

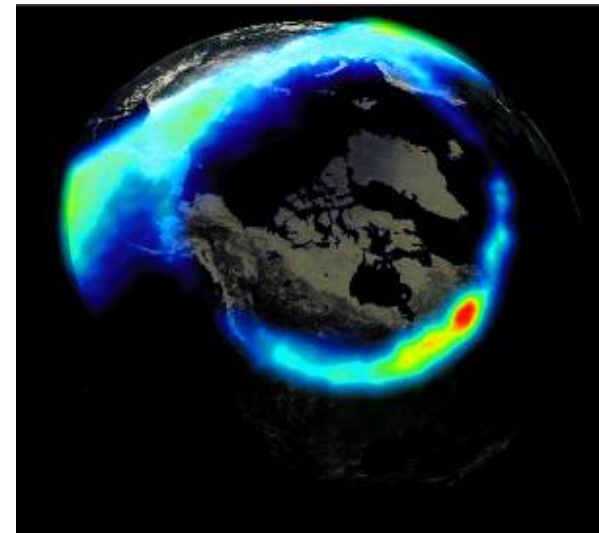
# Aurora

Charged particles from solar wind or solar flares get caught in B-field lines in Earth's B-field, funneled to the poles

Appears as circle surrounding magnetic pole  
(NASA's *Polar Sat.*)



[apod.nasa.gov](http://apod.nasa.gov)



[gsfc.nasa.gov](http://gsfc.nasa.gov)

Aurora-related youtube vids (copy/paste into your browser):

[http://www.youtube.com/watch?v=\\_Ehb38zzpgo](http://www.youtube.com/watch?v=_Ehb38zzpgo)

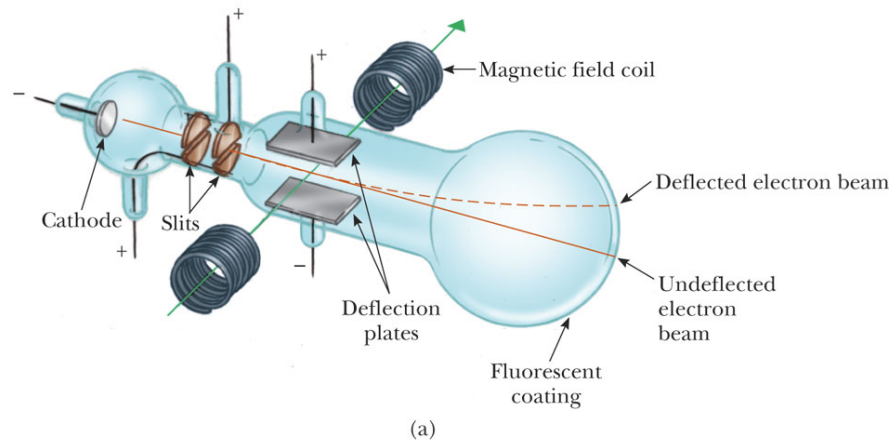
<http://www.youtube.com/watch?v=DUzT4skIUWc>  
(time-lapse view from the ground; with timestamps)

<http://www.youtube.com/watch?v=7AmyfuJDMIY>  
(featuring Saturn's auroras, observed with Cassini;  
however, there's an error in the animation -- it shows  
aurora around Earth's geographic, not magnetic, poles;  
warning: obnoxious ads)

