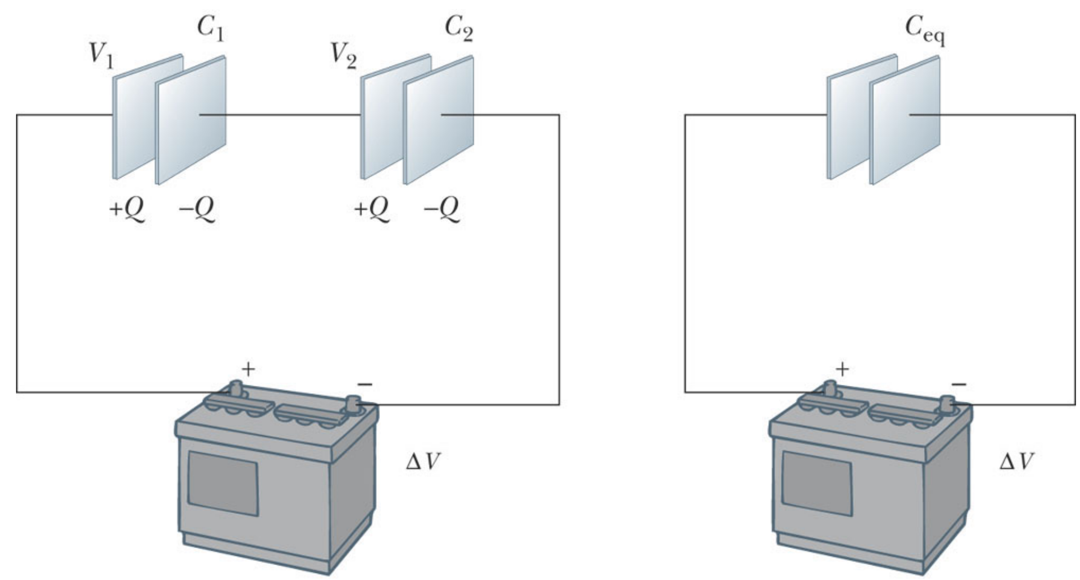
Connecting Capacitors in Series and in Parallel

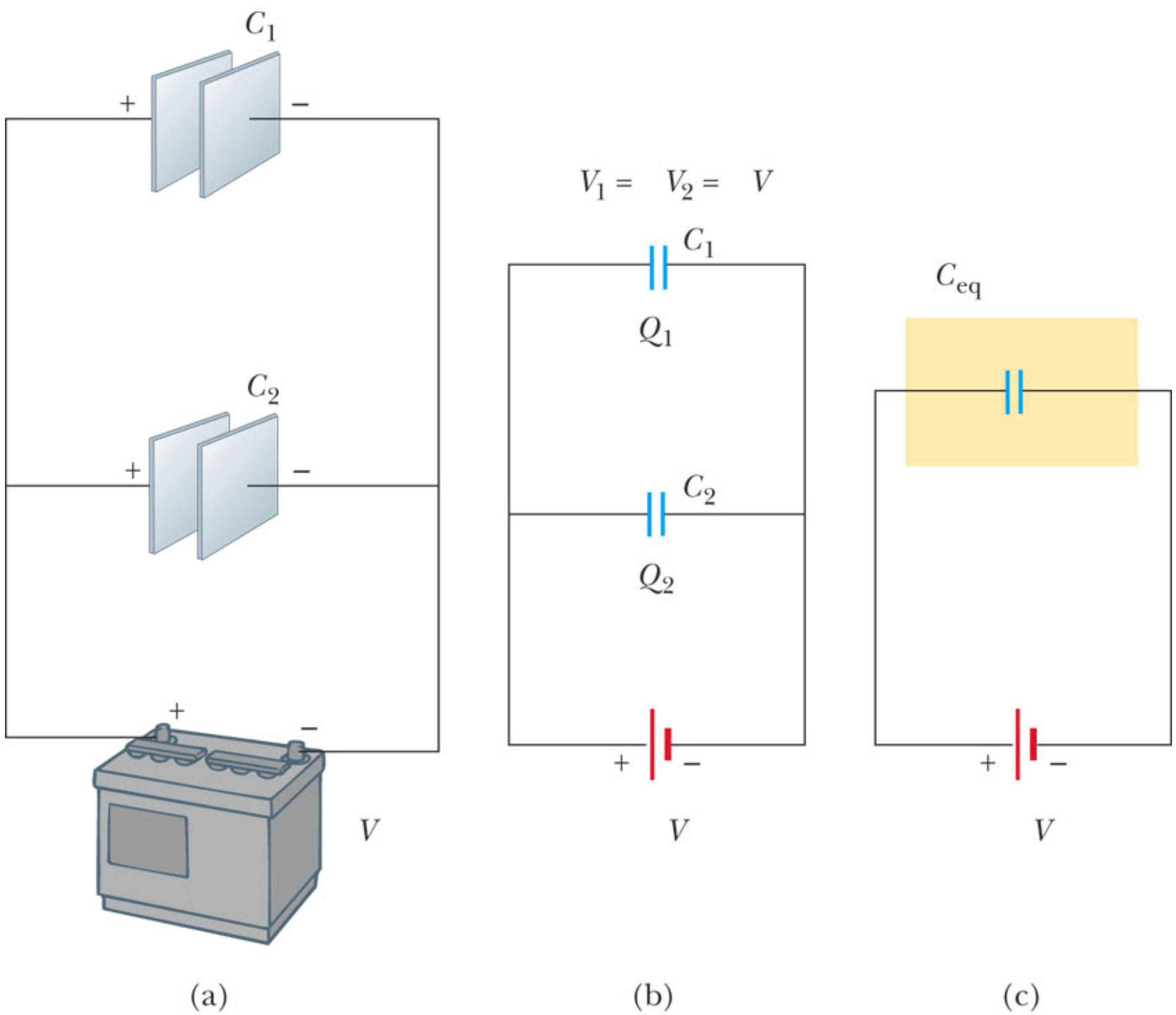
Goal: find "equivalent" capacitance of a single capacitor (simplifies circuit diagrams and makes it easier to calculate circuit properties)

Find C_{eq} in terms of C_1, C_2,… to satisfy $C_{eq} = Q/\Delta V$

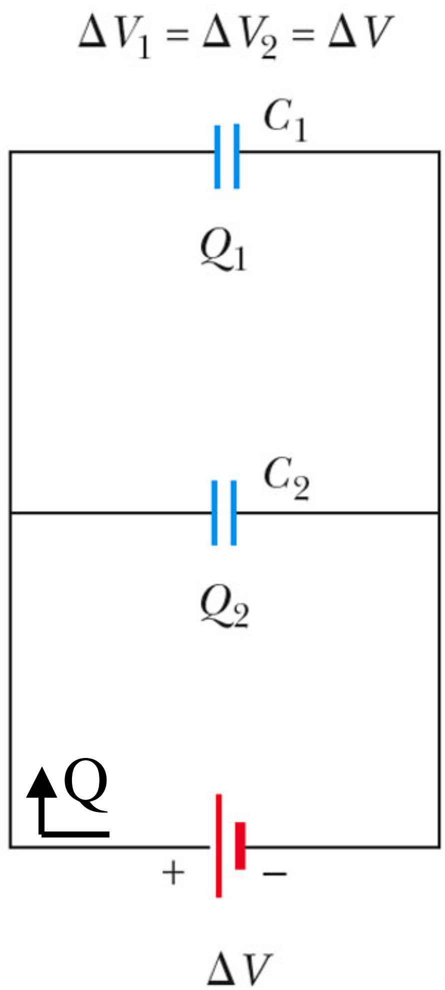


(a) (b)

