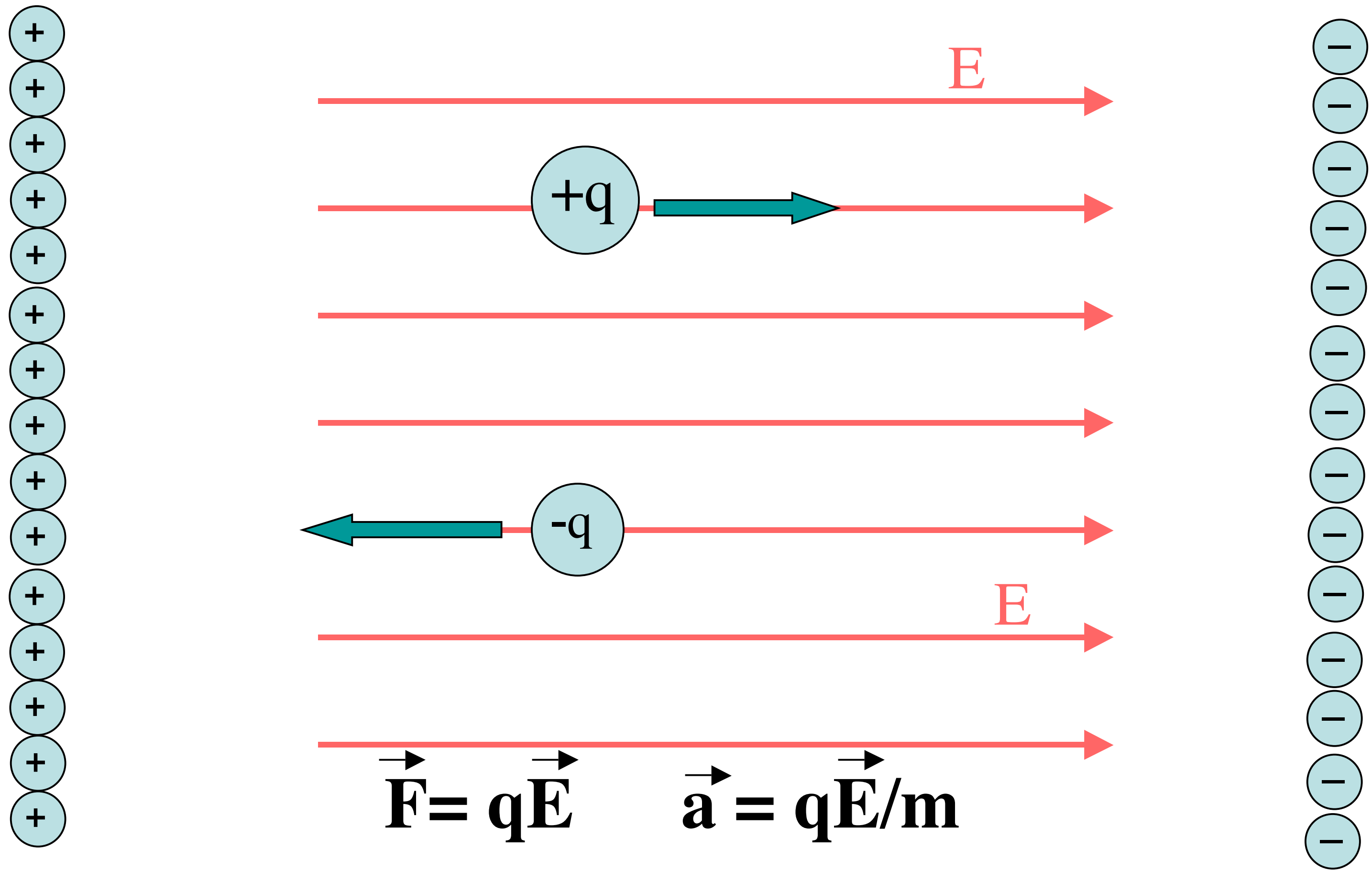
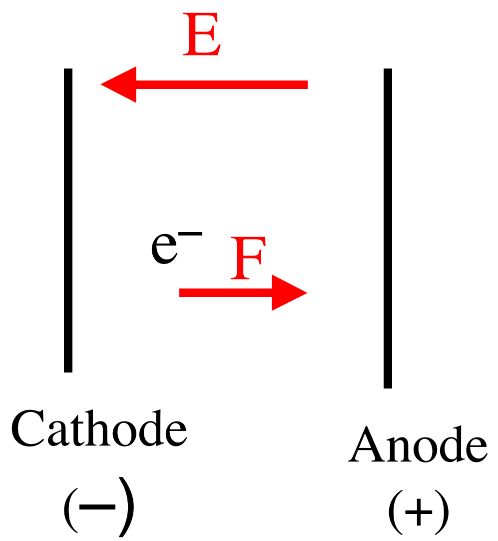
19.7: Motion of Charged Particles in a Uniform E-Field