# Charge/Mass ratio of particles

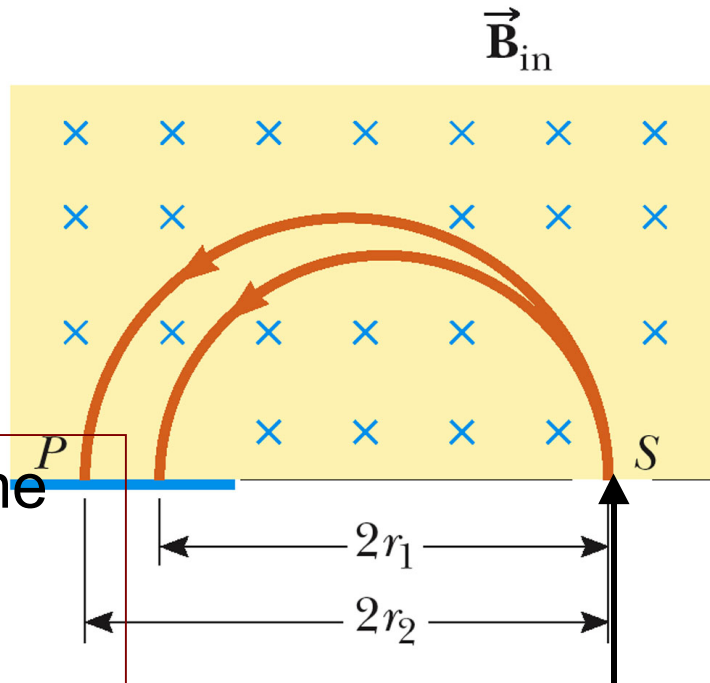
J.J. Thomson, 1897

e<sup>-</sup>'s accelerated in cathode, pass through regions of perp. E & B fields. Deflection measured.



# Mass Spectrometers

$$r = \frac{mv}{qB}$$



3. Molecular ions injected into B-field at velocity  $v$

4. Ions with the same charge become separated by mass onto photographic plate: compute charge/mass ratio

1. A sample is ionized (impacting with a  $e^-$  beam).  
2. Positive ions are accelerated by an E-field through veloc selector

Example: A mass spectrometer has a velocity selector at its inlet such that only  $q=+1$  ions with  $v = 1 \times 10^5$  m/s are permitted inside the mass spectrometer, where the B-field is 0.2 T. A mixture of gas containing  $\text{CO}_2^+$  is injected. But some of the  $\text{CO}_2$  contains Carbon-14. What radii are  $^{12}\text{CO}_2^+$  and  $^{14}\text{CO}_2^+$  rotated through, and what is their separation on the photographic plate?

*Reminder: mass is for whole molecule.*

*$F_B$  works on the singular positive charge only.*

Assume  $m_p = m_n$  for simplicity; ignore masses of electrons since they're 1800x less massive than protons/neutrons

$$\text{Mass } (^{12}\text{CO}_2^+) = (12+16+16) \cdot 1.67 \times 10^{-27} \text{kg} = 44 \cdot 1.67 \times 10^{-27} \text{kg} = 73.5 \times 10^{-27} \text{kg}$$

$$\text{Mass } (^{14}\text{CO}_2^+) = (14+16+16) \cdot 1.67 \times 10^{-27} \text{kg} = 46 \cdot 1.67 \times 10^{-27} \text{kg} = 76.8 \times 10^{-27} \text{kg}$$

*Reminder: mass is for whole molecule.*

*$F_B$  works on the singular positive charge only.*

$$r(^{12}\text{CO}_2^+) = mv/qB = (73.5 \times 10^{-27} \text{ kg} * 10^5 \text{ m/s}) / (1.6 \times 10^{-19} \text{ C} * 0.2 \text{ T}) = 23.0 \text{ cm}$$

$$r(^{14}\text{CO}_2^+) = mv/qB = (76.8 \times 10^{-27} \text{ kg} * 10^5 \text{ m/s}) / (1.6 \times 10^{-19} \text{ C} * 0.2 \text{ T}) = 24.0 \text{ cm}$$

The diameters of the circles traced out will be 46.0 and 48.0 cm, respectively.

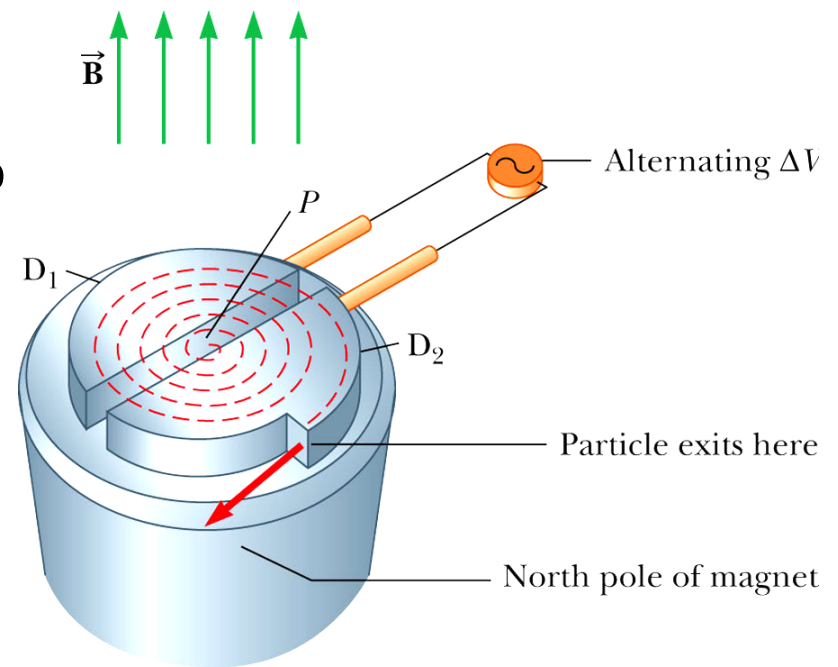
The separation on the photographic plate will be 2.0 cm.