Capacitors in Parallel



Capacitors in Parallel



Note that both capacitors are held are same potential difference ΔV:

$\Delta V_1 = \Delta V_2 = \Delta V$

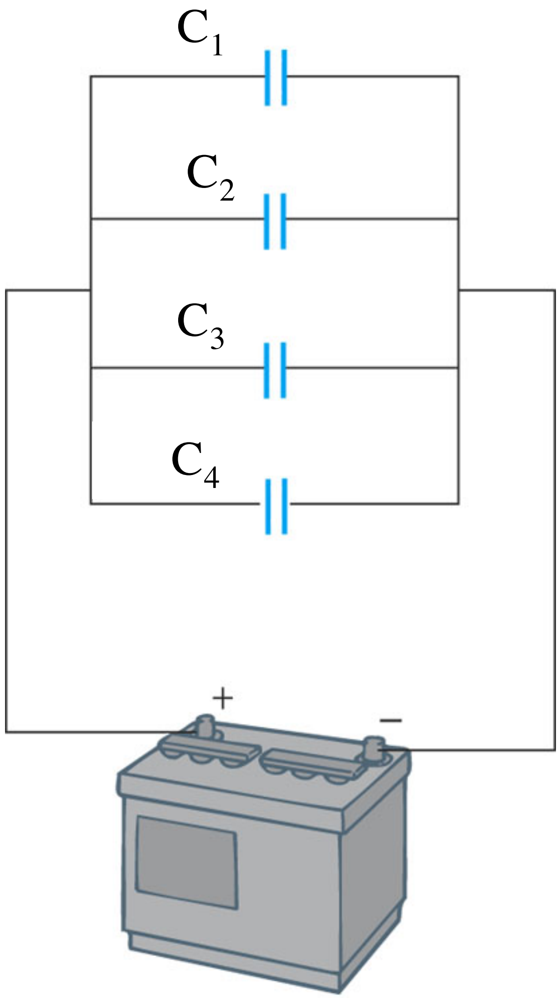
Total charge $Q = Q_1 + Q_2$

$Q = C_1\Delta V + C_2\Delta V$

$C_{eq} = Q/\Delta V = (C_1\Delta V + C_2\Delta V)/\Delta V$

$\mathbf{C_{eq} = C_1 + C_2}$

Capacitors in Parallel



For N capacitors in parallel:

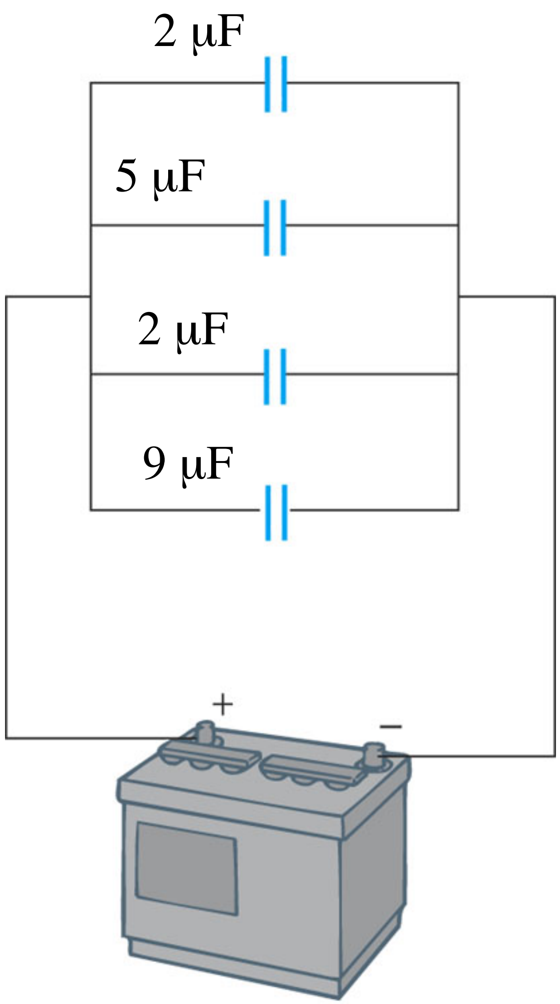
$$C_{eq} = C_1 + C_2 + C_3 + \dots + C_N$$

C_{eq} bigger than any individual C

akin to having larger area:

$\text{Area}(C_{eq}) = \text{Area1} + \text{Area2} + \dots$

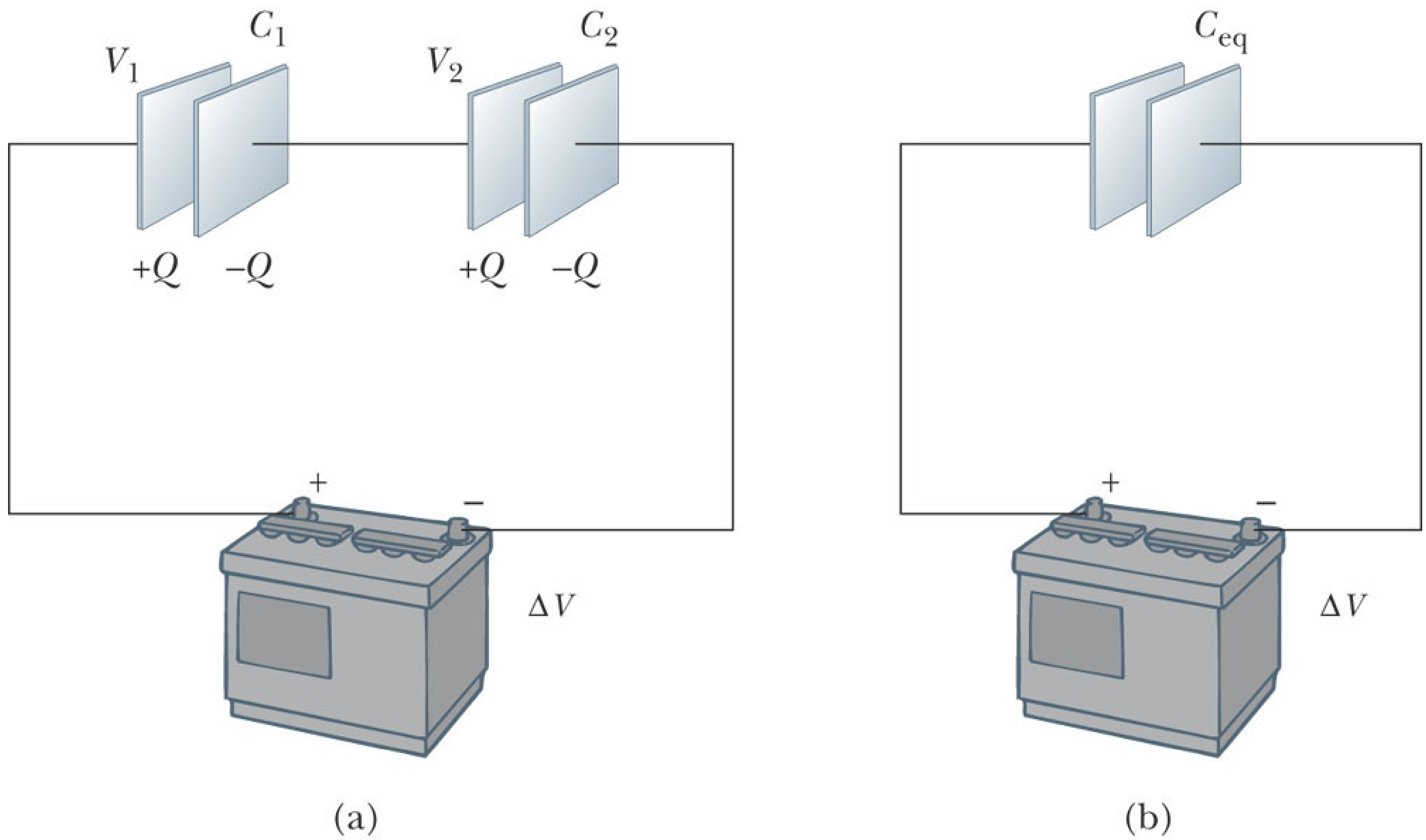
Capacitors in Parallel



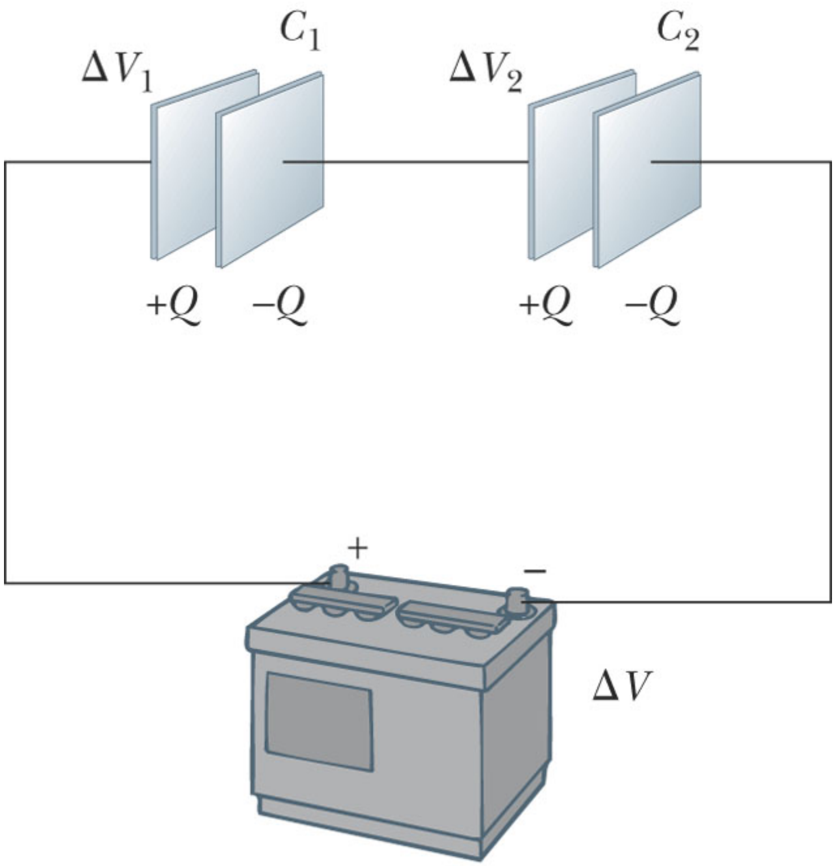
Example: Calculate $\mathbf{C_{eq}}$

$$\mathbf{C_{eq} = 2+5+2+9\ \mu F = 18\mu F}$$

Capacitors in Series



Capacitors in Series



(a)

Note both capacitors' left plates have equal positive charge, +Q (and right plates each have –Q)

Also: $\Delta V = \Delta V_1 + \Delta V_2$

$\Delta V = Q/C_1 + Q/C_2$

$C_{eq} = Q/\Delta V$

$1/C_{eq} = \Delta V/Q$

$1/C_{eq} = (Q/C_1 + Q/C_2)/Q$

$$\mathbf{1/C_{eq} = 1/C_1 + 1/C_2}$$

Capacitors in Series

For N capacitors in series:

$1/C_{eq} = 1/C_1 + 1/C_2 + 1/C_3 + \dots + 1/C_N$

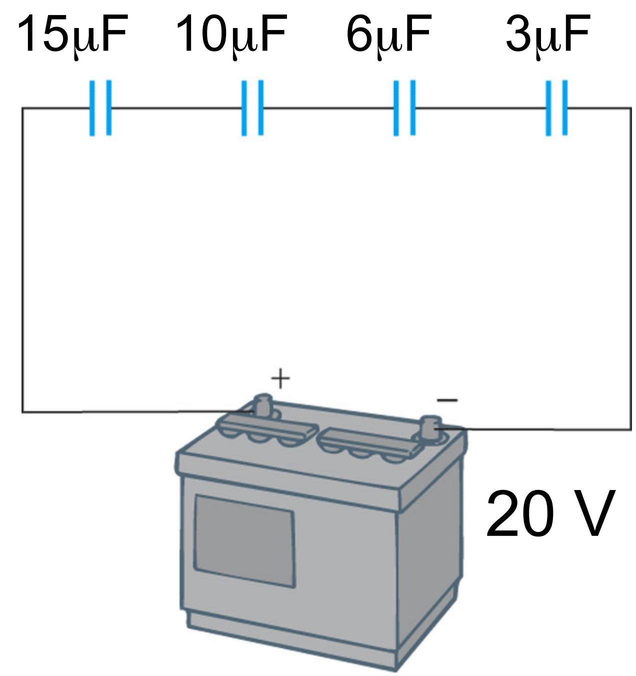
C_{eq} smaller than any individual C

Capacitors in Series

Example: Find C_{eq}.

$1/C_{eq}$ =
$1/(15\mu F)+1/(10\mu F)+1/(6\mu F)+1/(3\mu F) = 0.667/\mu F$

$C_{eq} = 1.5\ \mu F$



Find Q on each capacitor:

$Q = C_{eq}\Delta V =$
$(1.5\times10^{-6}F)(20V) = 30\mu C$

Capacitors in Series

Find the voltage drop across each capacitor:

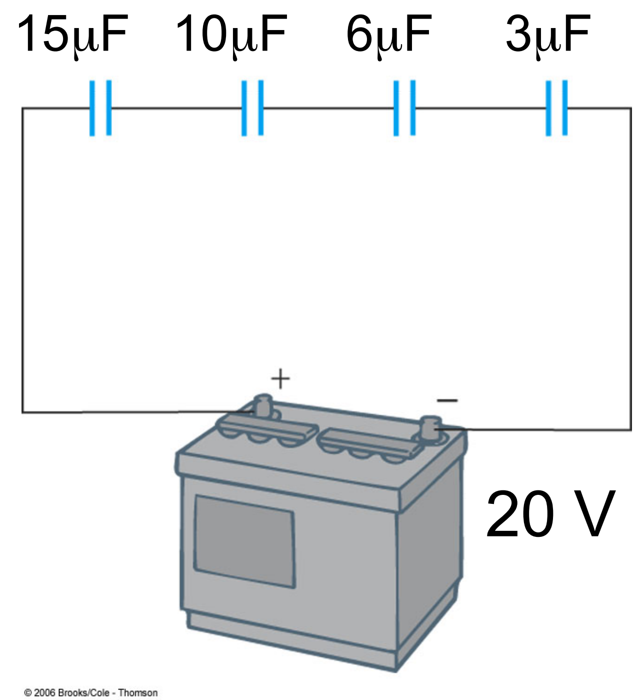
$\Delta V_1 = Q/C_1 = 30\mu C/15\mu F = 2V$

