Physics 1B: Electricity & Magnetism, Fall 2010

Quiz #2, Oct 19, 2010

This is version ***A****!*

Useful coefficients: $g = 9.8$ m s^{-2}; Unit of elementary charge $e = 1.6 \times 10^{-19}$ C
Mass of proton, m_p = Mass of neutron, $m_n = 1.67 \times 10^{-27}$ kg; Mass of electron, $m_e = 9.11 \times 10^{-31}$ kg
Coulomb's constant, $k_e = 8.99 \times 10^9$ N m^2 C^{-2}
Permittivity of free space $\epsilon_0 = \frac{1}{4\pi k_e} = 8.85 \times 10^{-12}$ C^2 N^{-1} m^{-2}
Prefixes: micro (μ) = 10^{-6}; nano (n) = 10^{-9}; pico (p) = 10^{-12}; fempto (f) = 10^{-15}

1. Points X, P_1, and P_2 are three separate points in space. When charge Q_1 is placed at point P_1, the resulting potential at point X is V_1. Then, when Q_2 is placed at point P_2, the resulting potential at point X becomes V_2. What is the potential at point X if charge Q_1 is then removed?
a. $V_2/2$
b. $V_1 - V_2$
c. $V_2 - V_1$
d. $(V_1 + V_2)/2$
e. V_2

SOLUTION: The total potential at any point in space is due to the sum of the potentials from each charge. The answer is $\mathbf{V_2 - V_1}$, as the potential due to only Q_1 (which is V_1) plus the potential due to only Q_2 (which is $V_2 - V_1$) equals the potential due to both (V_2).

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2. Consider a uniform electric field of 500 N/C pointing in the $+\hat{x}$ direction. You wish to move the nucleus of a completely-ionized carbon atom (12 protons, 12 neutrons) a distance of 10 cm in the $-\hat{x}$ direction. How much work do you need to provide?
a. 600 eV
b. 500 eV
c. 250 eV
d. 60 eV
e. zero

SOLUTION: The magnitude of the amount of work needed is
$W = qEd = 12(1.6 \times 10^{-19}C)(500N/C)(0.1m) = 9.6 \times 10^{-17}$ J $= 600$ eV.
Note that the amount of work is greater than zero (i.e., you must supply work), because you're trying to move the nucleus (with excess positive charge) from a region of relaively lower potential to relatively higher potential (against the E-field lines and against the desire of the nucleus which, in this field, will want to go in the +x direction).

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3. An ion is released from rest and moves due to the force from an electric field from a position in the field having a potential of 1.44 V to a position having a potential of 2.88 V. The ion:
a. must have a positive charge.
b. must have a negative charge.
c. can have either a positive or negative charge.
d. must have a charge exactly equal to +2e
e. must have a charge exactly equal to –2e

SOLUTION: Positively-charged particles want to move to regions of lower potential, by definition. Negatively-charged particles, then, want to move to regions of higher potential. The answer is "b."

Choice e must be rejected, because we're given no information which could constrain the magnitude of the charge (such as potential energy difference, or work the electric field does in moving the charge); based on the information given, only the sign of the charge can be constrained.

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4. Consider the following configurations of protons and electrons, with charges spaced evenly (e.g., located on the x-axis at positions x=0, x=1d, and x=2d).

a. $+ \quad + \quad -$

b. $+ \quad - \quad +$

c. $+ \quad - \quad -$

d. $+ \quad + \quad +$

Which configuration is the most stable?

a. a
b. b
c. c
d. d

SOLUTION: Let's label the charges from 1 to 3, going from left to right.
There are three interactions in each group: two with distance of 1d (1–2, 2–3), one with distance of 2d (1–3). P.E. in units of k_e/d are:

	1–2	2–3	1–3	Total
a.	(+1e)(+1e)/1	(+1e)(–1e)/1	(+1e)(–1e)/2	$(-1/2)e^2$
b.	(+1e)(–1e)/1	(–1e)(+1e)/1	(+1e)(+1e)/2	$(-3/2)e^2$
c.	(+1e)(–1e)/1	(–1e)(–1e)/1	(+1e)(–1e)/2	$(-1/2)e^2$
d.	(+1e)(+1e)/1	(+1e)(+1e)/1	(+1e)(+1e)/2	$(+5/2)e^2$

As choice d has a positive total potential energy, it is the most unstable of the four and will not be discussed further.

Given that there is a mixture of positive and negative charges in each of the a, b, and c configurations, it should not be a surprise that the total P.E. for each choice is negative for those choices.

But **choice b**'s total potential energy is closest to $-\infty$, hence it's the most stable configuration of those three.

Another of thinking about this problem is more qualitative: In choice c, there are two electrons; they'll repel each other and want to try to more as far apart from each other as they can. In choice a, there are two protons, and they will repel each other as well. In choice b, there are two protons, but they are *already* farther apart from each other than the two protons in choice a. This means there is relatively less tendency in configuration b for the system to push itself apart.

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5. Electrons in an X-ray machine are accelerated from rest through a potential difference of 70 kV (value taken from the X-ray machine in my dentist's office). What is the kinetic energy of each of these electrons in eV after they've been accelerated?
a. 70 eV
b. 112 eV
c. 140 eV
d. 70 keV
e. 112 keV

SOLUTION: After acceleration from rest, the energy of each electron in units of eV is the potential V times the charge in units of e. A single electron thus has energy **70,000 eV**.