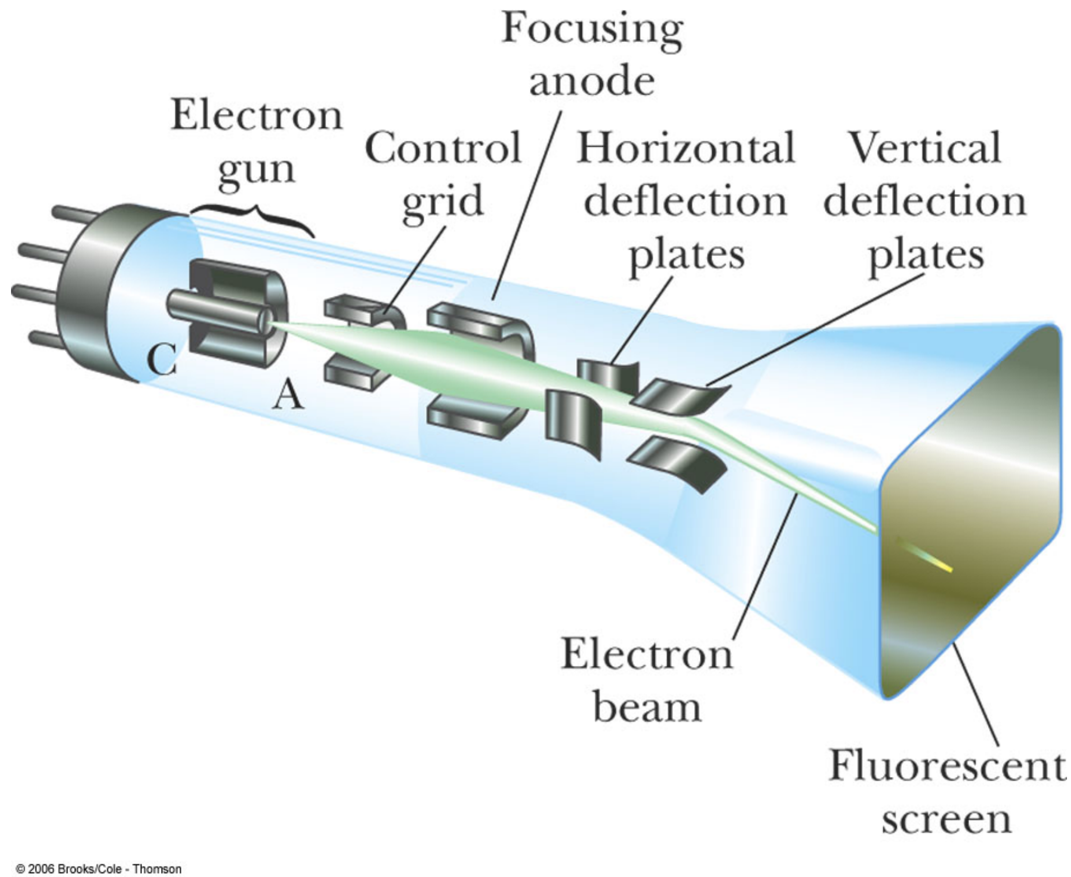
E-field exerts force on a charge

Consider an array of + charges and an array of – charges:



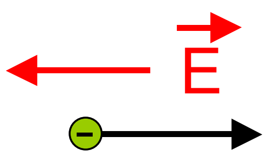
Cathode Ray Tube





Accelerating electrons in a constant E-field

A single electron is accelerated from rest in a constant electric field of 1000 N/C through a distance of 3 cm. Find the electric force on the electron, and calculate its final velocity (m_e = 9.1x10^{-31} kg)



$F = qE = m_e a$

$F = qE = (1.6\text{x}10^{-19}\ \text{C})(1000\text{N/C})$
$= 1.6\text{x}10^{-16}\ \text{N}$

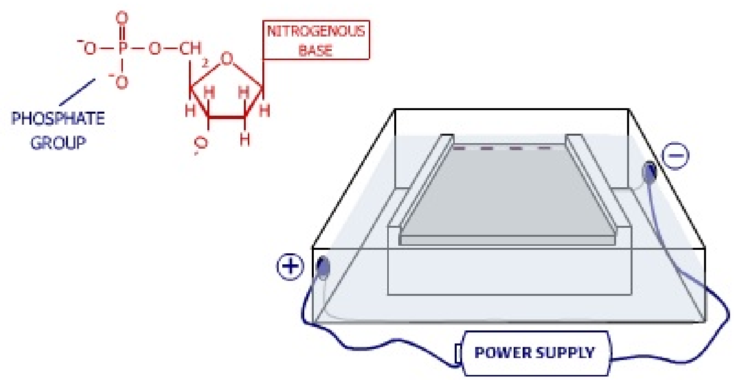
$$v^2 = v_0^2 + 2ad$$
$$\rightarrow v = \sqrt{2ad} = \sqrt{2(F/m_e)d} = \sqrt{2(qE/m_e)d}$$

$$v = \sqrt{\frac{2(1.6\times10^{-19}C)(1000N/C)0.03m}{9.1\times10^{-31}kg}}$$
$$v = 3.2 \times 10^6\ \text{m/s}$$