$\Delta V_2 = Q/C_2 = 30\mu C/10\mu F = 3V$

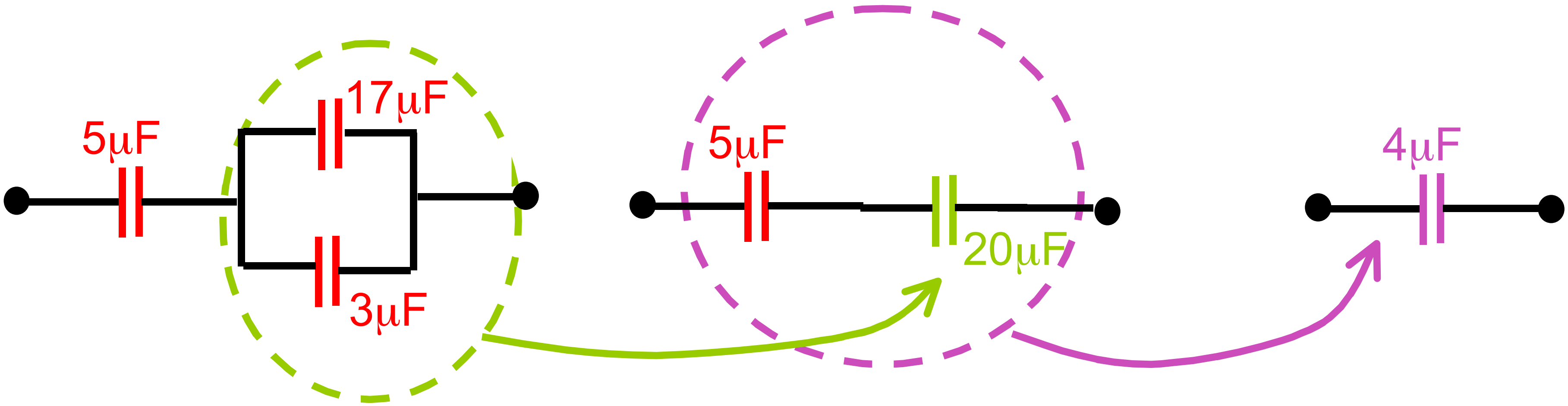
$\Delta V_3 = Q/C_3 = 30\mu C/6\mu F = 5V$

$\Delta V_4 = Q/C_4 = 30\mu C/3\mu F = 10V$

Notice that
$\Delta V_1+\Delta V_2+\Delta V_3+\Delta V_4=\Delta V$



Capacitors in Parallel AND in SERIES

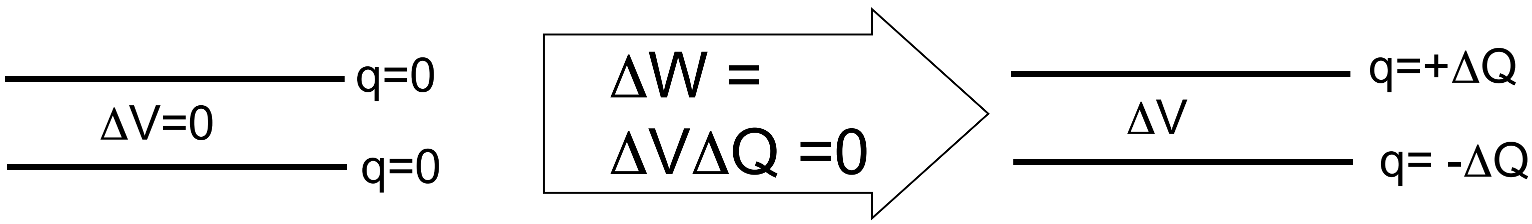


Energy stored in a capacitor

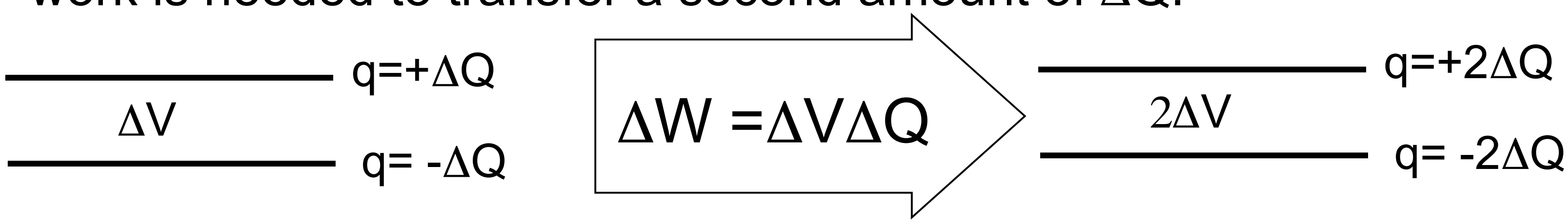
How much work does it take to charge up a capacitor?

Start with neutral plates, transfer a tiny amount of charge, ΔQ: Amount of work you need to do will equal the amount of charge times the potential difference currently across the plates

Energy stored in a capacitor

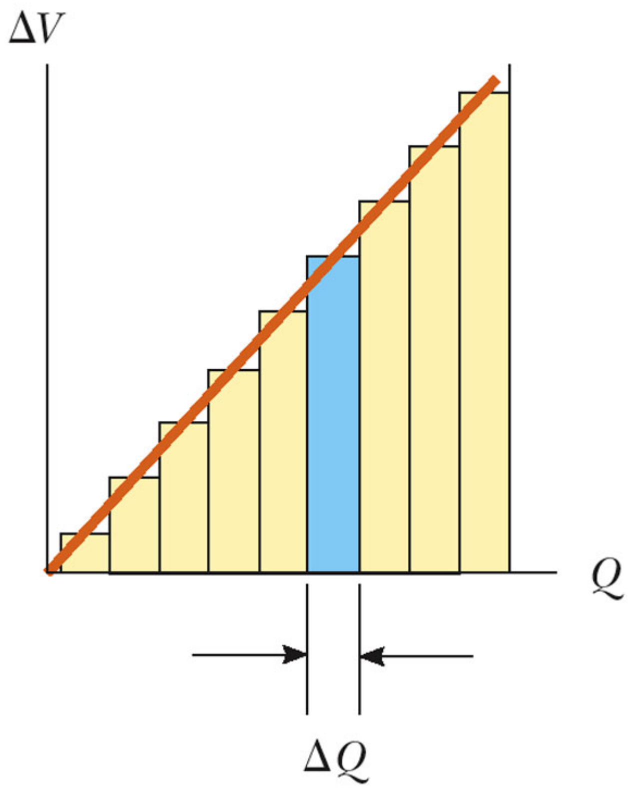


Once one ΔQ has been transferred, ΔV has increased, so additional work is needed to transfer a second amount of ΔQ:



To transfer a third ΔQ, you'll need to do work ΔW = (2ΔV)ΔQ….