*Reminder: mass is for whole molecule.*

*$F_B$  works on the singular positive charge only.*

# The Cyclotron

Used to accelerate charged particles to very high speeds, bombard other particles, induce nuclear reactions



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## 22.5: Force on a Current-carrying Conductor

Total force = Force on each charge  $\times$  # of charges in the wire

$$\vec{F}_B = (q \vec{v}_{dr} \times \vec{B}) (n A \ell)$$

$$F_B = (q v_{dr} B \sin\theta) (n A \ell) \quad (A \times \ell = \text{vol.}; n = \text{charges/vol})$$

But recall that  $I = \Delta Q / \Delta T$

and  $\Delta Q / \Delta T = (\text{q of each charge}) \times (\text{\# of charges per second})$

$$\begin{aligned} \text{\# of charges per second} &= A n v_{dr} \\ &(\text{m}^2) (\text{m}^{-3}) (\text{m/s}) \end{aligned}$$

$$\text{So } I = n A v_{dr} q$$

$$\vec{F}_B = I \vec{\ell} \times \vec{B}$$

$$F_B = B I \ell \sin(\theta) \quad \text{or} \quad F_B / \ell = B I \sin(\theta)$$

## More general form: Force on a Wire, Arbitrary Shape

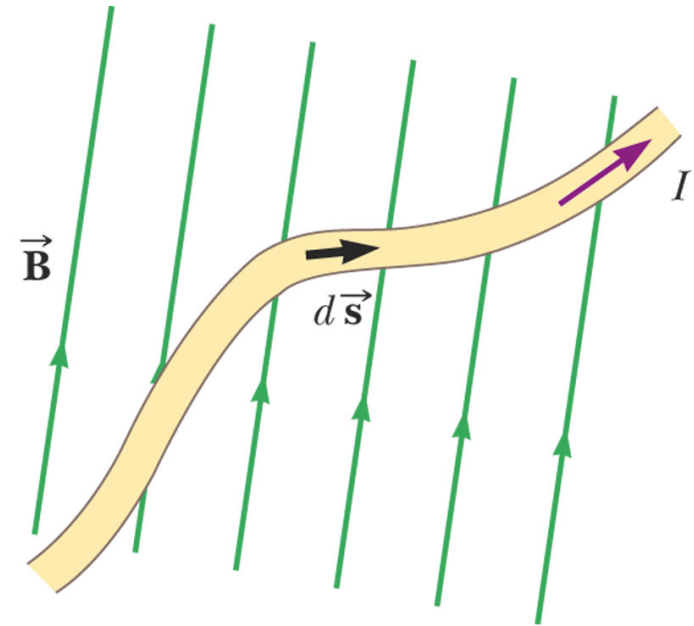
- Consider a small segment of the wire,  
 $d\vec{s}$

