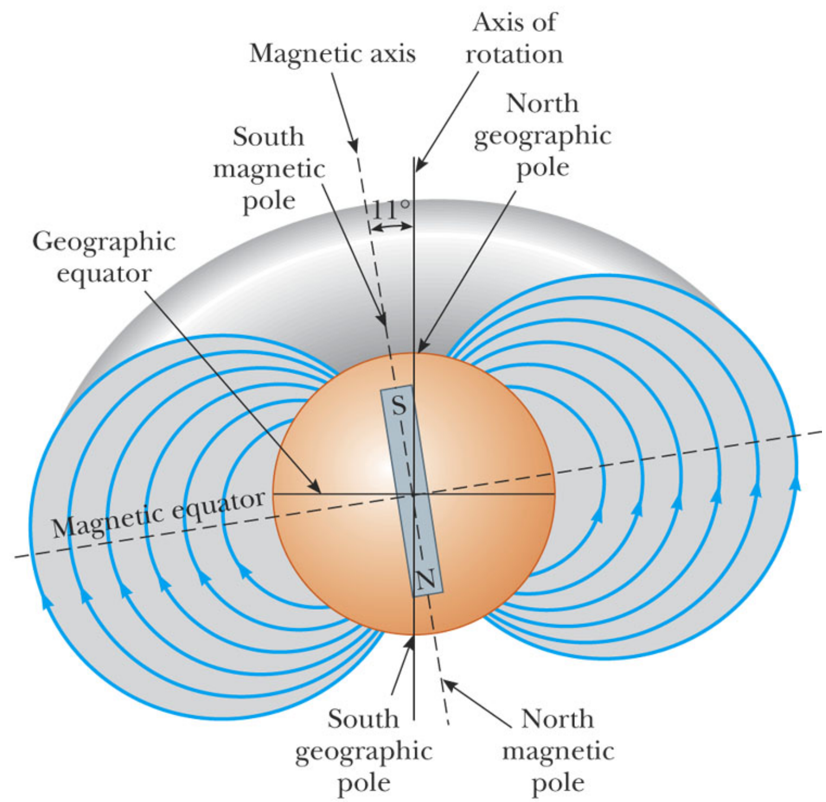
The N pole of a magnet got its name because it points roughly towards Earth's *north* pole. It points towards NE Canada, since that's the Earth's South magnetic pole.

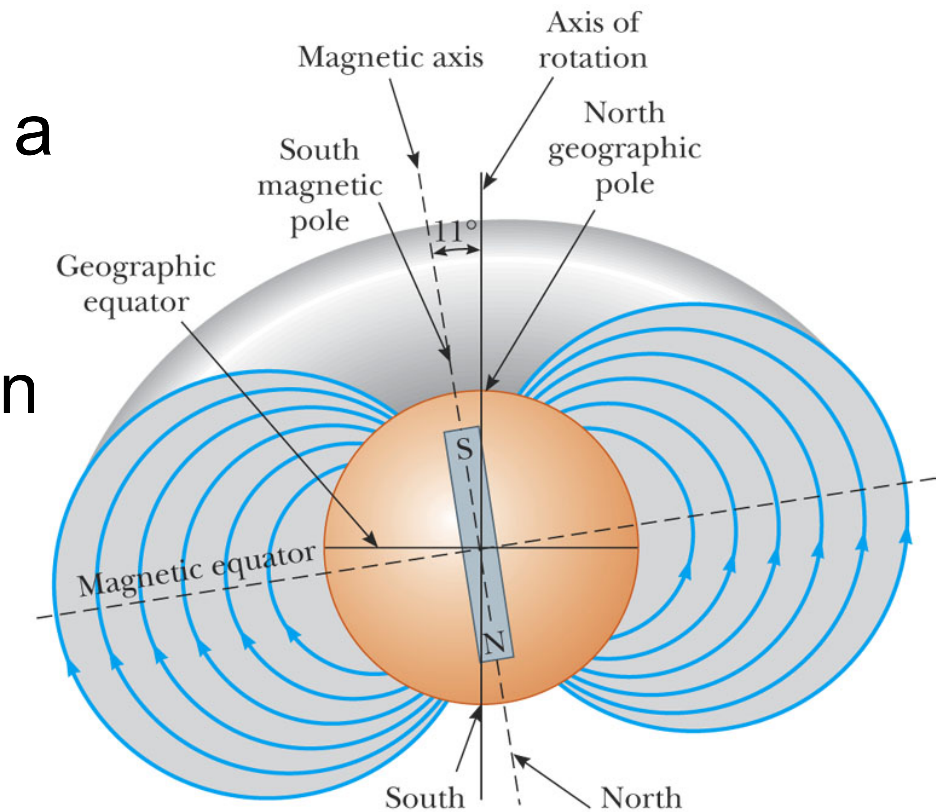
Earth’s B-field is actually upside down, and the dipole axis is offset slightly (~11º) from Earth's rotation axis.