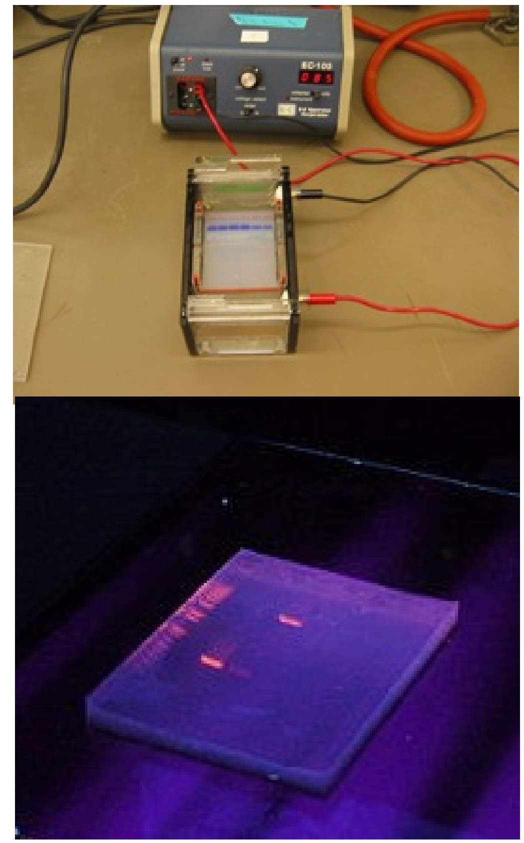
Electrophoresis

Separation of DNA segments (q ~ –1000 e due to O–'s in phosphate backbone of DNA chain) in an E-field ~ 1000 N/C.