Energy stored in a capacitor



Total work = shaded area

$$W = \int_0^Q \frac{q}{C}\, dq = \frac{1}{C}\int_0^Q q\, dq = \frac{1}{C}\frac{Q^2}{2}$$

Total Work = U = 1/2 ($Q\,\Delta V$) =
1/2 $C(\Delta V)^2$ = $Q^2/(2C)$

Energy density

- Energy stored can also be expressed in terms of the energy density (energy per unit volume)

$$u_E = {}^{1}/_{2}\ \varepsilon_o\ E^2$$

Defibrillators

A fully charged defibrillator contains U = 1.2 kJ of energy stored in a capacitor with C = 1.1×10^{-4} F. Find the voltage needed to store this amount of energy.

$U = 1/2\ C\ (\Delta V)^2$

$\Delta V = \sqrt{2\ U / C} = \sqrt{(2)(1200J) / 1.1\times10^{-4}\ F} = 4670\ V$

In a discharge through a patient, 600 J of electrical energy are delivered in 2.5 ms. What’s the average power delivered during this time?

Avg. Power = Energy/time = 600J/0.0025sec = 2.4×10^{5} W

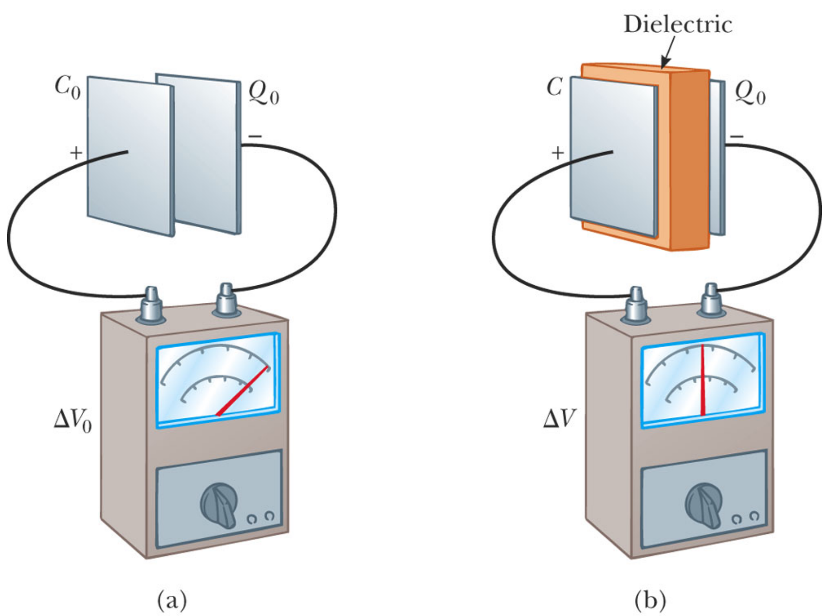
Recap: when d is halved, what happens to the energy stored?

$d \rightarrow d/2$, with battery attached: ΔV=same; $E = \Delta V/d \rightarrow 2E$; $Q \rightarrow 2Q$; $C \rightarrow 2C$; $U \rightarrow 2U$

$d \rightarrow d/2$, after battery removed: Q=same; $E = \sigma/\epsilon_0$=same; $\Delta V = Ed \rightarrow \Delta V/2$; $C = Q/\Delta V = \frac{\epsilon_0 A}{d} \rightarrow 2C$; $U \rightarrow U/2$.

Dielectrics

Insulators placed in the gap to increase capacitance by a factor κ: ceramic, paper, glass, plastic, water, teflon,…



Originally:
ΔV_0, C_0, Q_0

When you insert a dielectric (no battery attached), the voltage is observed to drop

$\Delta V = \Delta V_0/\kappa$

κ always > 1 (κ=1 for vacuum)

Q_0 remains the same (charge can't flow anywhere)

$C = Q/\Delta V = Q/(\Delta V_0/\kappa) = \kappa Q/\Delta V_0$

$\mathbf{C = \kappa C_0}$

Dielectrics

TABLE 20.1

Dielectric Constants and Dielectric Strengths of Various Materials at Room Temperature

Material	Dielectric Constant κ	Dielectric Strength (V/m)
Vacuum	1.000 00	—
Air	1.000 59	3×10^6
Bakelite®	4.9	24×10^6
Fused quartz	3.78	8×10^6
Pyrex® glass	5.6	14×10^6
Polystyrene	2.56	24×10^6
Teflon®	2.1	60×10^6
Neoprene rubber	6.7	12×10^6
Nylon	3.4	14×10^6
Paper	3.7	16×10^6
Strontium titanate	233	8×10^6
Water	80	—
Silicone oil	2.5	15×10^6

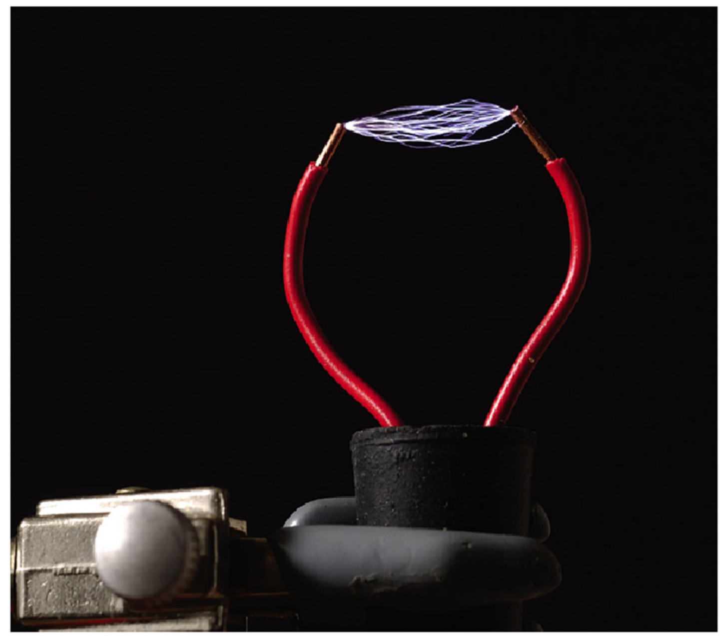
Dielectrics

$C = \kappa\varepsilon_0 (A/d)$

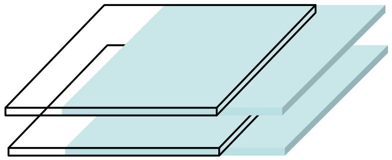
For any given d, there's a maximum electric field that can occur inside the dielectric above which conduction will occur.

