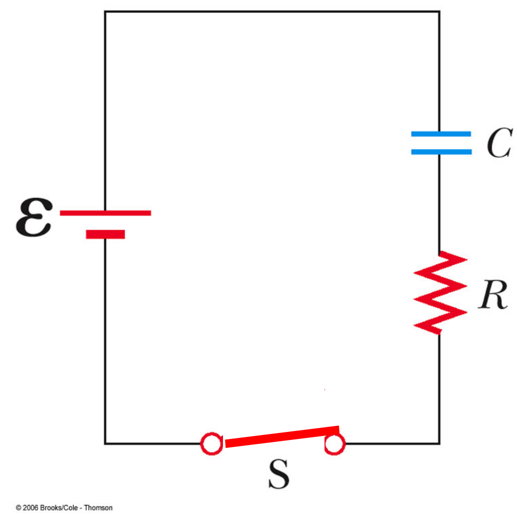
So ΔV_R is decreasing over time.

And I through the resistor = $\Delta V_R/R$ is also decreasing.

RC circuit: charging

After a very long time:

Charge accumulates until Q reaches its maximum:



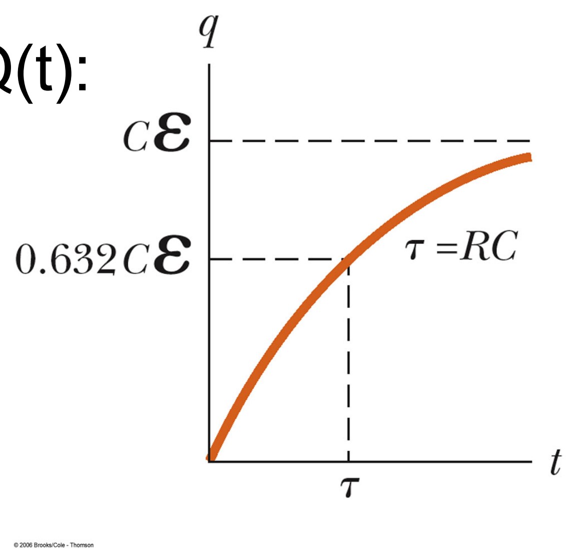
ΔV_C goes to ε. Total Q on the capacitor goes to Cε.

$+\varepsilon = \Delta V_C + \Delta V_R$

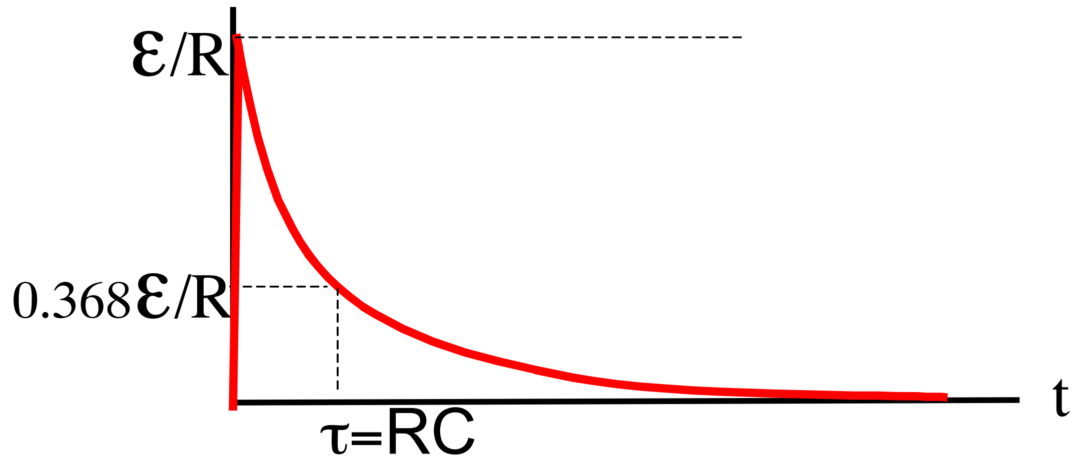
ΔV_R goes to zero. And I goes to zero.