Moves through pores in gel towards anode; smaller segments travel further



Source: http://dnalc.org



http://web.mit.edu/7.02/virtual_lab/RDM/RDM1virtuallab.html

V_{init} of charge perp. to E-field

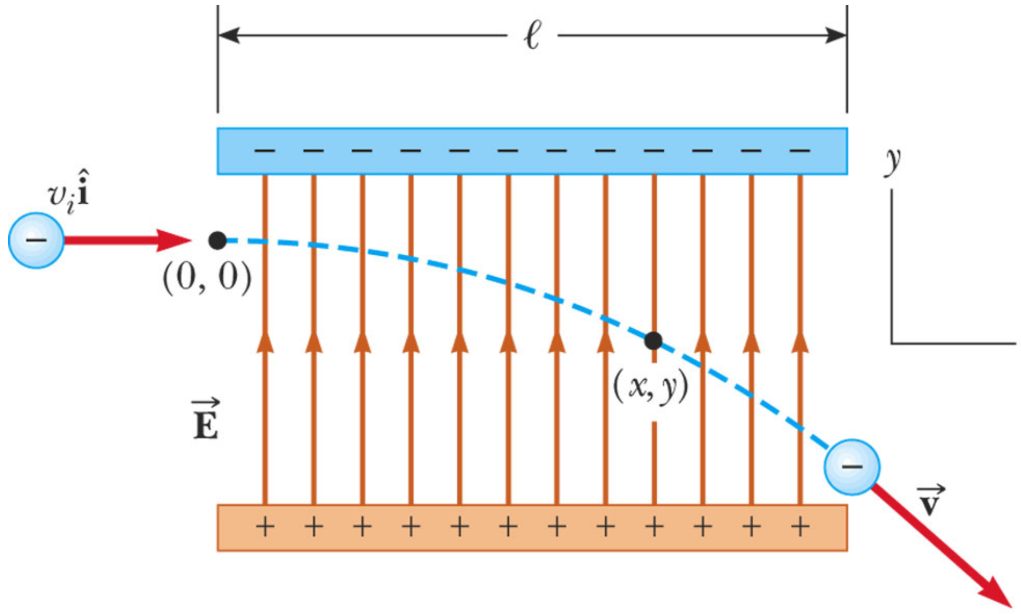
$v_x = v_{init} = \text{constant}$

$v_y = a_y t = -eEt/m_e$

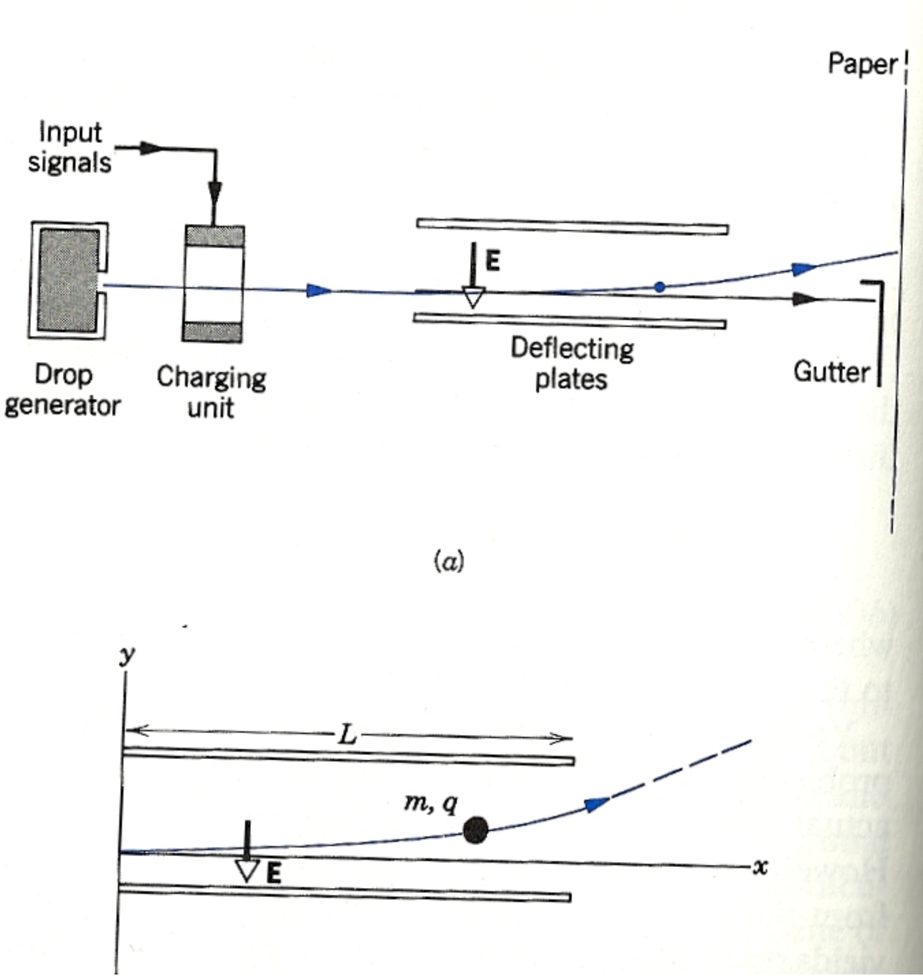
$x_f = v_{init} t$

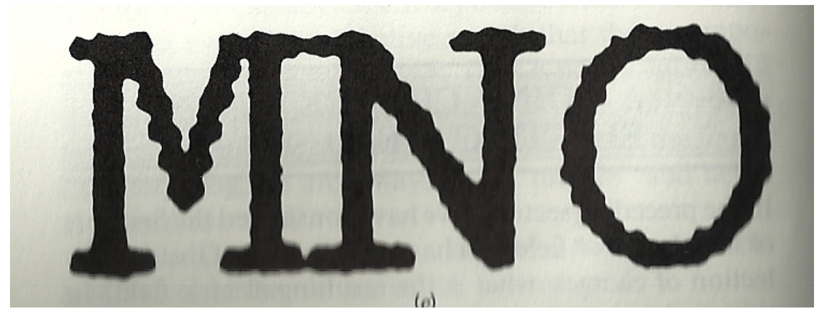
$y_f = 1/2\ a_y t^2 =$
$-(1/2)eEt^2/m_e$

Example 19.7



Application: Ink-jet printers





Tiny drop of ink is shot through charging unit, where a negative charge (typ. ~ –1000e) is applied. An E-field is then applied to deflect the drop through the proper angle.

19.8 – 19.10 Electric Flux & Gauss' Law

OVERVIEW:

Gauss' Law: relates electric fields and the charges from which they emanate

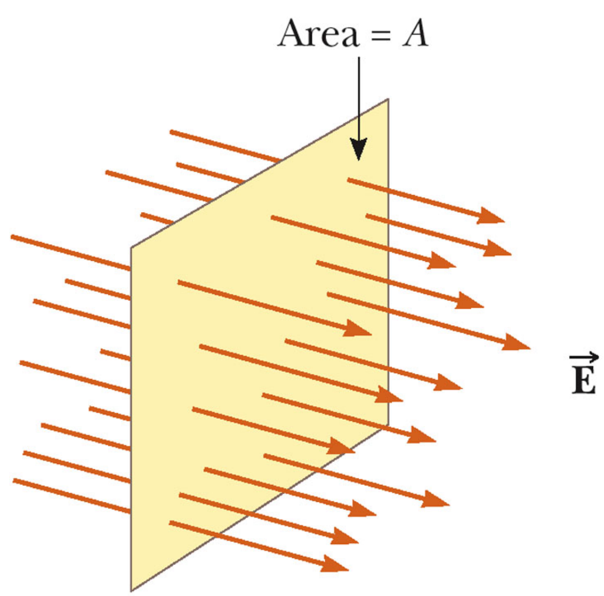
Technique for calculating electric field for a given distribution of charge

Relates the total amount of charge to the "electric flux" passing through a closed surface surrounding the charge(s).

