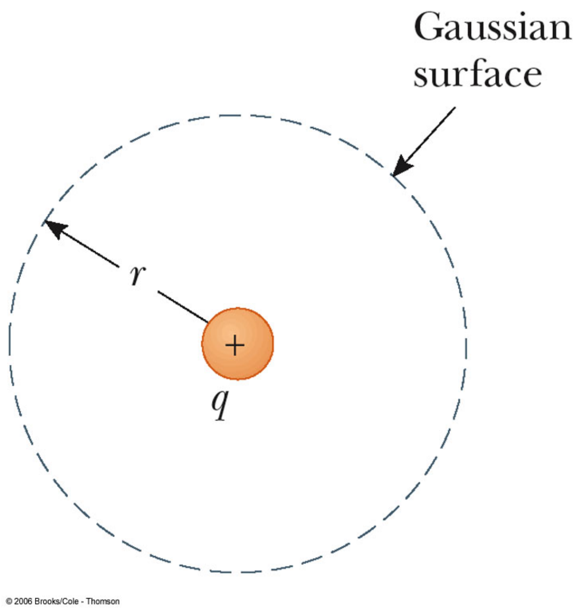
Gauss' Law



At radius r: $E = \frac{k_e q}{r^2}$

$\Phi_E = E \times Area = \frac{k_e q}{r^2} \times (4\pi r^2)$

Define $\epsilon_0 = \frac{1}{4\pi k_e} = 8.85 \times 10^{-12} \frac{C^2}{Nm^2}$

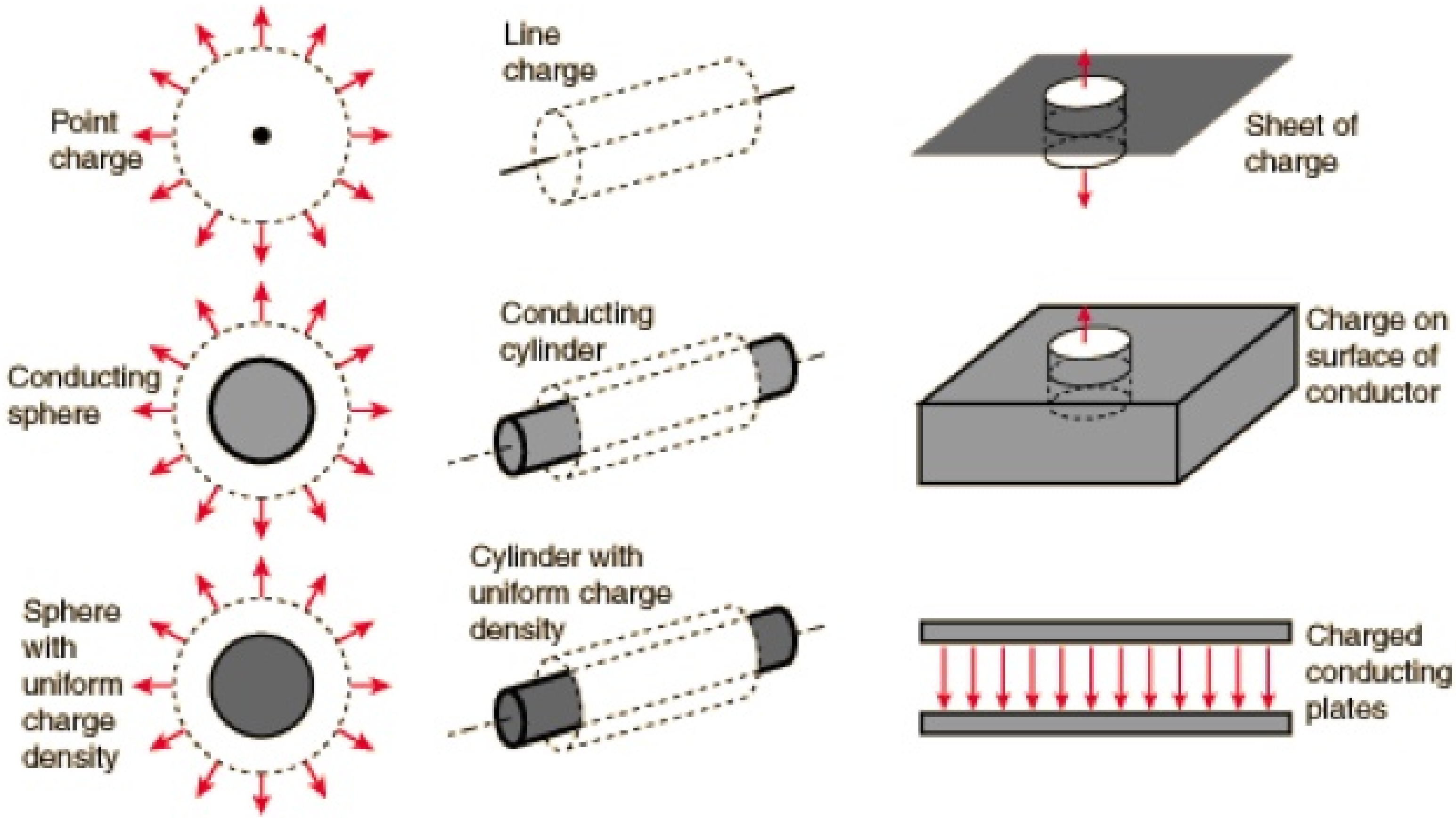
ϵ_0 = permittivity of free space