"Dielectric strength"

For air ($\kappa = 1.00059$), this is $E = 3\text{x}10^6$ V/m



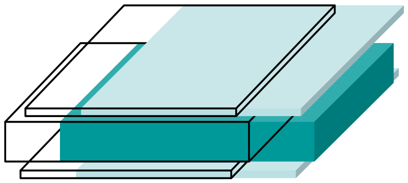
Permittivity of a dielectric



Consider a capacitor not connected to a battery: $E_0 = \Delta V_0/d$

Add dielectric $E = \Delta V/d = (\Delta V/\kappa)/d$

$E = E_0/\kappa$



For a capac., $E_o = \sigma/\varepsilon_0$

$E = (\sigma/\varepsilon_0)/\kappa = \sigma/\varepsilon$

$\varepsilon = \kappa\varepsilon_o$

Permittivity is increased compared to vacuum

Example:

You have a capacitor with plates of area = 20 cm^2, separated by a 1mm-thick layer of teflon. Find the capacitance and the maximum voltage & charge that can be placed on the capacitor.

Find κ from Table 20.1: For teflon, κ=2.1

$C = \kappa\varepsilon_0 (A/d)$

$C= 2.1(8.85x10^{-12} C^2/Nm^2)(20x10^{-4} m^2)/(10^{-3} m) = 3.7x10^{-11}$ F = 37pF

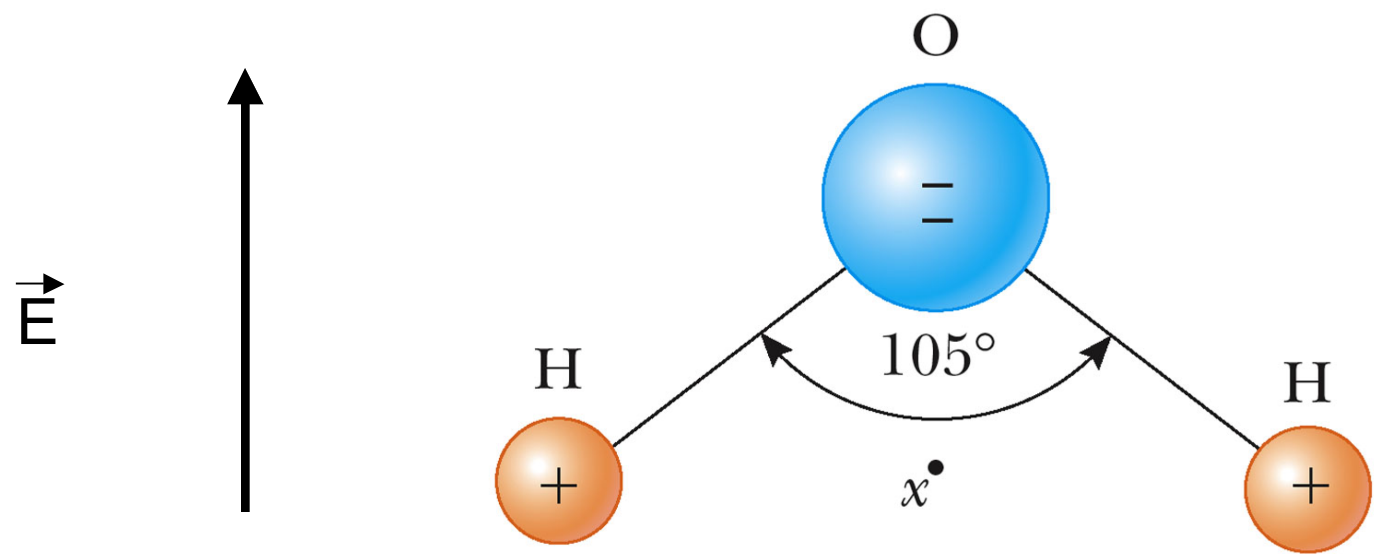
Diel. Strength is also found in Table 20.1: $E_{max} = 6x10^7$ V/m

$\Delta V_{max} = E_{max}d = (6x10^7 V/m)(0.001m)=6x10^4V$

$Q_{max} = C\Delta V_{max} = (37x10^{-12} F)(6x10^4 V)= 2.2x10^6 C$

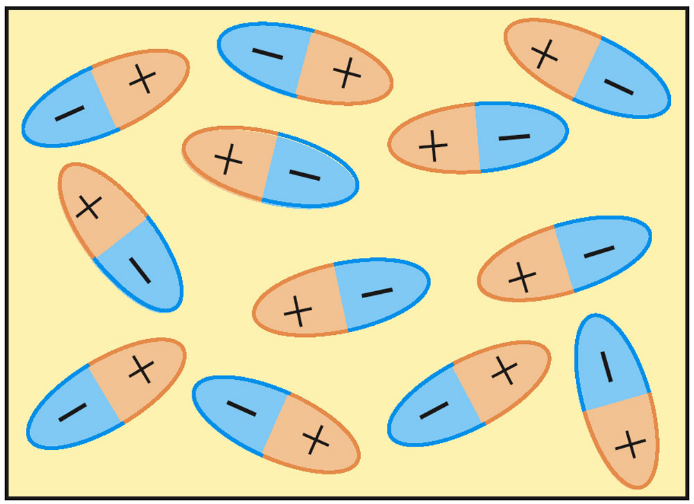
Molecular basis for dielectric constant

Relies on POLARIZATION: In some molecules, there's a separation between average positions of + and – charges



Molecular basis for dielectric constant

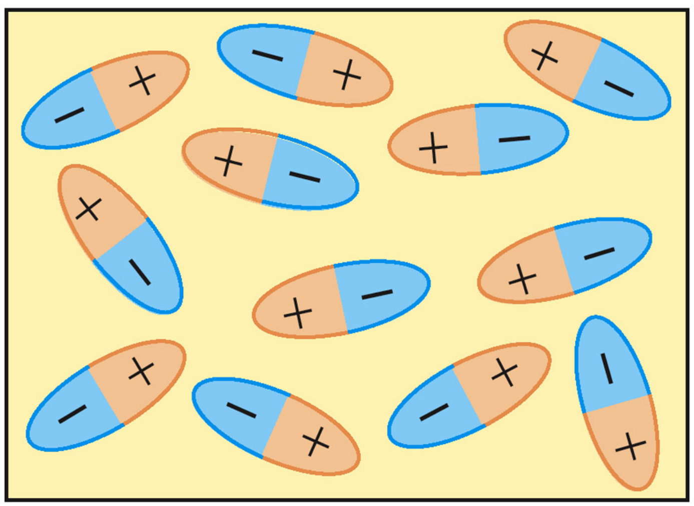
No external E-field: Molecules are randomly oriented