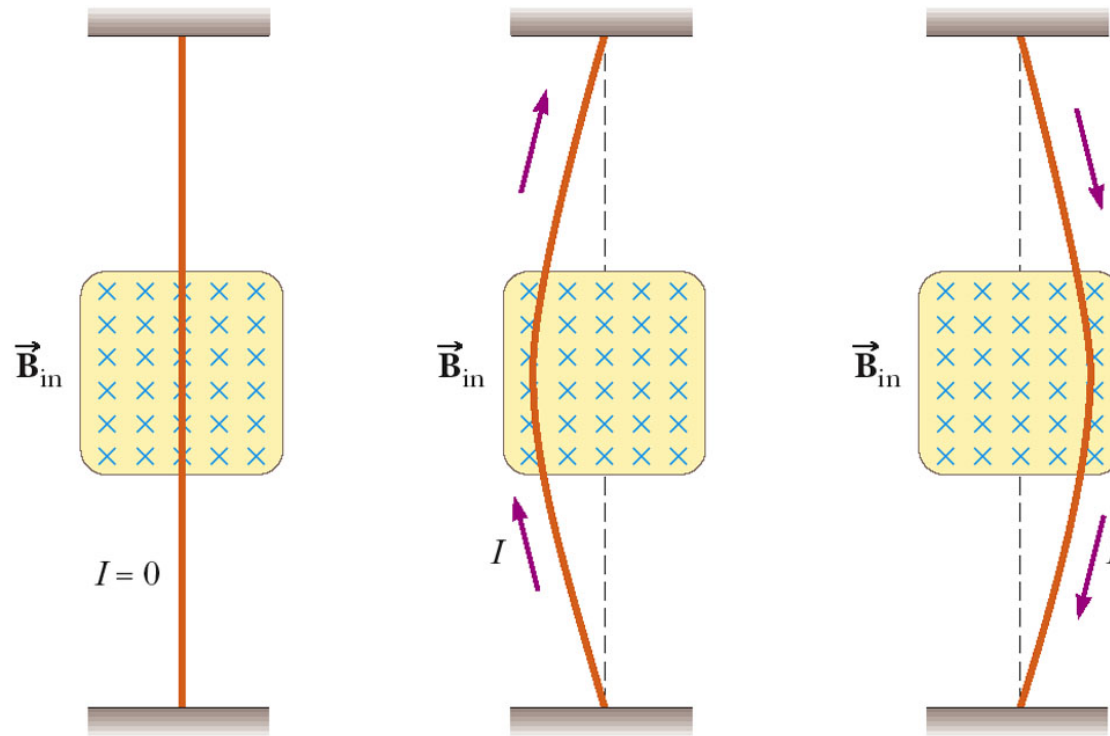
- The force exerted on this segment is

$$d\vec{F}_B = I d\vec{s} \times \vec{B}$$

- The total force is

$$\vec{F}_B = I \int_a^b d\vec{s} \times \vec{B}$$





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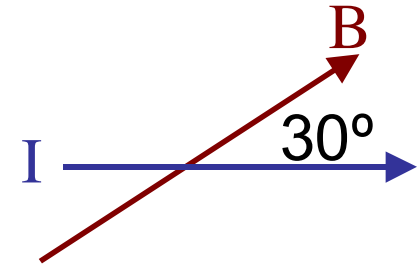
Reminder: Direction of deflection is given by right hand rule:

$I$  = thumb

$B$  = fingers

$F_B$  = out of your palm

Example: Your magnet produces a B-field with strength 0.01 T in the plane of the page as shown. A wire carries 1 Amp from left to right, such that  $\theta = 30^\circ$ . What's the force per unit length (and its direction)?



$$F_B / \ell = B I \sin 30^\circ = 0.01 \text{ T} \times 1 \text{ A} \times 0.5 = 0.005 \text{ N/m (out of page)}$$

So it's not much.

Suppose the current is 6 Amps and the B-field is 3 T.

$$F_B / \ell = 9 \text{ N/m (which is slightly more substantial)}$$

Consider a power transmission line carrying 100 Amps from W to E. What's the total force on 100m of wire due to the Earth's magnetic field (assume a North component only)?

In which direction is the wire deflected (assume charge carriers are e<sup>-</sup>'s)?

Reminder: the Earth's B-field is 0.5 Gauss =  $0.5 \times 10^{-4}$  T

$$F_B = B I \ell \sin\theta$$
$$= (0.5 \times 10^{-4} \text{ T})(100\text{A})(100\text{m}) = 0.5 \text{ N.}$$

So, again, it's not much.

