Physics 1B: Electricity & Magnetism, Fall 2010

Quiz #1, Oct 5, 2010

*This is version **A**!*

Useful coefficients: $g = 9.8$ m s^{-2}
Mass of proton, $m_p = 1.67 \times 10^{-27}$ kg; Mass of electron, $m_e = 9.11 \times 10^{-31}$ kg
Unit of elementary charge $e = 1.6 \times 10^{-19}$ C
Coulomb's constant, $k_e = 8.99 \times 10^9$ N m^2 C^{-2}
Permittivity of free space $\epsilon_0 = \frac{1}{4\pi k_e} = 8.85 \times 10^{-12}$ C^2 N^{-1} m^{-2}

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1. A 1-cm^2 ($= 10^{-4}$ m^2) area of the surface of a balloon is charged by rubbing, and 10^{10} electrons are stripped off, leaving a net positive charge of $10^{10} \times e$ (assume uniform across the 1-cm^2 area). For a point P located 1.0×10^{-7} m from the surface of the balloon, this 1-cm^2 area effectively looks like an infinite plane of charge. Estimate the E-field at point P. (Hint: calculate the surface charge density σ in units of C/m^2.)
   a. 1×10^{15} N/C
   b. 9×10^4 N/C
   c. 9×10^7 N/C
   d. 1×10^7 N/C
   e. 9×10^5 N/C

SOLUTION: The net positive charge is quantified as $10^{10} \times e = 1.6 \times 10^{-9}$ C.
The surface charge density σ = (total charge) / (area) = 1.6×10^{-9} C / 10^{-4} m^{-2} = 1.6×10^{-5} C/m^2.
The electric field from an infinite plane is $E = \frac{\sigma}{2\epsilon_0} = \frac{1.6\times10^{-5}Cm^{-2}}{2\times8.85\times10^{-12}C^2N^{-1}m^{-2}} = 9 \times 10^5$ N/C.
Notice that the distance between point P and the balloon, 1.0×10^{-7} m, does not enter into the above equation – it was chosen only to establish that because the 1-cm^2 area is so much larger than this distance, the E-field can be approximated as that from an infinite plane of charge.
As an aside, the electric field in this case is getting close to the electric field needed to ionize air and allow air to become a conductor, at which point sparks will be visible ($E = 3 \times 10^6$ N/C).

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2. Two identical balls have the same amount of charge, but the charge on ball A is positive and the charge on ball B is negative. The balls are placed on a smooth, level, frictionless table whose top is an insulator. Which of the following is true?
   a. Since the force on A is equal but opposite to the force on B, they will not move.
   b. They will move together with constant acceleration.
   c. Since the force on both balls is negative they will move in the negative direction.
   d. Since the forces are opposite in direction, the balls will move away from each other.
   e. None of the above is correct.

SOLUTION: The two balls will start moving towards each other, but their acceleration will NOT be constant. The acceleration a of each ball is related to the magnitude of the instantaneous electrostatic force F_e via $F_e = ma$, but F_e at any given instant depends on the distance of separation r of the two balls, which is decreasing as the balls roll towards each other. So as r decreases, F_e increases, and the instantaneous acceleration will also be increasing (not constant). So the answer is "none of the above."

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3. A ping-pong ball covered with a conducting graphite coating has a total mass of 2 grams and carries an excess charge of -0.1μC. What is the magnitude and direction of the electric field needed to exactly counterbalance the downward weight of the ball? ($g = 9.8$ m s^{-2})

a. 2.0×10^5 N/C, upward
b. 2.0×10^5 N/C, downward
c. 2.0×10^8 N/C, upward
d. 2.0×10^{-1} N/C, downward
e. 2.0×10^8 N/C, downward

SOLUTION: We need to counterbalance the gravitational force $\vec{F_g}$ pulling the ball downward with an electrostatic force $\vec{F_e}$ pulling the ball upward. Because the charge is negative, $\vec{F_e}$ and $\vec{E}$ will point in opposite directions, so we need an electric field whose field lines point **downward**.
We have $F_g = mg$ where g is the gravitational acceleration and $F_e = qE$, and the magnitudes of the two forces are equal.
mg = qE
E = mg/q = (0.002kg)(9.8 m s^{-2})/(1×10^{-7}C) = 2.0×10^5 N/C

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4. The following charges are all enclosed by a Gaussian surface: +6C, +4C, –2C, and –4C. What is the net electric flux exiting the surface?

a. $+4\text{C}/\epsilon_0$
b. $+16\text{C}/\epsilon_0$
c. $-4\text{C}/\epsilon_0$
d. $-16\text{C}/\epsilon_0$
e. $+2\text{C}/\epsilon_0$

SOLUTION: From Gauss' Law, the net flux will be equal to the total charge enclosed divided by ϵ_0.
For net positive charge enclosed, there will be electric field lines exiting the closed surface, and the net electric flux will be positive.
The net electric flux exiting the surface here is $(+6\ +4\ -2\ -4)\text{C}/\epsilon_0 = \mathbf{+4C/}\epsilon_0$