Earth's magnetic field has a vertical component at the surface:
Points upwards in southern hemisphere.
Points downwards in northern hemisphere.

Parallel to surface at magnetic equator.

Source: Current/ convection in hot Fe liquid core



Units of B-fields

Tesla [SI] and Gauss [cgs]

1 Tesla = 10^4 Gauss.

$$T = \frac{Wb}{m^2} = \frac{N}{C \cdot (m/s)} = \frac{N}{A \cdot m}$$

Typical B-field strengths:

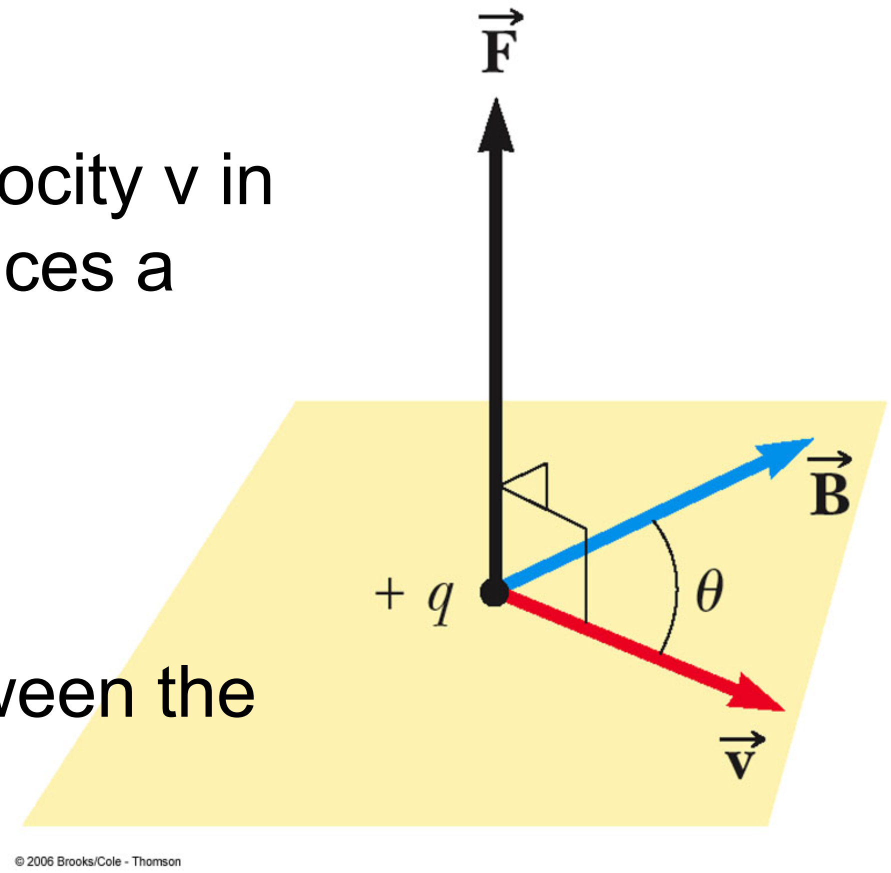
Earth's B-field at surface: 0.5×10^{-4} T = 0.5 G
Refrigerator magnet: ~0.005 T
Bar magnets: 0.01 T
MRI machine: 1-5 T
Laboratory magnet: 5 T
Superconducting magnet: 20-30 T

Magnetic field produces a force on a moving charge

A charge q moving with velocity v in a magnetic field B experiences a force $\vec{F}$ with magnitude:

$F = q\ v\ B\ \sin(\theta)$

θ is the smallest angle between the vectors $\vec{v}$ & $\vec{B}$



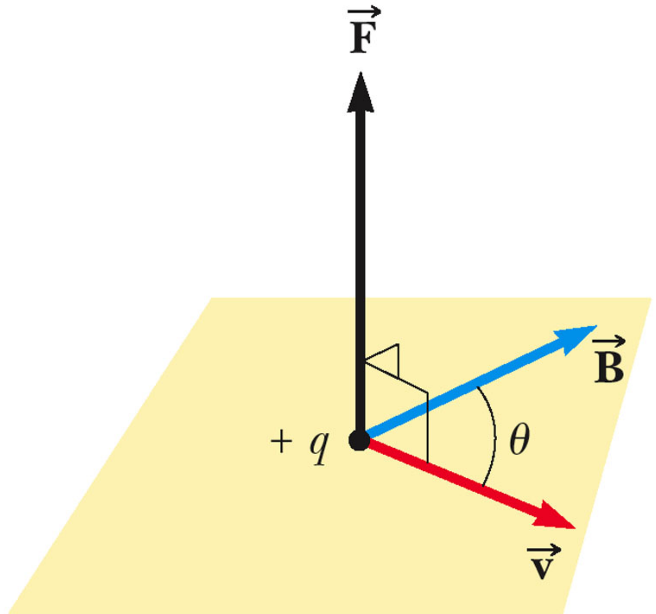
$\vec{F}$ is perpendicular to BOTH $\vec{v}$ and $\vec{B}$