Φ_E through any closed surface is equal to the net charge enclosed, Q_{encl}, div. by ε_0

$$\mathbf{\Phi_E = Q_{encl} / \varepsilon_0}$$

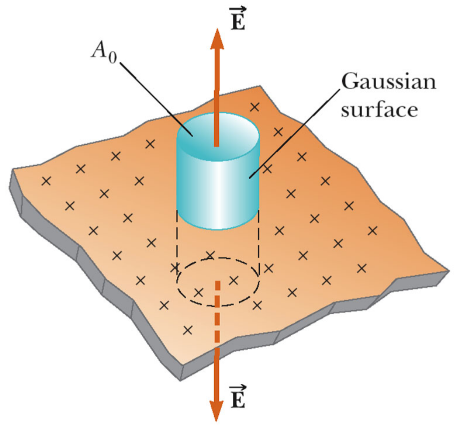
Sample Gaussian surfaces

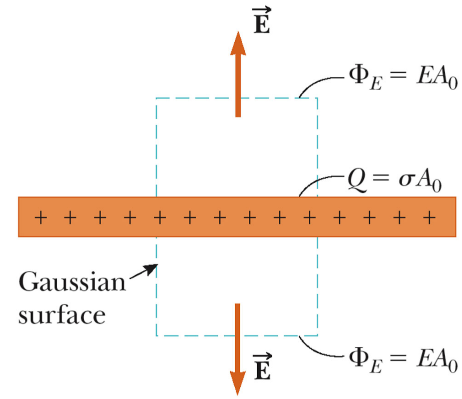
Hint: Choose surfaces such that $\vec{E}$ is ⊥ or || to surface!