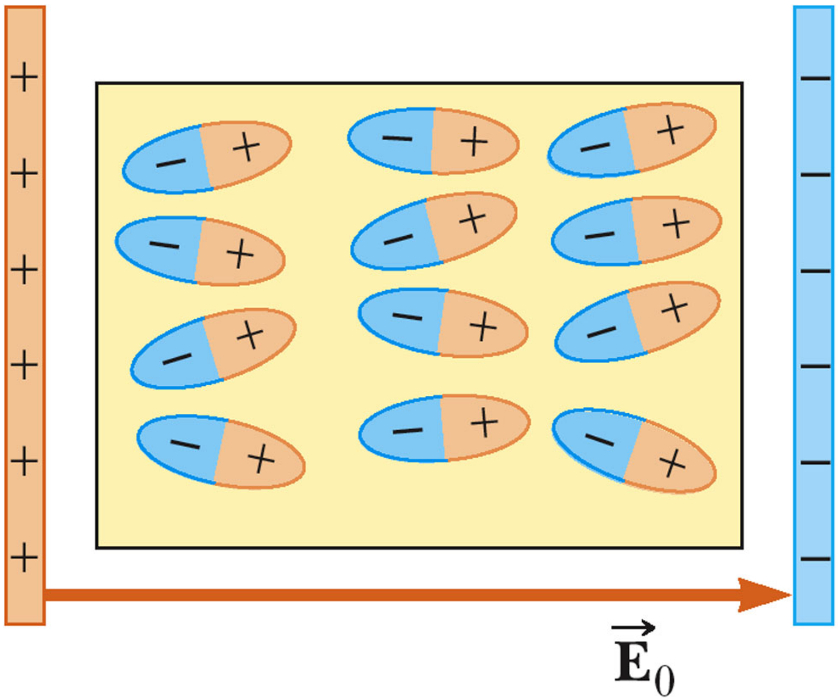


Molecular basis for dielectric constant

No external E-field: Molecules are randomly oriented

Apply external E-field: molecules orient themselves to partially align with the field





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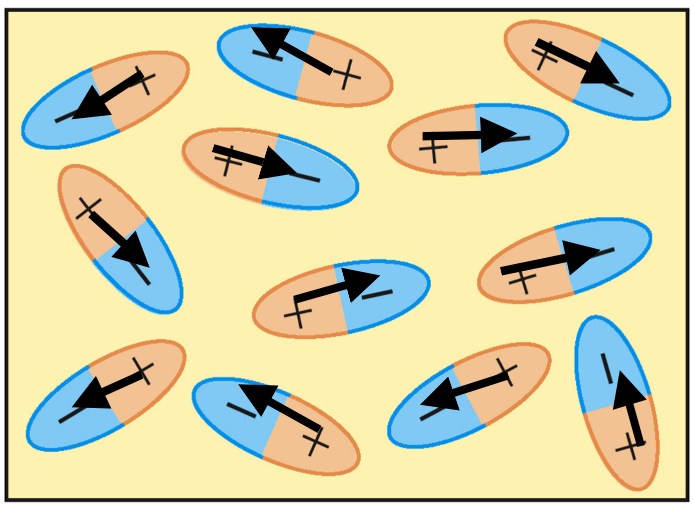
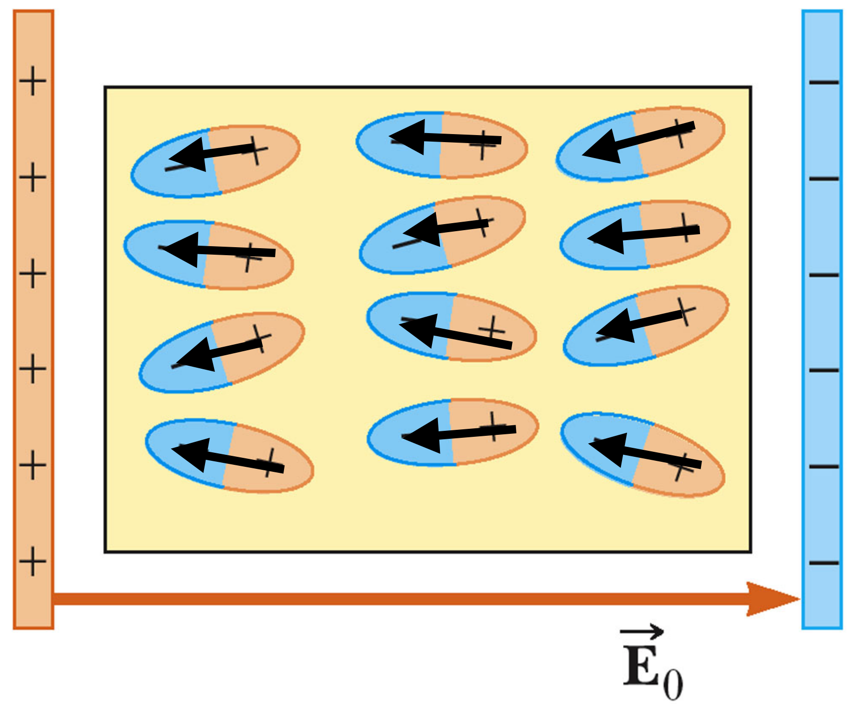




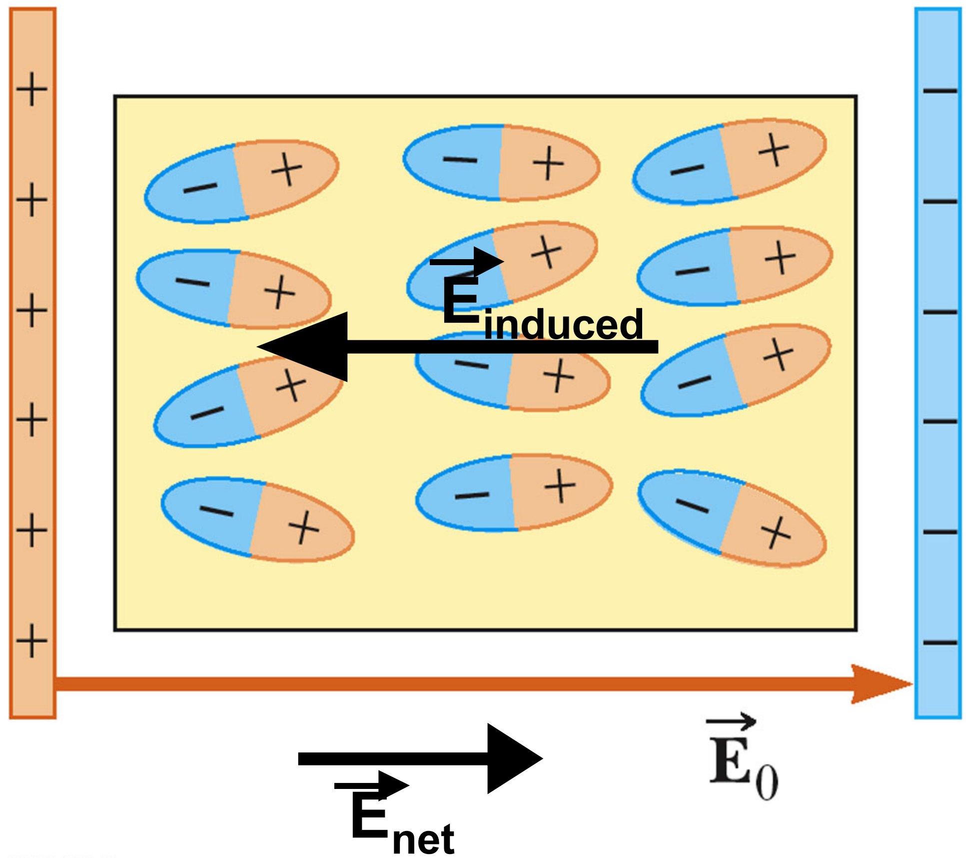
Molecular basis for dielectric constant

Dielectric produces its own E-field ($E_{induced}$)

$E_{net} = E_0 - E_{induced}$

Negative poles of molecules attract more +Q onto positive plate, etc…. so capacitor can hold more charge

(and if a battery's attached, more charge CAN be added)



Dielectric Example 1

Example: You have a capacitor with capacitance C_0, charge it up via a battery so the charge is +/- Q_0, with ΔV_0 across the plates and E_0 inside. Initially $U_0 = 1/2C_0(\Delta V_0)^2 = Q_0^2/2C_0$. Then, disconnect the battery, and then insert a dielectric with dielectric constant κ.