Electric Flux Φ_E

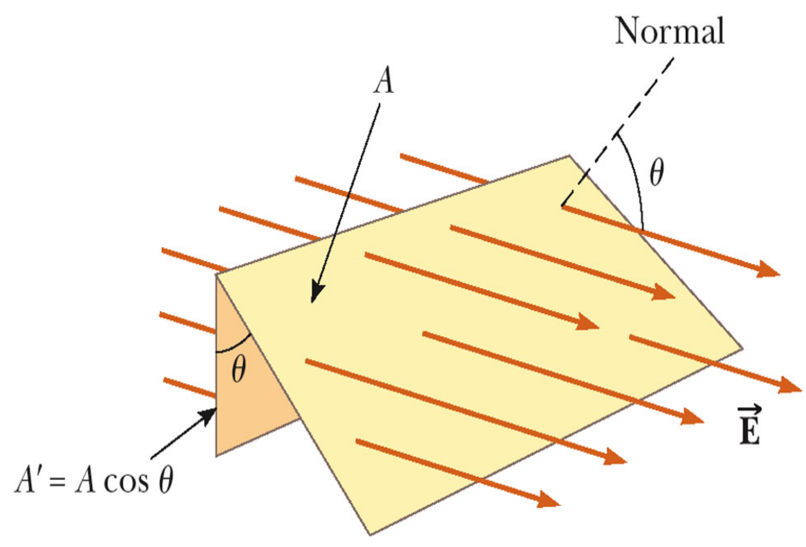
Consider a uniform E-field and an area A ⊥ to E-field lines:

$$\Phi_E = E\,A$$

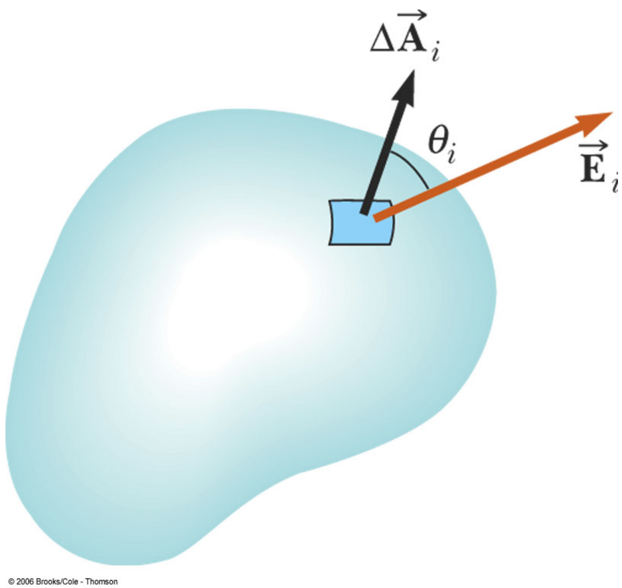


If E-field lines make angle θ to normal of plane:

$$\Phi_E = E\,A\cos\theta$$



Φ_E



- In the more general case, look at a small area element

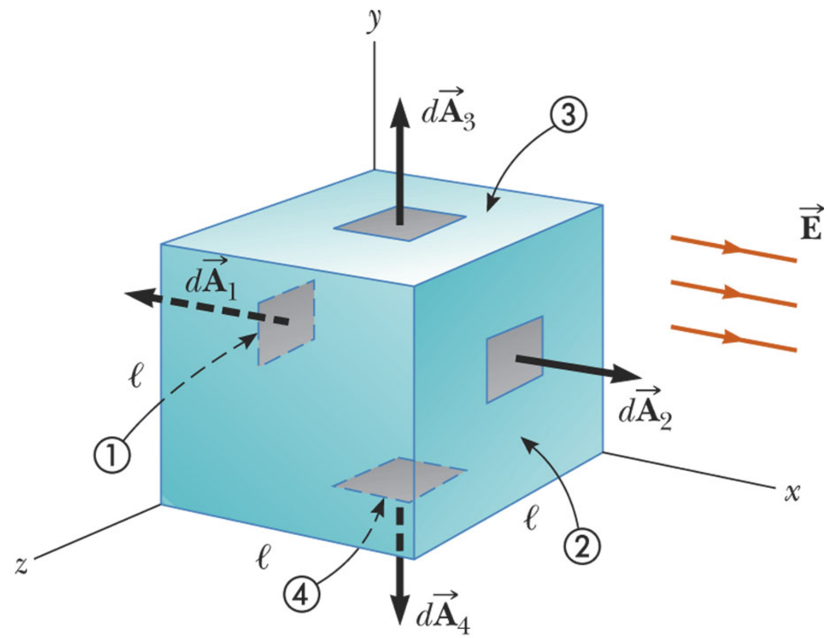
$$\Delta\Phi_E = E_i A_i \cos\theta_i = \vec{\mathbf{E}}_i \cdot \Delta\vec{\mathbf{A}}_i$$