Gauss' Law: A sheet of charge

Define σ = charge per unit area





$$\Phi_E = EA = Q_{encl}/\epsilon_0$$

A = area of top + bottom surfaces = 2 A_0

$$Q_{encl} = \sigma A_0$$

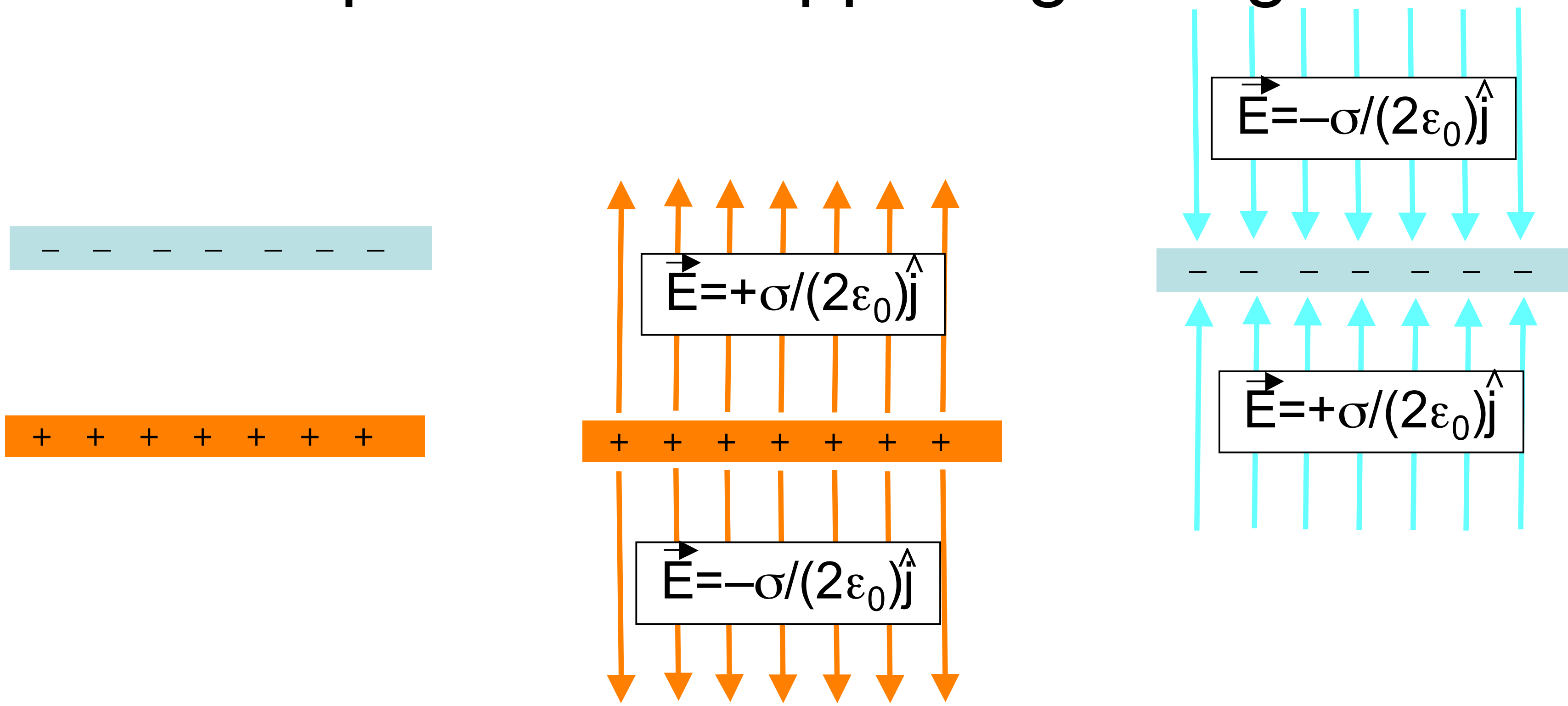
$$EA = \frac{\sigma A_0}{\epsilon_0}$$

$$E = \frac{\sigma A_0}{2A_0\epsilon_0}$$

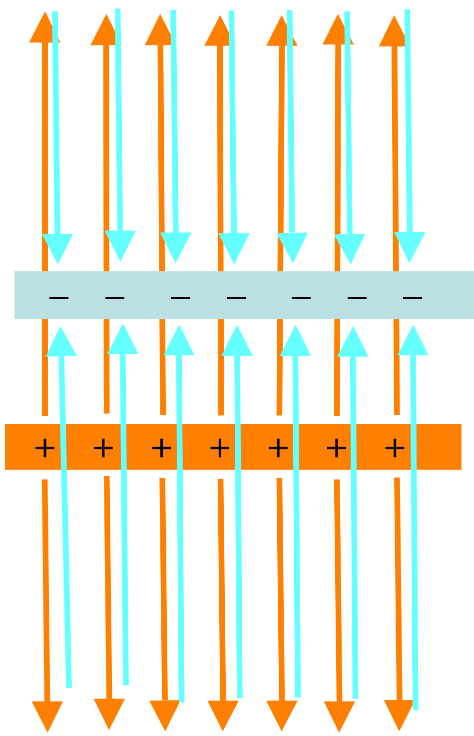
$$E = \frac{\sigma}{2\epsilon_0}$$

This is the magnitude of $\vec{E}$.
$\vec{E}$ points away from the the plane.
$\vec{E} = +\frac{\sigma}{2\epsilon_0}$ above the plane
$\vec{E} = -\frac{\sigma}{2\epsilon_0}$ below the plane