RC circuit: charging

Q(t):



I(t):



	t=0		$t \to \infty$
ΔV_C	0	$\varepsilon(1-e^{-(t/\tau)})$	ε
Q	0	$C\varepsilon(1-e^{-(t/\tau)})$	$C\varepsilon$
ΔV_R	ε	$\varepsilon(e^{-(t/\tau)})$	0
I	ε/R	$(\varepsilon/R)(e^{-(t/\tau)})$	0

Exponential decay

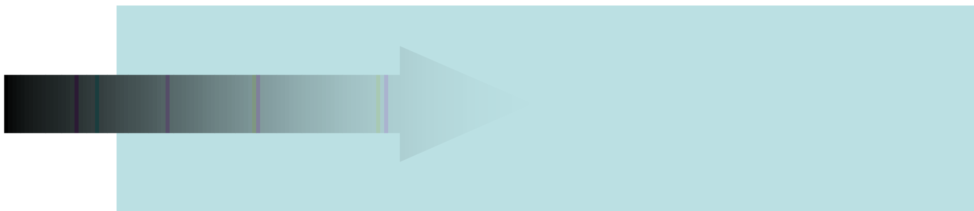
Rate of decay is proportional to amount of species.

Other applications: Nuclear decay, some chemical reactions

Atmospheric pressure decreases exponentially with height

If an object of one temperature is exposed to a medium of another temperature, then the temperature difference between the object and the medium undergoes exponential decay.

Absorption of electromagnetic radiation by a medium (intensity decreases exponentially with distance into medium)



Time constant τ = RC

RC is called the time constant: it's a measure of how fast the capacitor is charged up.

It has units of time:
RC = (V/I)(q/V) = q/I = q / (q/t) = t

At t = RC, Q(t) and $\Delta V_C(t)$ go to 1 – 1/e = 0.63 of the final values

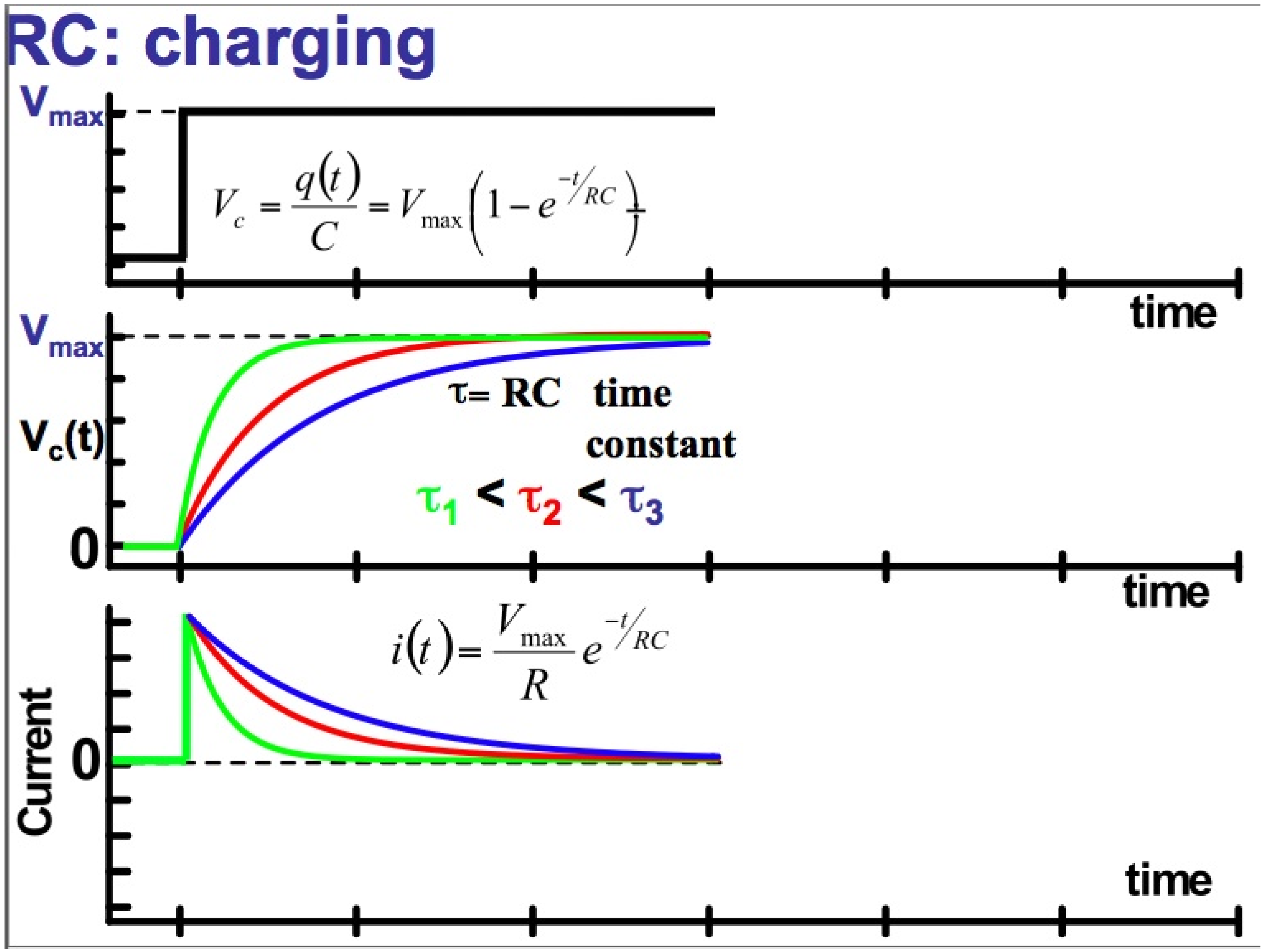
At t= RC, I(t) and $\Delta V_R(t)$ go to 1/e of the initial values