- In general, this becomes

$$\Phi_E = \sum \vec{\mathbf{E}}_i \cdot \Delta\vec{\mathbf{A}}_i = \int_{surface} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$$

Electric Flux Φ_E Through a Cube

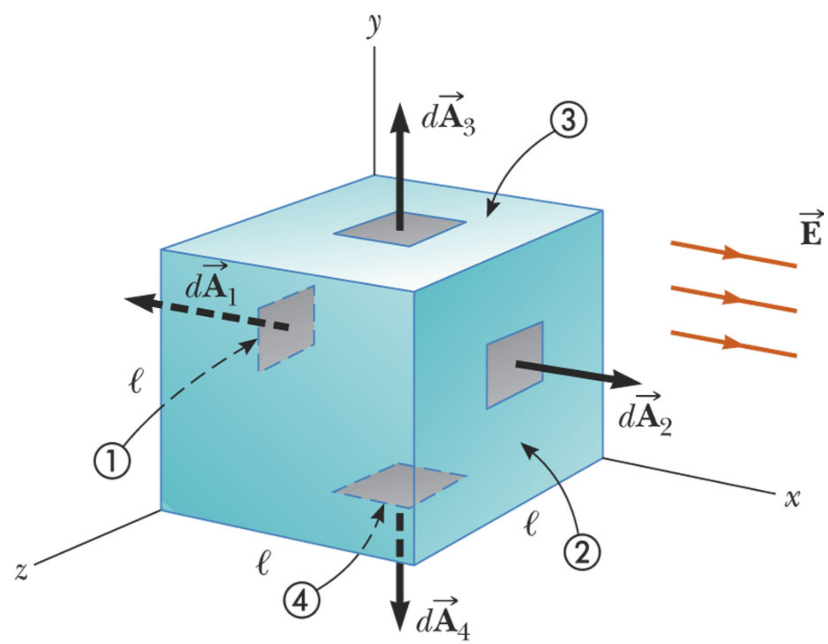
Uniform E-field parallel to x-axis: What's the net elec. flux Φ_E through the cube?



Electric Flux Φ_E Through a Cube

Uniform E-field parallel to x-axis: What's the net elec. flux Φ_E through the cube?

Normal vector points outward for a closed surface



Electric Flux Φ_E Through a Cube

Surface1: $\vec{E}$ antiparallel to $\vec{A}$

$\Phi_E = E\,A\cos(180°) = -EL^2$

Surface 2: $\vec{E} \parallel \vec{A}$

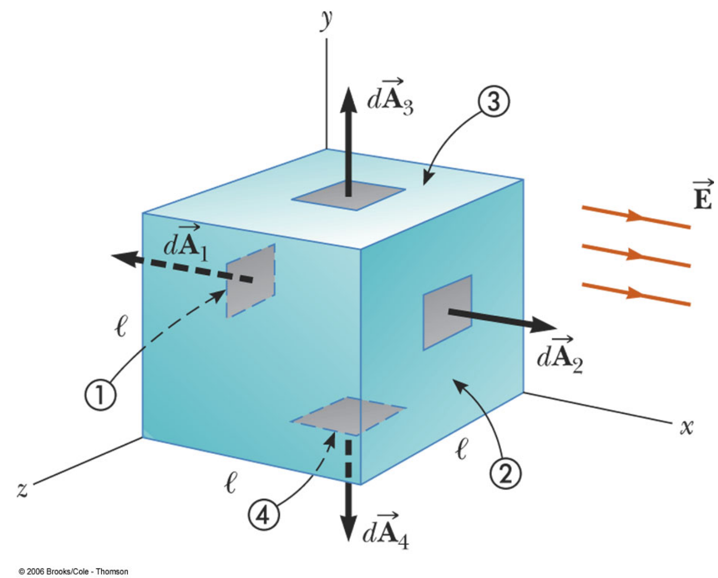
$\Phi_E = E\,A\cos(0°) = +EL^2$

Top & Bottom: $\vec{E} \perp \vec{A}$

$\Phi_E = E\,A\cos(90°) = 0$

Each side:

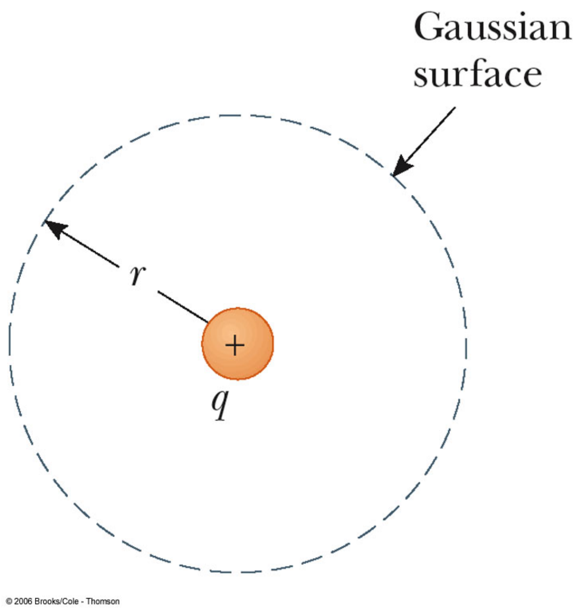
$\Phi_E = E\,A\cos(90°) = 0$



Net Φ_E =
0+0+0+0+ EL^2 - EL^2 = **0**

The net electric flux through any closed surface will be zero if there is no charge enclosed inside!

Gauss' Law



At radius r: $E = \frac{k_e q}{r^2}$

$\Phi_E = E \times Area = \frac{k_e q}{r^2} \times (4\pi r^2)$

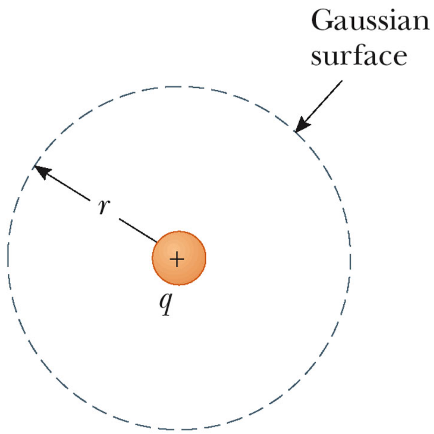
Define $\epsilon_0 = \frac{1}{4\pi k_e} = 8.85 \times 10^{-12} \frac{C^2}{Nm^2}$

ϵ_0 = permittivity of free space

$\Phi_E = Q_{encl} / \varepsilon_0$

Φ_E through any closed surface is equal to the net charge enclosed, Q_{encl}, div. by ε_0

Gauss' Law



Φ_E does not depend on radius of sphere: just the charge enclosed ($1/r^2$ dependence of E cancelled by r^2 dependence of A)

Gauss' Law: describes how charges create electric fields

Gaussian surfaces: not a real surface -- does not have to coincide with the surface of a physical object

Gauss' Law

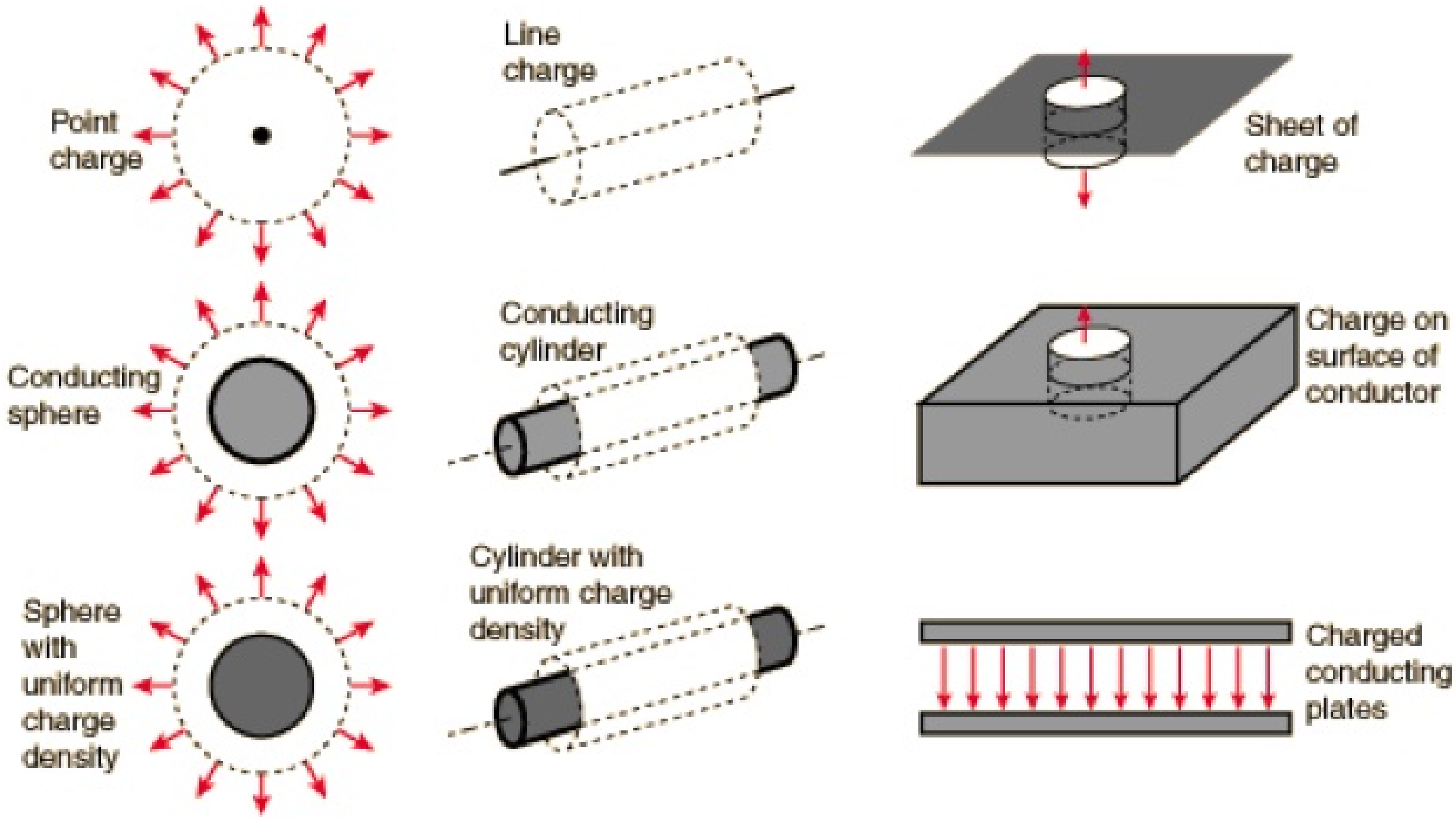
Eqn 19.22: more generalized form of Gauss' Law