Direction of deflection = downward

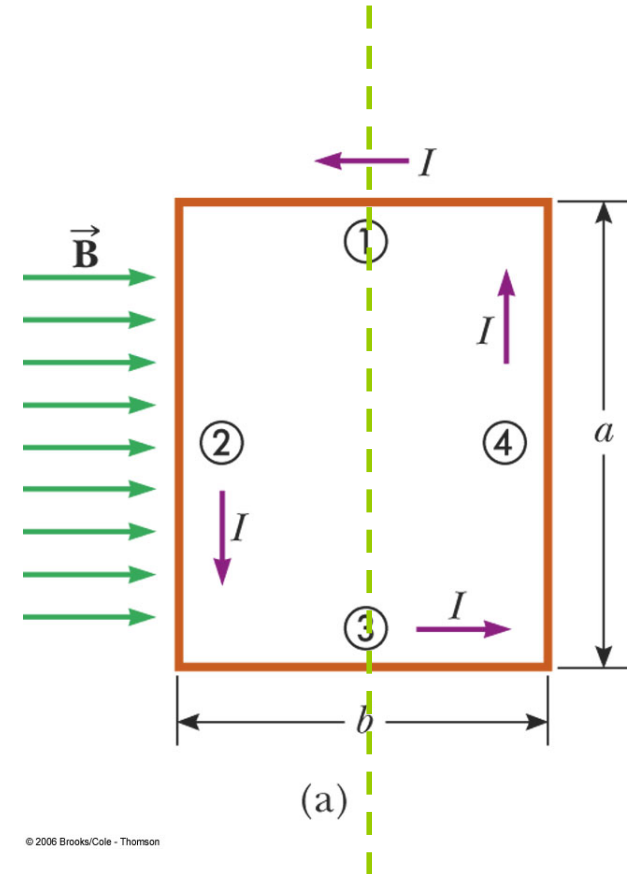
# Force on a square loop of current in a uniform B-field.

$$F_{\text{top}} = 0 \quad \theta = 0; \sin\theta = 0; \text{ so } F_B = 0$$

$$F_{\text{bottom}} = 0$$

$$F_{\text{left}} = I a B \text{ (out of page)}$$

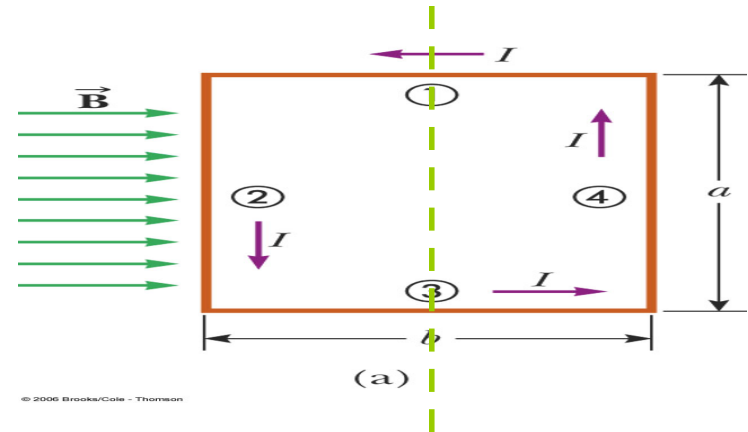
$$F_{\text{right}} = I a B \text{ (into page)}$$



Assume loop is on a frictionless axis

What's the TORQUE  
on the current loop?

*Fig. 22.19b in text is the view  
along the axis, from the  
bottom towards the top.*



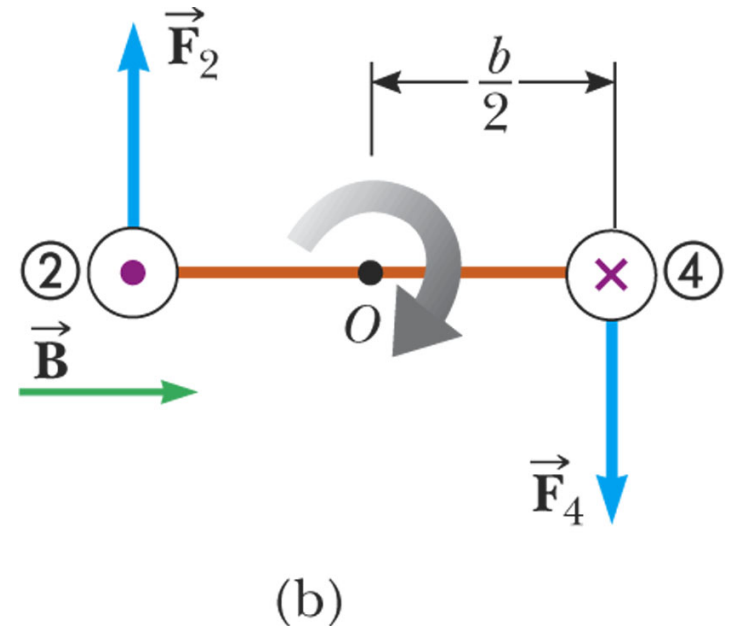
Reminder: torque =  $\vec{F} \times \vec{r} =$   
 $F r \sin\theta$