$F = qvB\sin(\theta)$

Magnitude of Force depends on both v and B:

If v=0 or B=0, then F=0

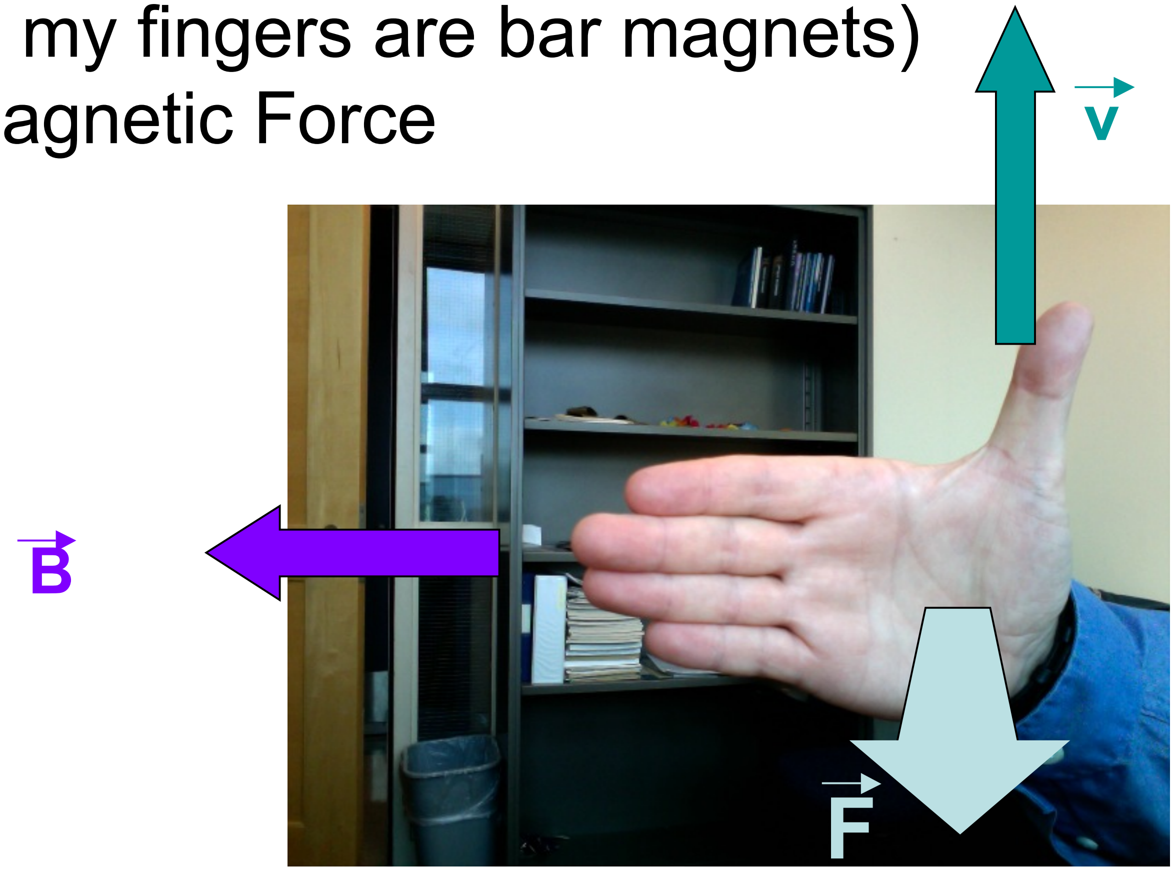
Force depends on angle θ:
If $\vec{B} \parallel \vec{v}$, F = 0
Force is max. when $\vec{v}$ and $\vec{B}$ are perpendicular



Because $\vec{F}$ is perp. to $\vec{v}$, magnetic forces cannot change a particle's speed, just its direction of motion.

Right-hand rule (my version)

Thumb = v (e.g., hitchhiking)
Fingers = B (like my fingers are bar magnets)
Out of Palm = Magnetic Force



Example: A proton is moving at 1×10^4 m/s from left to right in a magnetic field of 0.4 T that's in the upward direction (in the plane of the page). Find the magnitude of the force vector. Find the direction. What would the force be if the particle was an e^-?

$F = B * q * v * \sin\theta = 0.4T * 1.6 \times 10^{-19} C * 10^4 m/s * \sin 90^\circ = 6.4 \times 10^{-16} N$

Direction of force: out of the page towards you.

For e^-: Magnitude of force is same, but in the opposite direction. Calculate direction of force same as for proton, then reverse the direction!