Time constant τ = RC

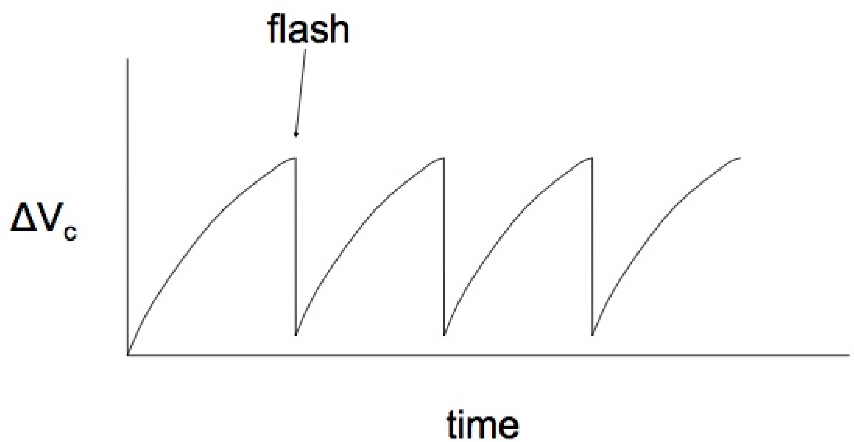
Think about why increasing R and/or C would increase the time to charge up the capacitor:

When charging up: τ will increase with C because the capacitor can store more charge.
Increases with R because the flow of current is lower.

Time constant τ = RC



You want to make a flashing circuit that charges a capacitor through a resistor up to a voltage at which a neon bulb discharges once every 5.0 sec. If you have a 10 microfarad capacitor what resistor do you need?



Solution: Have the flash point be equal to 0.63 $\Delta V_{C,max}$

$\tau = RC \rightarrow R = \tau/C = 5s/10^{-5}F = 5 \times 10^5$ Ohms

This is a very big resistance, but 5 seconds is pretty long in "circuit" time