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5. Four objects, A, B, C and D, are located in the x-y plane. They each have a net charge of $+4\mu$C, $+3\mu$C, $+1\mu$C, and $+1\mu$C, respectively. Object A is at located at the origin (x=0,y=0). Objects B, C, and D are located at (x = +0.5 cm, y = 0), (x = 0, y = –0.5 cm), and (x=-0.3cm, y=+0.3cm), respectively. Which of the following best represents the magnitude of the electric field at a point P very far away along the +y-axis, at (x = 0, y = +0.9 m) (90 cm from the origin)?

a. 9×10^3 N/C
b. 3×10^6 N/C
c. 1×10^5 N/C
d. 3×10^2 N/C
e. 1×10^9 N/C

SOLUTION: The key here is the phrase "best represents" – we can do some simplifications here. Note that we're trying to measure the electric field at a distance r which is much, much greater than the separations of the four charges clustered close to the origin. So at

point P, the total electric field from all four charges will be very similar to the electric field from a *single* charge that is located very close to the origin and whose charge is equal to the sum of the four charges above, 4μC + 3μC + 1μC + 1μC = 9μC.
The electric field will be approx. equal to $\frac{k_e Q}{r^2} = \frac{9\times10^9 Nm^2C^{-2} 9\times10^{-6}C}{(0.9m)^2} = 1 \times 10^5$ N/C.

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6. What happens to the net electric flux Φ_E through the surface of a sphere (closed Gaussian surface) when its radius is tripled? Assume that no new charges become encompassed by the sphere when the radius is increased.

   a. It increases by a factor of 9.
   b. It is tripled.
   c. It remains the same.
   d. It will be 1/3 the original value.
   e. It will be 1/9 the original value.

SOLUTION: The answer is the same for the case where the Gaussian surface encloses some charge or encloses no charge – there is no change in Φ_E.
In the first case, Gauss' Law related the electric flux to the charge enclosed WITHOUT having a dependence on radius. Recall for the case of a single point charge that the r^2 dependence of the area of the sphere on radius is exactly cancelled by the $1/r^2$ dependence of the electric field strength.
In the latter case, the net electric flux through ANY closed Gaussian surface which encloses no charge is zero – and changing the radius of our Gaussian sphere will thus have no impact on Φ_E.

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7. Object A is illustrated with 10 electric field lines coming out from it, Object B has 20 lines coming out, Object C has 25 lines coming out, and Object D has 30 lines coming out. Which pair of these charges will have the largest electrostatic force between them if placed one cm apart?

   a. A and B
   b. B and C
   c. A and D
   d. C and D
   e. More information is needed.

SOLUTION: The magnitude of the electrostatic force is $F_e = \frac{k_e q_1 q_2}{r^2}$, i.e., it's proportional to the product of the magnitudes of each charge. Recall that the number of field lines terminating or beginning on any object is directly proportional to the amount of charge on that object. Since objects C and D have the two largest values of charge, these two will yield the strongest electrostatic force between them.

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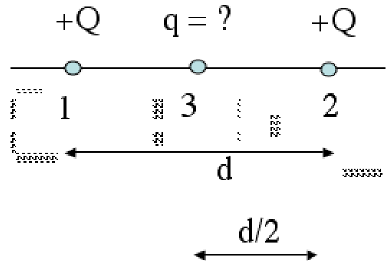
8. Two objects (#1 & #2) with equal charge, each +Q, are separated by some distance d. What third charge would need to be placed halfway between the two charges so that the net force on either charge #1 or #2 would be zero?

   a. –2Q
   b. –Q
   c. –Q/2
   d. –Q/4
   e. –Q/8

SOLUTION: Label the charges as shown: objects 1 and 2 each have charge +Q and are



separated by a distance d. Let's say the third object has some unknown charge q_3.

The force on object 2 due to the electric field associated with object 1 is $\vec{F_{12}}$. The force on object 2 due to the electric field associated with object 3 is $\vec{F_{32}}$. These two vectors must sum up to be zero.

$\vec{F_{12}} + \vec{F_{32}} = 0.$

$\frac{k_e QQ}{d^2} + \frac{k_e Q q_3}{(d/2)^2} = 0$

Cancel out the k_e and one Q from each side:

$\frac{Q}{d^2} + \frac{q_3}{(d/2)^2} = 0$

$\frac{Q}{d^2} + \frac{4q_3}{d^2} = 0$

$Q + 4q_3 = 0$

$q_3 = -Q/4$

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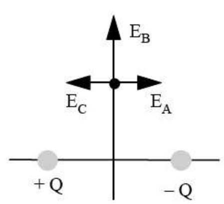
9. Two point charges, +Q and –Q, are located two meters apart and there is a point P



equidistant from the two charges as indicated. Which vector best represents the direction of the electric field at that point?

a. Vector E_A
b. Vector E_B
c. Vector E_C
d. The electric field at that point is zero.
e. None of the above

SOLUTION: The electric field lines originate on the positive charge and terminate on the negative charge. Everywhere along the line which is equidistant from both charges, the field lines will be pointing exactly to the right. The answer is Vector E_A. See Figure 19.17 in the text.