F_B vs F_E

F_E is always parallel or anti-parallel to E-field;

F_B is always perpendicular to B-field

F_E acts on a particle independently of the particle's veloc;

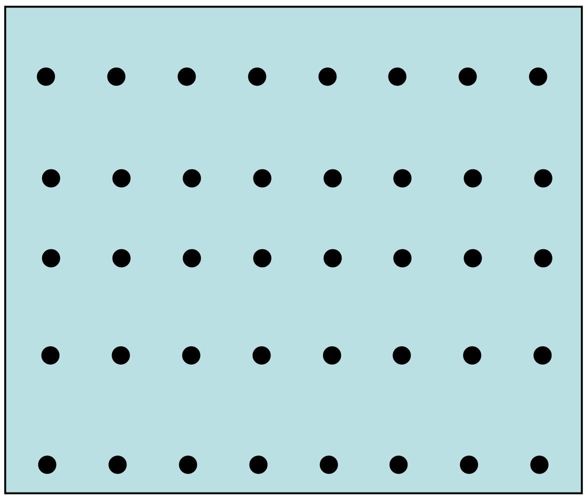
F_B depends on velocity

F_E does work in displacing a charged particle

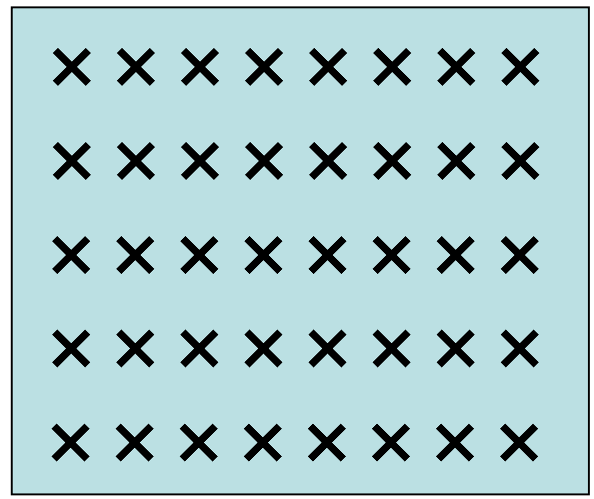
F_B does no work (particle's kinetic energy unchanged)

B-field notation

out of page:



into page:



think of the points/tails of arrows

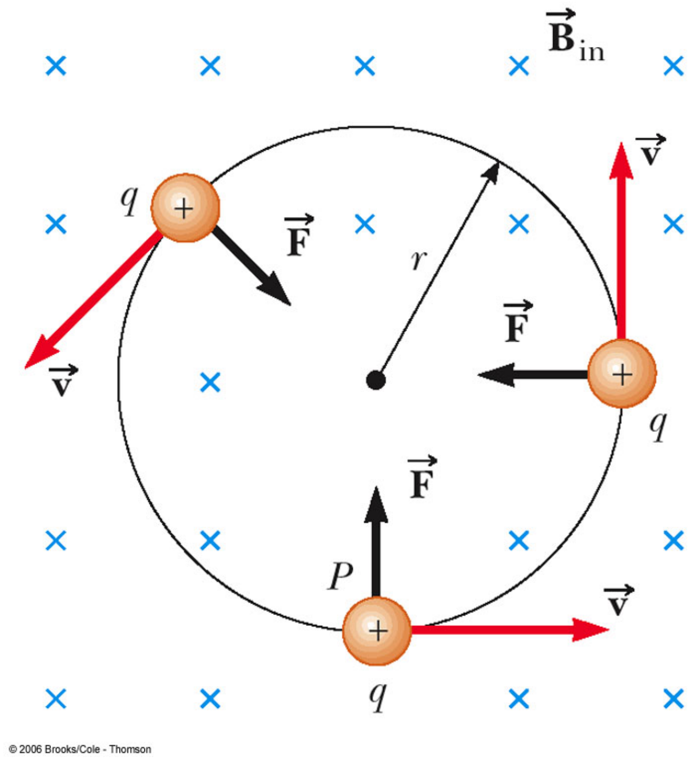
22.3: Motion of a charged particle in a magnetic field

$\vec{F}$ is in a plane perpendicular to $\vec{B}$.

Particle's path remains in plane perpendicular to $\vec{B}$.

$$F = qvB = \frac{mv^2}{r}$$

$$r = \frac{mv}{qB}$$



A proton with velocity $v=1\times10^6$ m/s is in a uniform B-field of 0.2 T. Find r:

$r = mV / qB =$
$(1.67\times10^{-27}\ kg * 1\times10^6\ m/s) / (1.6\times10^{-19}\ C * 0.2\ T) =$
$0.052\ m = 5.2\ cm$

Cyclotron Frequency

Angular Speed $\omega = \frac{v}{r} = \frac{qB}{m}$

a.k.a. cyclotron frequency. Units = radians/sec.