$$\tau = F_{\text{left}} a/2 + F_{\text{right}} a/2 =$$

$$(B b I + B b I) a/2 = B I A$$

A = area;  $\theta=90^\circ$  here

Note direction of torque:  
clockwise



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For 1 loop:

$$\tau = BIA \sin\theta$$

$$\tau_{\max} = BIA$$

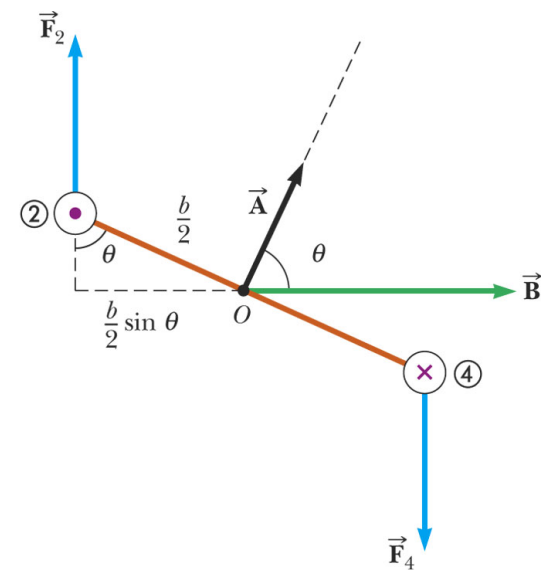
For N turns: Total current = NI

$$\tau = BIAN \sin\theta$$

Magnetic Moment  $\vec{\mu} = IAN$

$\vec{\mu}$  always points perp. to the plane of the loops (points along the normal)

$$\tau = \mu B \sin\theta$$





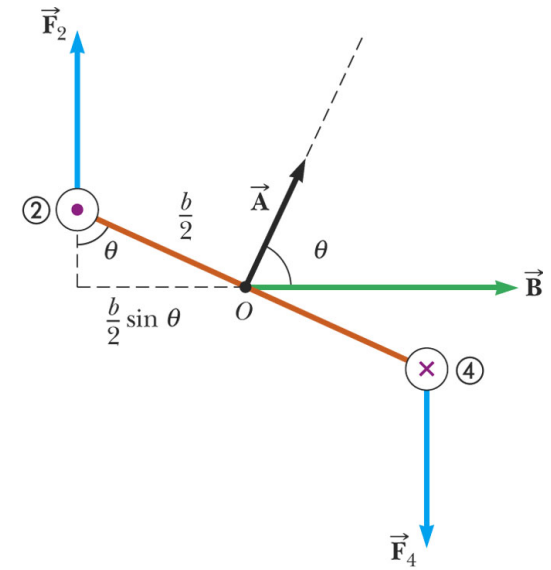
$$\vec{\tau} = I \vec{A} \times \vec{B}$$

- The product  $I\vec{A}$  is defined as the **magnetic dipole moment**,  $\vec{\mu}$  of the loop (for ANY loop shape)
- SI units: A m<sup>2</sup>
- Torque in terms of magnetic moment:

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

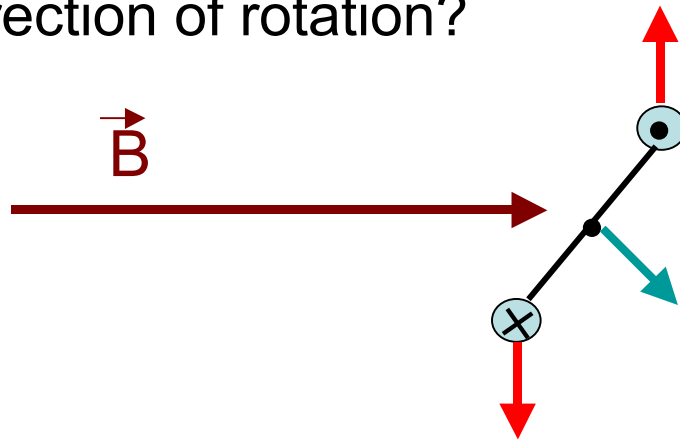
For a coil with N turns of wire:

$$\vec{\mu} = NI\vec{A}$$



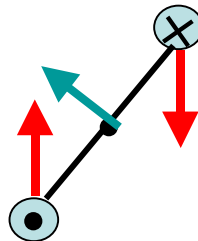
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A coil consisting of 100 turns, each carrying 3A of current and having an area 0.2 m<sup>2</sup>, is oriented such that its normal makes a angle of 45° with a B-field of 0.5T. Find the total torque on the coil. What's the direction of rotation?