2 planes with opposing charges



2 planes with opposing charges



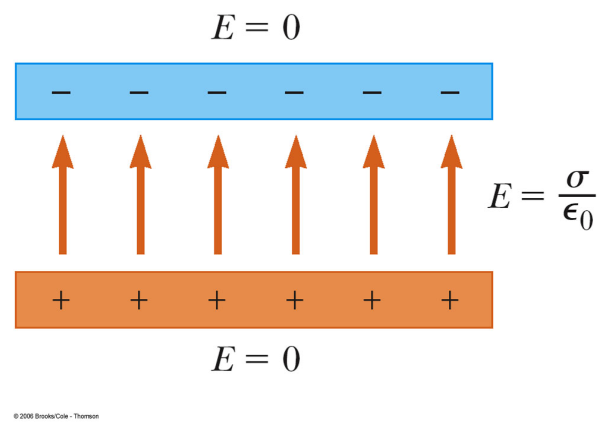
E=0 outside

E=0 outside

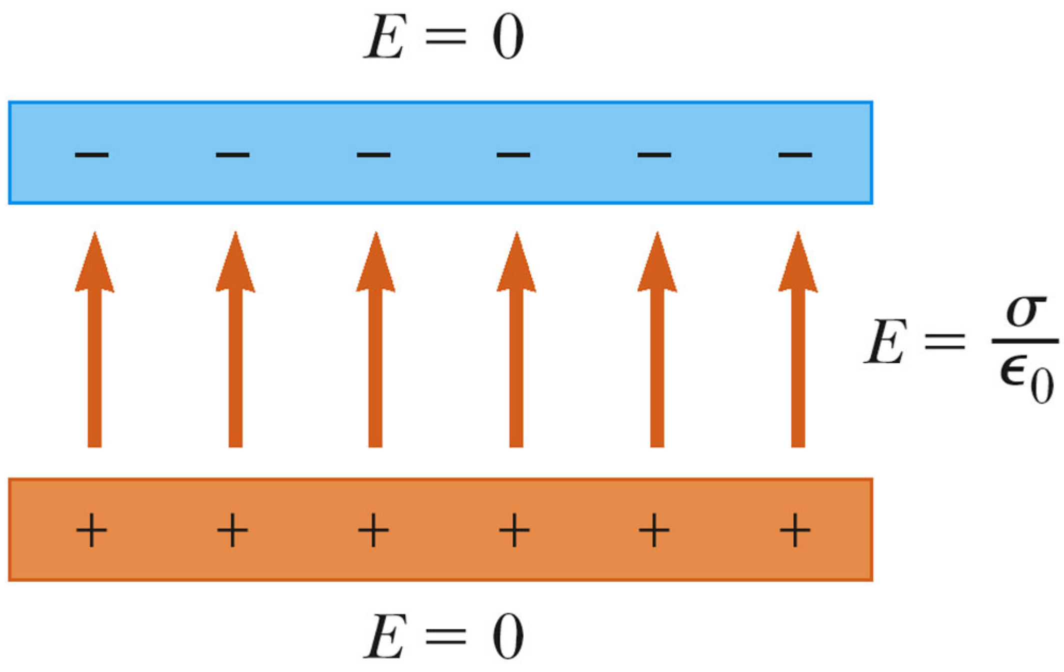
Inside:

$\vec{E} = +\sigma/(2\varepsilon_0)\hat{\mathbf{j}} + \sigma/(2\varepsilon_0)\hat{\mathbf{j}}$