Freq f in cycles/sec = $v / (2\pi r) = \omega / 2\pi$

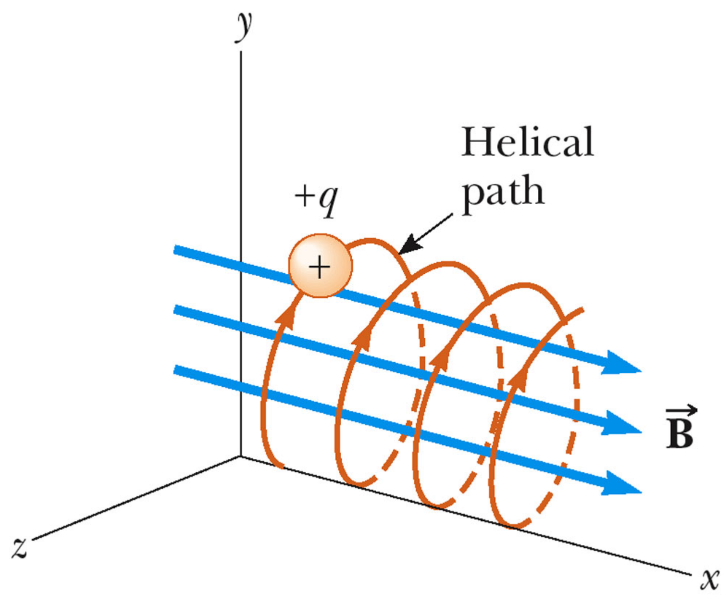
Time to complete 1 cycle

$$T = \frac{2\pi r}{v} = \frac{2\pi}{\omega} = \frac{2\pi m}{qB}$$

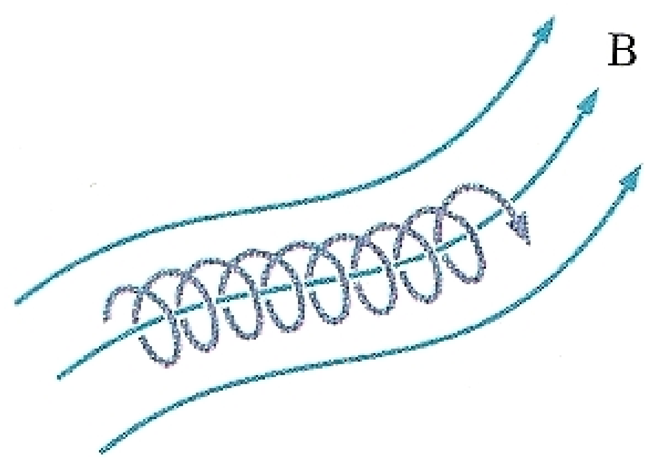
When $\vec{B}$ and $\vec{v}$ are not exactly perpendicular:

Motion || to B-field is unaffected.
Motion perp. to B-field is circular.

So a charge will follow a HELICAL path around field lines (helix axis || to B-field lines).



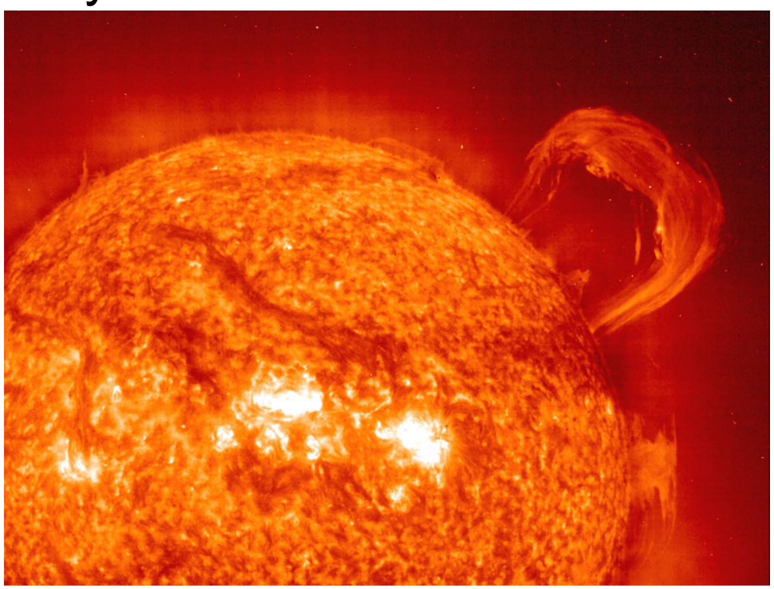
Projection of motion onto the y-z plane is still a circle. Cyclotron frequency equations refer only to motion in the y-z plane.