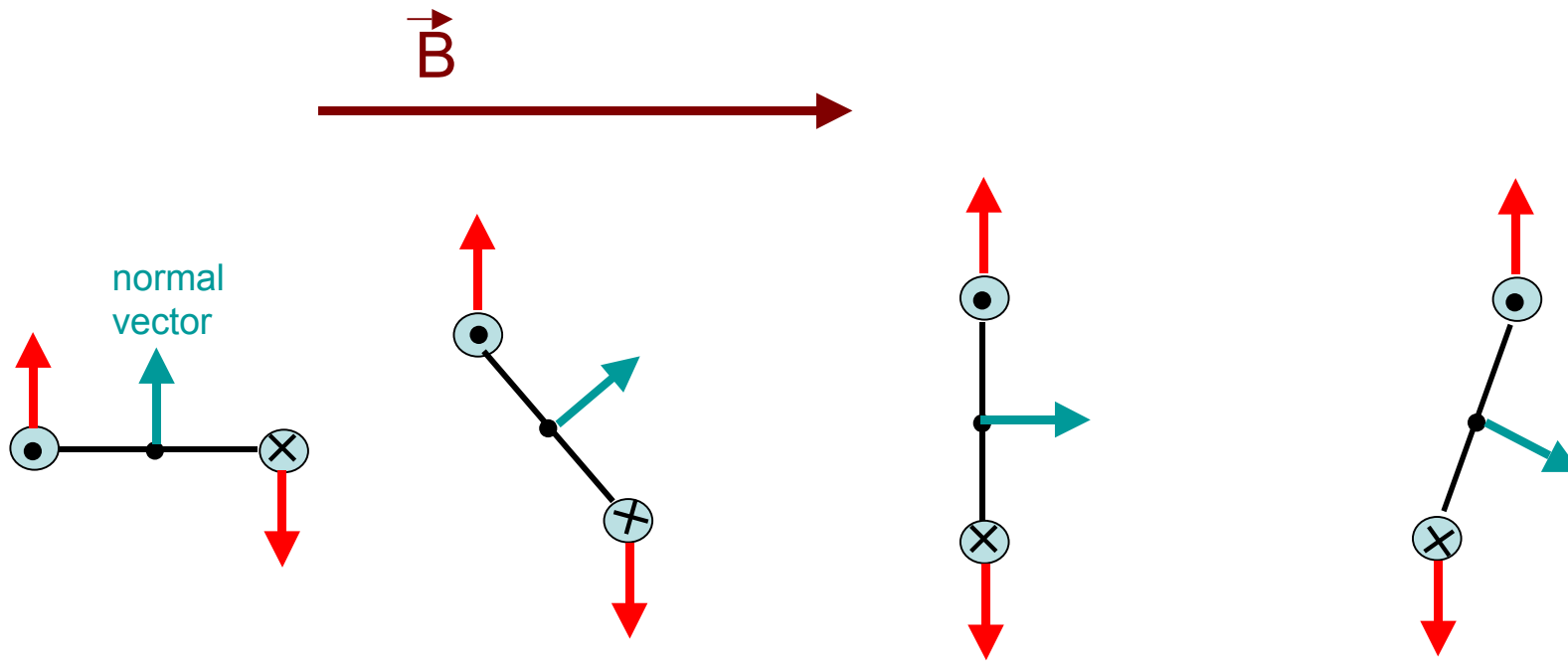


$$\tau = BIAN\sin\theta = (0.5\text{T})(3\text{A})(100)(0.2\text{m}^2)\sin 45^\circ = 21.2 \text{ Nm}$$

What would happen if the current were flowing in the opposite direction?