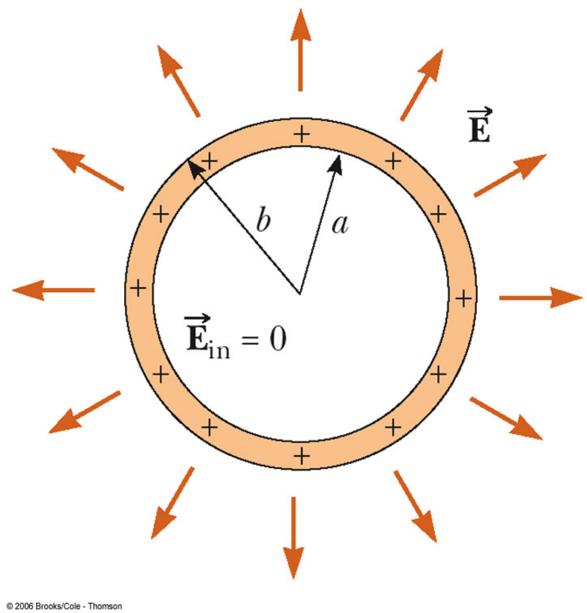
$= \sigma/\varepsilon_0\,\hat{\mathbf{j}}$

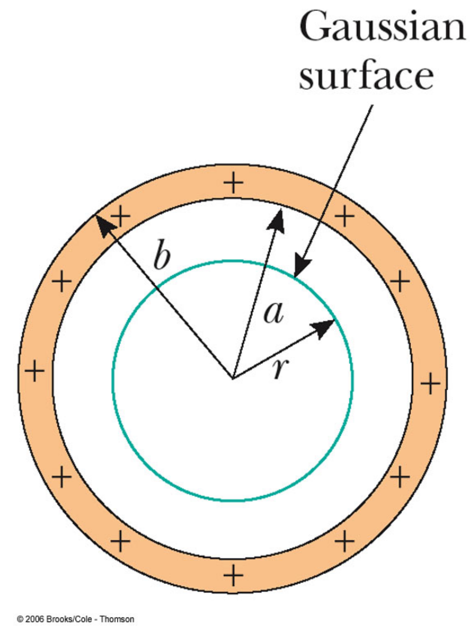


2 planes with opposing charges



Gauss' Law: Charged Spherical Shell

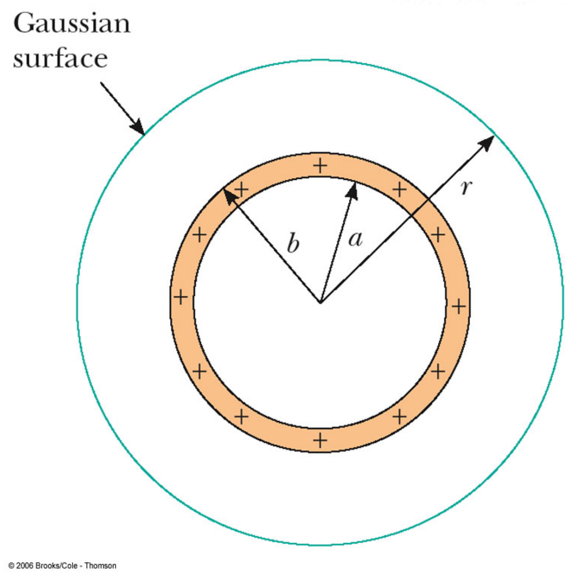




At r < a: $\vec{E} = 0$.

Gauss' Law: Charged Spherical Shell

$$\text{At r > b, } \Phi_E = EA = E4\pi r^2 = Q_{encl}/\epsilon_0$$



Divide both sides by area:

$$E = \frac{Q_{encl}}{4\pi\varepsilon_0 r^2}$$

At r > b, $\vec{E}$ looks like that from a single point charge Q

Ex. 19.11, then 19.10

E-field a distance r from an infinite line of charge (charge per length λ): Deriving $E(r) = 2k_e\lambda/r$

E-field associated with a solid sphere of charge. Total charge = Q, distributed uniformly. Radius a.

Outside sphere ($r > a$), $E(r) = k_eQ/r^2$

Inside sphere ($r<a$), $E(r) = k_eQr/a^3$

(inside, enclosed charge increases with radius as r^3, and area of the Gaussian surface increases with r^2 -- so E is proportional to r.)