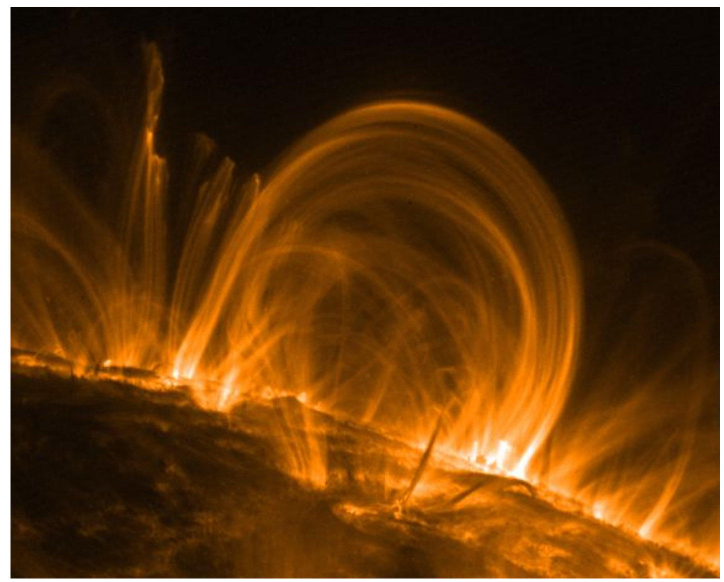
Component of motion along x-axis unaffected (accel along x-axis=0)

Example:
Solar Prominences: Charged particles in sun's corona move in helices along B-field lines, emit light & map out those B-field lines

Solar prominence viewed by SOHO:



Solar prominence viewed by TRACE:

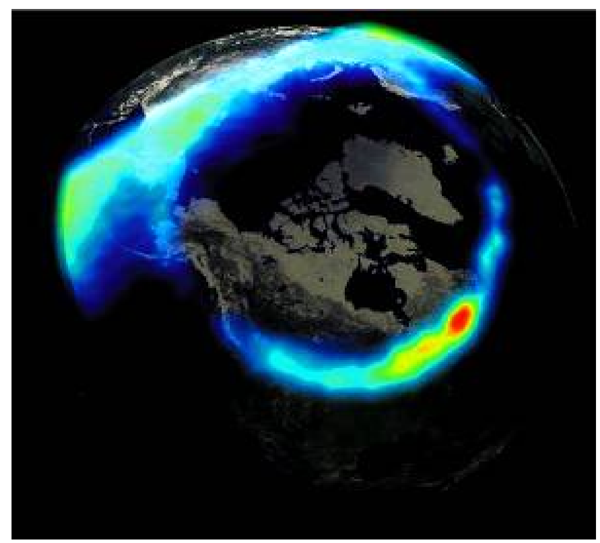


Aurora

Charged particles from solar wind or solar flares get caught in B-field lines in Earth's B-field, funneled to the poles

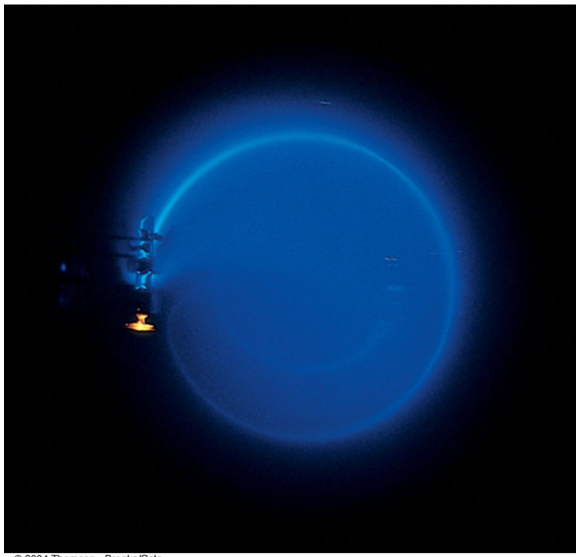
Appears as circle surrounding magnetic pole (NASA's *Polar* Sat.)

apod.nasa.gov



gsfc.nasa.gov

Example 22.3



Measure B from deflection, velocity of electrons

22.4 Applications involving charged particles moving in a B-field

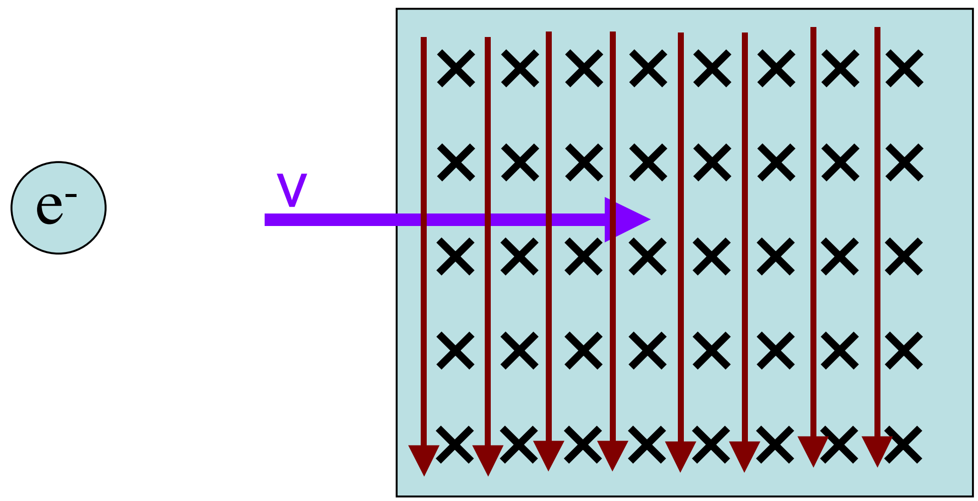
Velocity Selector

Perpendicular E and B fields can be used to select charged particles having a specific velocity