What are C_f, U_f, Q_f, E_f, and ΔV_f?

Isolated system, so $Q_f = Q_0$.

$\Delta V_f = \Delta V_0/\kappa$

$E_f = E_0/\kappa$

$C_f = Q_f/\Delta V_f = \kappa C_0$

$U_f = 1/2\ C_f\ (\Delta V_f)^2 = U_0/\kappa$

Dielectric Example 2

Example: You have a capacitor with capacitance C_0, charge it up via a battery so the charge is +/- Q_0, with ΔV_0 across the plates and E_0 inside. Initially $U_0 = 1/2C_0(\Delta V_0)^2 = Q_0^2/2C_0$. Then, while keeping the connection to the battery, insert a dielectric with dielectric constant κ.

What are C_f, U_f, Q_f, E_f, and ΔV_f?

Battery maintains a constant potential diff.: $\Delta V_f = \Delta V_0$

$E_f = E_0$ ($E_0 = \sigma_0/\varepsilon_0$; $E_f=\sigma_f/\varepsilon = \kappa\sigma_0/\kappa\varepsilon_0$)

$Q_f = \kappa Q_0$.

$C_f = Q_f/\Delta V_f = \kappa C_0$

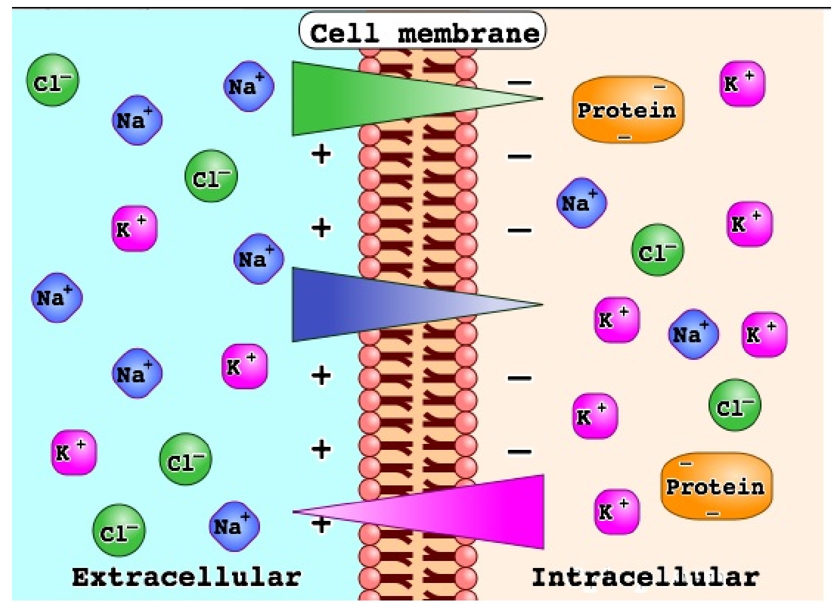
$U_f = 1/2\ C_f\ (\Delta V_f)^2 = \kappa U_0$

Capacitance of Biological Membranes

Neurons:

Outside: Na^+, Cl^-

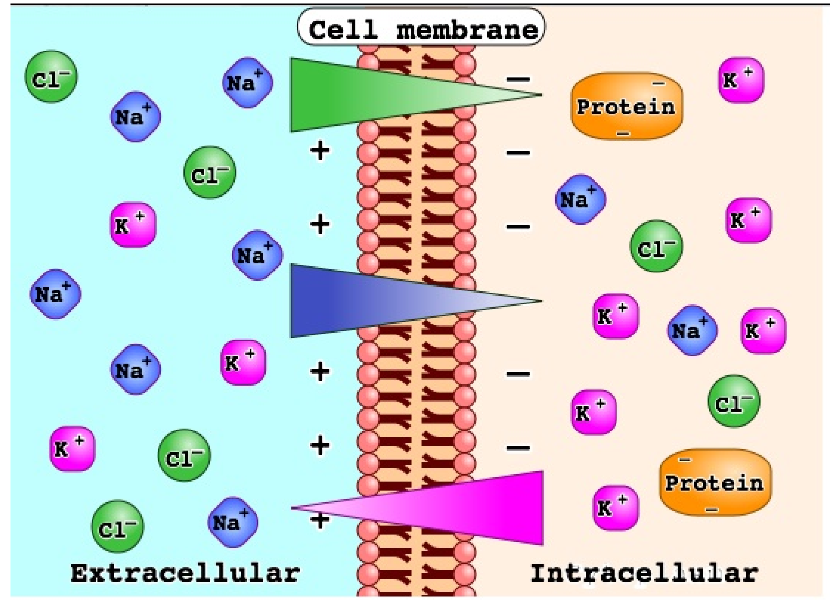
Inside: K^+, neg. organics



No electric pulse: equal amounts of Na^+ and K^+ on either side

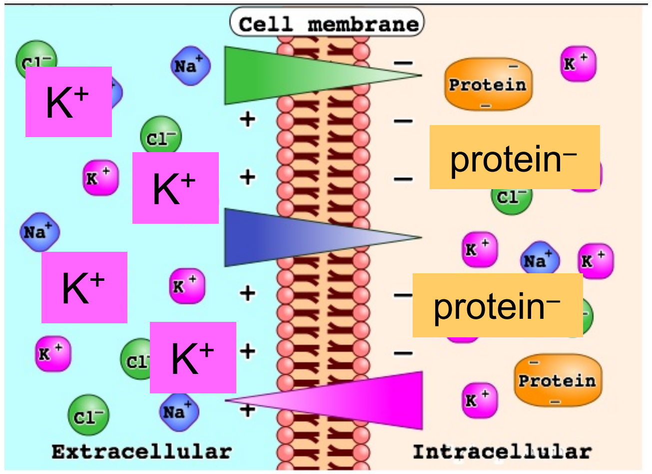
Capacitance of Biological Membranes

Electric pulse: K^+ ions leave the semi-permeable membrane and migrate outside (for every 3 K^+'s that leave, 2 Na^+'s enter). Neg. proteins are too big to go anywhere.



Creates net positive charge outside, net negative inside: ΔV can be as high as ~ 0.05 – 0.10V

Capacitance of Biological Membranes



Typical d is a few nm.

Typical Capacitance is ~ 2 μF per square cm of membrane.

Estimate κ, assuming d = 3nm:

$C = \varepsilon_0 \kappa (A/d) \quad \rightarrow \quad C/A = \varepsilon_0 \kappa / d$

$\kappa = (C/A)d / \varepsilon_0 = (2\text{x}10^{-6}\ \text{F/cm}^2)(10^4\ \text{cm}^2/\text{m}^2)\ (3\text{x}10^{-9}\ \text{m}) / (8.85\text{x}10^{-12}\ \text{C}^2/\text{Nm}^2) = 6.8$

Compare to the teflon (κ=2.1) capacitor, where C/A was 37pF/20cm^2 = 1.9pF/cm^2. Here we have much smaller d (and higher κ).