Same magnitude of  $\tau$ , but rotation is now CW



#1:  $\theta = 90^\circ$

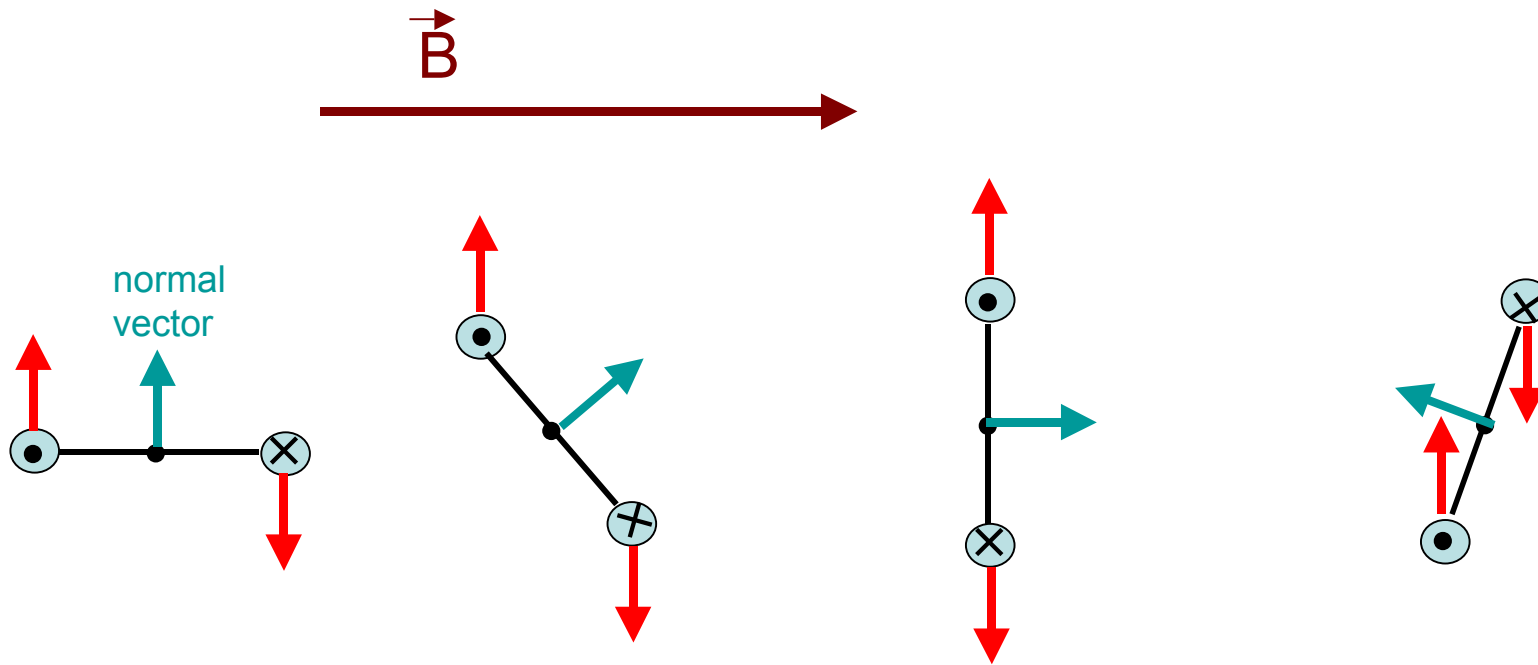
#2:  $0^\circ < \theta < 90^\circ$

#3:  $\theta = 0^\circ$

#4:  $90^\circ < \theta < 180^\circ$

torque acts to align plane of loop perpendicular to B-field (align normal vector with B-field), as in #3

(if released from rest in this position, it won't rotate)



#1:  $\theta = 90^\circ$

#2:  $0^\circ < \theta < 90^\circ$

#3:  $\theta = 0^\circ$

#4:  $90^\circ < \theta < 180^\circ$

As loop is rotating, what would happen if we switched the direction of current immediately after #3?

The loop would continue to rotate clockwise!

# Electric motors

- If direction of current is switched every time  $\tau$  is about to change sign, then  $\tau$  will never change sign!
- Loop will rotate nonstop: we have an electric motor (electrical energy converted to mechanical (rotational) energy)!
- Fans, blenders, power drills, etc.
- Use AC current (sign changes naturally), or if you only have DC current available....

How do you switch the sign of current every half cycle? Use a "commutator"