$F_B = evB$ (downward for e^-)

$F_E = eE$ (upward for e^-)

When $F_B = F_E$, forces cancel: $evB = eE$



E: downward

B: into page

$\underline{v = E/B}$: electron with this velocity will be undeflected

Lorentz Force

- In many applications, the charged particle will move in the presence of both magnetic and electric fields
- In that case, the total force is the sum of the forces due to the individual fields
- In general: $\vec{\mathbf{F}} = q\vec{\mathbf{E}} + q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$
  – This force is called the Lorenz force
  – It is the vector sum of the electric force and the magnetic force

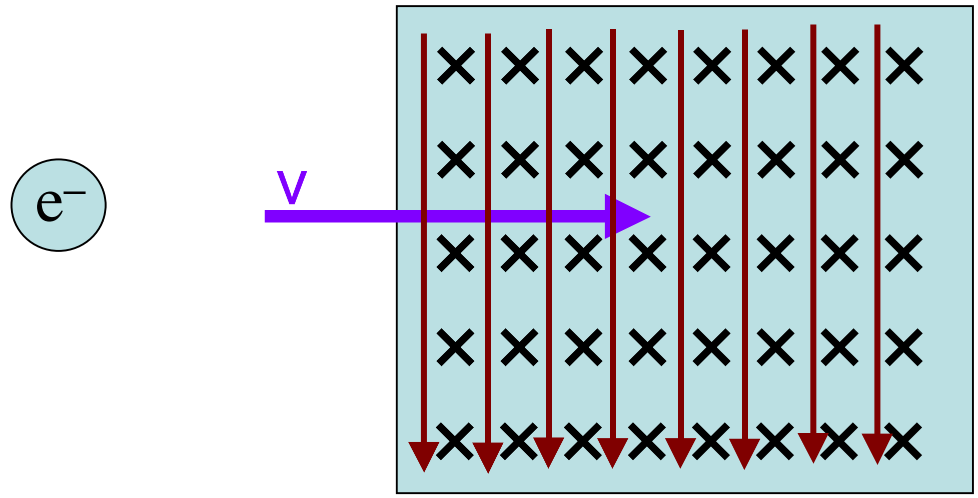
Velocity Selector

Example: A velocity selector has perpendicular electric and magnetic field of E= 1000 V/m and B= 0.3 T. Find the velocity of the electrons that pass through undeflected. What would happen to faster electrons? Slower?

$v = E/B =$

$1000 V/m / 0.3 T =$

3.3×10^4 m/s



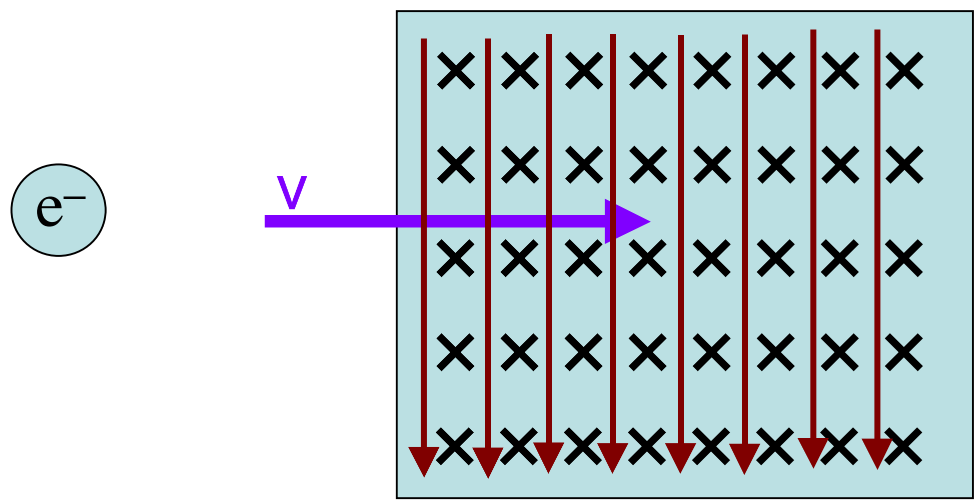
E: downward

B: into page

Velocity Selector

For electrons traveling faster than this velocity, the magnitude of the magnetic force is larger than that of the electric force (because F_B is proportional to v); $F_E < F_B$. Since F_B points downward, the too-fast electrons will be directed downward.

For electrons traveling too slowly, the magnetic force is insufficient to counter balance the upward electric force; $F_E > F_B$. These electrons travel upwards and do not make it out the right side.