$$\Phi_E = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{in}}{\varepsilon_o}$$

In practice, Gaussian surfaces || or$\perp$ to $\vec{E}$ will greatly simplify calculations.

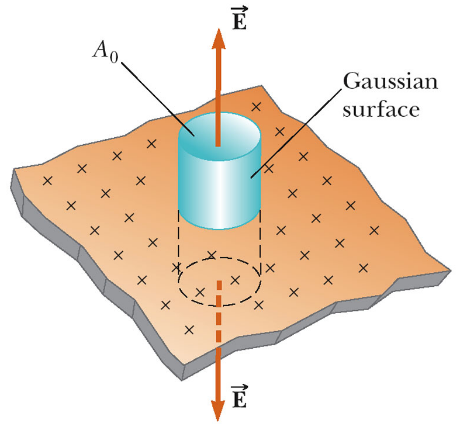
Sample Gaussian surfaces

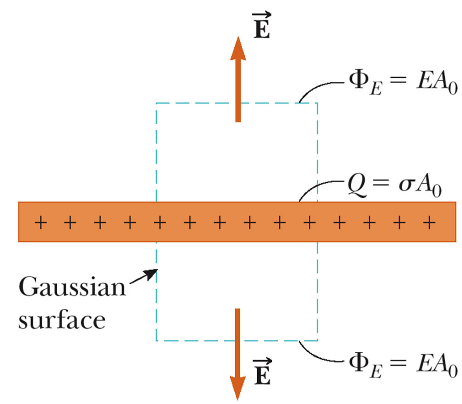
Hint: Choose surfaces such that $\vec{E}$ is ⊥ or || to surface!



Gauss' Law: A sheet of charge

Define σ = charge per unit area





$$\Phi_E = EA = Q_{encl}/\epsilon_0$$

A = area of top + bottom surfaces = 2 A_0

$$Q_{encl} = \sigma A_0$$

$$EA = \frac{\sigma A_0}{\epsilon_0}$$

$$E = \frac{\sigma A_0}{2A_0\epsilon_0}$$

$$E = \frac{\sigma}{2\epsilon_0}$$

This is the magnitude of $\vec{E}$.
$\vec{E}$ points away from the the plane.
$\vec{E} = +\frac{\sigma}{2\epsilon_0}$ above the plane
$\vec{E} = -\frac{\sigma}{2\epsilon_0}$ below the plane

Quiz #1: Tues. Oct 5

45 mins long, multiple choice. ~8-10 questions

Arrive promptly; we start exactly at 09:30!

You bring scantrons (X-101864-PAR only!), #2 pencils, erasers, current student ID, scientific calculator, ‘3x5’ index card: you supply all the equations, we supply constants.

No cell phones, iPhones, or any other notes allowed.

Quiz will cover chapter 19, from section 19.1 up through 19.10 (including examples 19.9 & 19.12, but not examples 19.10 & 19.11)

Quiz #1: Tues. Oct 5

Quiz #1: Tues. Oct 5

Really, arrive early -- we need to distribute exam code forms before the quiz can start.

Reminder: I have office hours today & Monday at 11:30.

Grigor has office hours today & Monday at 2:00

Discussion/problem session, Monday evening at 6:00

Physics Tutorial Center, 3-8 pm tonight, Sun and Mon.