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6. A parallel-plate capacitor has a plate area of 1×10^{-3} m^2 and a capacitance of 8.85 pF. Its plates are connected to the terminals of a 10 Volt battery. Find the magnitude of the electric field between the plates.
a. 1×10^1 V/m
b. 1×10^3 V/m
c. 1×10^4 V/m
d. 1×10^5 V/m
e. 1×10^7 V/m

SOLUTION: The capacitance of a parallel-plate capacitor is $C = A\epsilon_0/d$, which can be rearranged to solve for the plate separation $d = A\epsilon_0/C$.

Plugging in values, we find $d = (10^{-3}\ \text{m}^{-2})(8.85 \times 10^{-12}\ \text{C}^2\ \text{N}^{-1}\ \text{m}^{-2})\ /\ (8.85 \times 10^{-12}\ \text{F}) = 10^{-3}\text{m} = 1$ mm.

The voltage difference ΔV between the plates will be equal to $E \times d$ (we can ignore the minus sign typically found in ΔV = –Ed and just worry about the magnitude of the drop), so we can solve for the magnitude of the electric field $E = \Delta$V/d = ΔV / $(A\epsilon_0/C)$ = 10V / 10^{-3}m = 10^4 V/m.

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7. A single electron is accelerated in a cathode ray tube from rest through a potential difference of 1250 V. Find its final velocity at the end of this process.
a. 5.9×10^5 m/s
b. 6.3×10^6 m/s
c. 4.9×10^5 m/s
d. 1.2×10^7 m/s
e. 2.1×10^7 m/s

SOLUTION: The change in potential energy is $\Delta U = q\Delta V = (1.6 \times 10^{-19} C)(1250V) = 2.0 \times 10^{-16} J$

Conservation of total energy dictates that this potential energy difference will be converted into kinetic energy:
K = $\frac{1}{2} m_e v_\text{f}^2$
$v_\text{f} = \sqrt{2K/m_e} = \sqrt{2\Delta U/m_e} = 2.1 \times 10^7$ **m/s**

- 8. Many familiar electrostatic phenomenona we enounter from having two insulating surfaces rubbing together are associated with generating surprisingly high voltage differences. Consider four objects placed on a planar grid at (x = 0, y = +10cm), (x = +10cm, y = 0), (x = 0, y = –10cm), & (x = –10cm, y = 0). Each object has an identical excess charge of –10 nC. Calculate the total absolute potential V at the origin (x = 0, y = 0) due to the associated electric fields (V=0 an infinite distance away).

a. –900 V
b. –36 V
c. –3600 V
d. +4000 V
e. +900 V

SOLUTION: The absolute potential (setting V=0 at r=∞) a distance r from a point charge Q is V= $k_e Q/r$.

When r = 10cm and Q = -5×10^{-9}C, V = $\frac{(9\times10^9 Nm^2C^{-2})(-10\times10^{-9}C)}{0.1m}$ =–900 V

The TOTAL potential at the origin will be 4 times this: **–3600 V**

This may seem high, but "doorknob sparks" usually involve potential differences of several thousands volts, and even very small "static sparks" can have ΔV of 1000 volts.

The electrical discharge (spark) in air occurs when you put two objects close enough together to generate an E-field above 3×10^6 V/m – this allows air to become a conductor and excess charge to jump the air gap and flow between objects.

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9. Point A is located on the origin (0,0) of a 2-dimensional Cartesian grid. Point B is located at (x,y) coordinates (3m, 4m). Throughout the region, the electric field is described by $\vec{E}$ = $(-2\hat{\mathbf{i}} + -1\hat{\mathbf{j}})$ N/C. What is the potential difference between points A and B, $V_B - V_A$? (Reminder: $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ are the unit vectors pointing in the +x and +y directions, respectively. And, a hint, if needed: think about the potential at (x=3m,y=0)).

a. 12 V
b. 6 V
c. –6 V
d. 10 V
e. –12 V

SOLUTION: We can break the electric field into its x- and y-components, and calculate the potential as a function of each of the x- and y-components.

$E_x = -2\hat{\mathbf{i}}$ N/C.

So we integrate with respect to x and multiple by –1 to obtain V_x = +2x Volts (Potential V(x,y) increases linearly with increasing x).

The potential difference between point A, the origin, and the point (x=3m, y=0) is then

$\Delta V_x = -E_x \times \Delta x = (+2V/m)(3m) = 6$ V.
$E_y = -1\hat{\mathbf{j}}$ N/C.
So we integrate with respect to y and multiple by –1 to obtain $V_y = +1y$ Volts (Potential V(x,y) increases linearly with increasing y).
The potential difference between the point (x=3m, y=0) and point B is then $\Delta V_y = -E_y \times \Delta y = (+1V/m)(4m) = 4$ V.
The total increase in potential from point A to (x=3m,y=0) and then up to point B is thus 10 V.
In conclusion, V(x,y) = +2x + 1y Volts.
At point A: V(0,0) = 0 Volts
At point B: V(3m,4m) = +2(3) + 1(4) volts = +10 volts.

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10. All but one of the statements below describe a property of a conductor in electrostatic equilibrium. Which statement is not correct?
a. The electrostatic field is zero everywhere inside the conductor.
b. A net charge on an isolated conductor resides entirely on its surface.
c. The electric field just outside any point on the surface of a charged conductor is perpendicular to the surface of the conductor.
d. The surface charge density is highest at locations where the radius of curvature is large, e.g., where the surface is essentially flat.
e. The magnitude of the electric field at a point just outside a charged conductor is σ/ϵ_0, where σ is the surface charge density at that point.

SOLUTION: Choice D is incorrect: σ is higher at relatively sharper points (smaller radius of curvature)

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