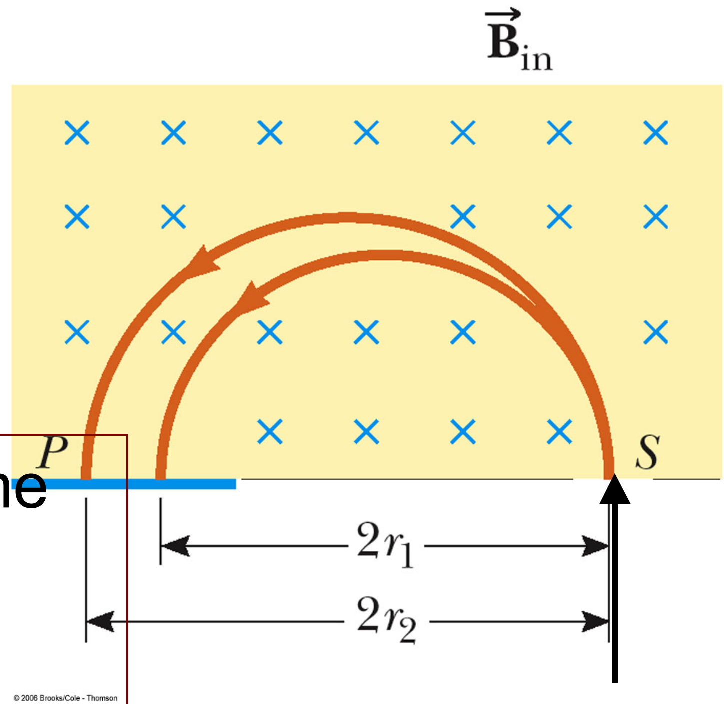


E: downward

B: into page

Mass Spectrometers

$$r = \frac{mv}{qB}$$



3. Molecular ions injected into B-field at velocity v

4. Ions with the same charge become separated by mass onto photographic plate: compute charge/mass ratio

1. A sample is ionized (impacting with a e⁻ beam).
2. Positive ions are accelerated by an E-field through veloc selector

Example: A mass spectrometer has a velocity selector at its inlet such that only q=+1 ions with $v = 1\times10^5$ m/s are permitted inside the mass spectrometer, where the B-field is 0.2 T. A mixture of gas containing CO_2^+ is injected. But some of the CO_2 contains Carbon-14. What radii are $^{12}CO_2^+$ and $^{14}CO_2^+$ rotated through, and what is their separation on the photographic plate?

Reminder: mass is for whole molecule.
F_B works on the singular positive charge only.

Assume $m_p = m_n$ for simplicity; ignore masses of electrons sicne they're 1800x less massive than protons/neutrons

Mass ($^{12}CO_2^+$) = $(12+16+16)*1.67\times10^{-27}$kg = $44 * 1.67\times10^{-27}$kg = 73.5×10^{-27} kg
Mass ($^{14}CO_2^+$) = $(14+16+16)*1.67\times10^{-27}$kg = $46 * 1.67\times10^{-27}$kg = 76.8×10^{-27} kg

Reminder: mass is for whole molecule.
F_B works on the singular positive charge only.

$r(^{12}CO_2^+) = mv/qB = (73.5\times10^{-27}$ kg * 10^5 m/s) / (1.6×10^{-19}C * 0.2T) = 23.0 cm

$r(^{14}CO_2^+) = mv/qB = (76.8\times10^{-27}$ kg * 10^5 m/s) / (1.6×10^{-19}C * 0.2T) = 24.0 cm

The diameters of the circles traced out will be 46.0 and 48.0 cm, respectively.

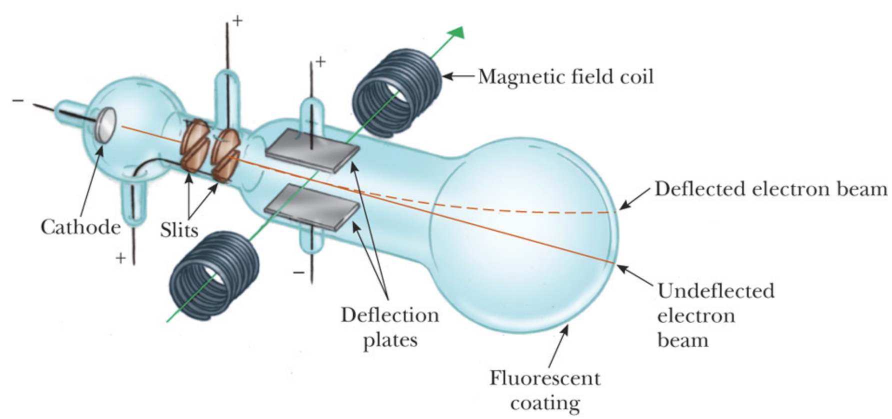
The separation on the photographic plate will be 2.0 cm.

Reminder: mass is for whole molecule.
F_B works on the singular positive charge only.

Charge/Mass ratio of particles

J.J. Thomson, 1897

e-'s accelerated in cathode, pass through regions of perp. E & B fields. Deflection measured.



(a)

The Cyclotron

Used to accelerate charged particles to very high speeds, bombard other particles, induce nuclear reactions