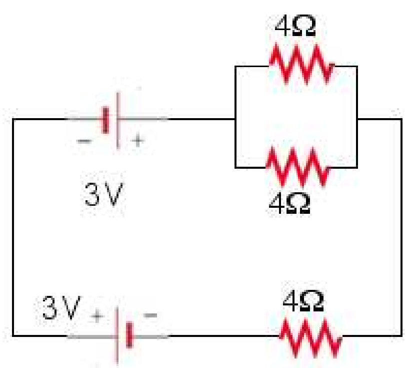
Physics 1B: Electricity & Magnetism, Fall 2010
Quiz #4, Nov. 18, 2010

This is version **A**!

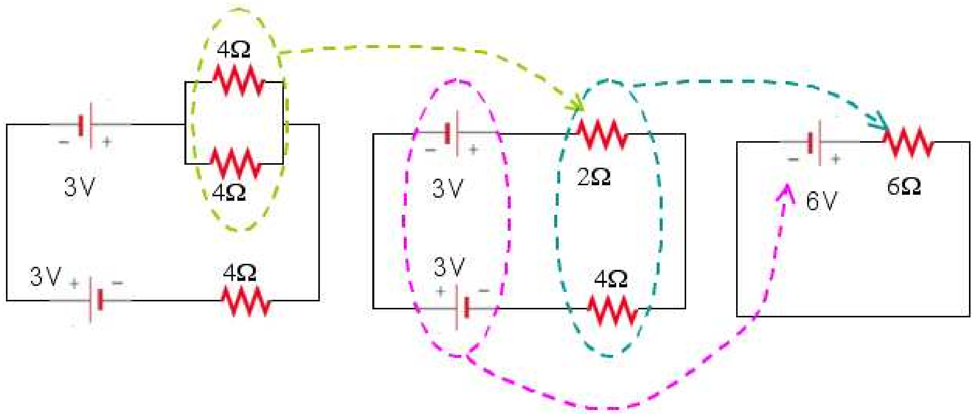
Useful coefficients: $g = 9.8$ m s^{-2}; Unit of elementary charge $e = 1.6 \times 10^{-19}$ C
Mass of proton, m_p = Mass of neutron, $m_n = 1.67 \times 10^{-27}$ kg; Mass of electron, $m_e = 9.11 \times 10^{-31}$ kg
Coulomb's constant, $k_e = 8.99 \times 10^9$ N m^2 C^{-2}
Permittivity of free space $\epsilon_0 = \frac{1}{4\pi k_e} = 8.85 \times 10^{-12}$ C^2 N^{-1} m^{-2}
Prefixes: milli (m) = 10^{-3}; micro (μ) = 10^{-6}; nano (n) = 10^{-9}; pico (p) = 10^{-12}; fempto (f) = 10^{-15}
1 eV = 1.6 $\times 10^{-19}$ J
1 T = 10^4 G

1. Here's a circuit:



What is the magnitude and direction of the resulting current?
a. 1.0 A, clockwise
b. 2.0 A, clockwise
c. 1.0 A, counter-clockwise
d. 2.0 A, counter-clockwise
e. 0.5 A, clockwise

Solution: Combine resistors using the Parallel Law to make the circuit a bit more simple: When two 4Ω resistors are in parallel, you can calculate the equivalent resistance as $1/R_{eq} = 1/(4\Omega) + 1/(4\Omega) = 1/(2\Omega)$, or $R_{eq} = 2\Omega$.



Now we have a circuit where everything's connected in series (see middle diagram). We can then use the series law to simplify the circuit even further: $R_{eq} = 2\Omega + 4\Omega = 6\Omega$.
We can also sum up the individual EMFs supplied by each battery to find the total EMF available to power the circuit. Both batteries' terminals are oriented such they want to drive current in a clockwise direction. The circuit is equivalent to being powered by a single battery with an EMF of 3V + 3V = 6V. Our final simplied circuit is shown on the right.
Using ϵ = IR for our simplified/equivalent circuit; we find I = ϵ/R = 6V/6Ω = 1.0 A.
Final answer = 1.0 A, clockwise

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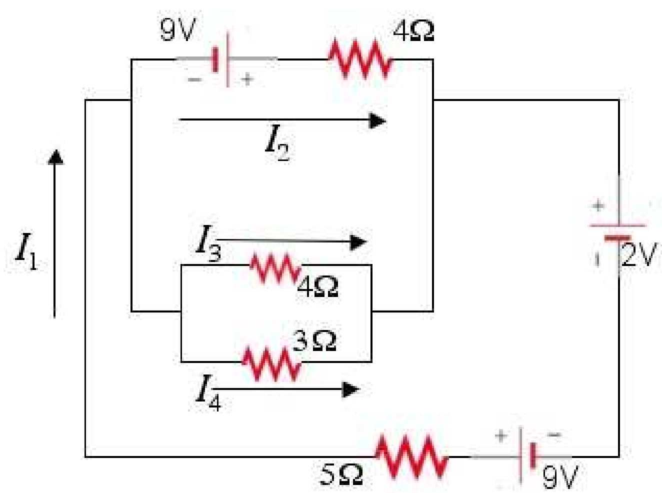
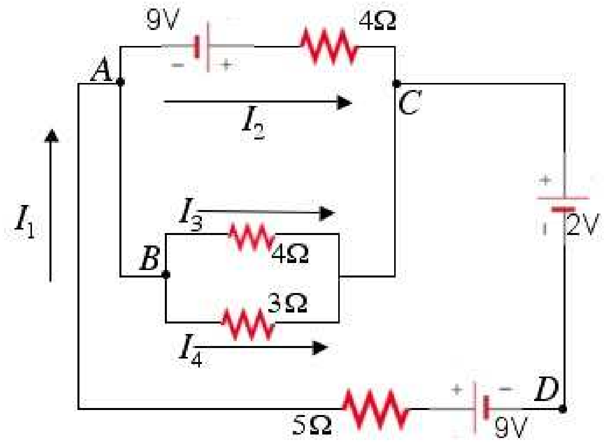
2. Consider the following circuit:



Which of the following equations is a valid result from applying Kirchhoff's Loop and/or Junction Rules?
a. $+9V + I_1 + 5\Omega - 2V = 0$
b. $I_2 = I_3 + I_4$
c. $+9V + (5\Omega)(I_1) + 9V + (4\Omega)(I_2) + 2V = 0$
d. $(4\Omega)(I_3) = (3\Omega)(I_4)$
e. $+2V - (4\Omega)(I_3) + (5\Omega)(I_1) - 9V = 0$

Solution: The Junction Rule will give us an equation in units of current (Amps). The Loop Rule will give an equation in units of voltage (Volts). So, first off, choice A should be rejected immediately, as its units are a non-sensible mix of current, resistance values, and voltages.

Let's re-draw the circuit with certain junctions/ points in the conducting wires labelled for clarity:



At point A, current I_1 splits up into I_2 and what I'll call I_{34}, the value of the current between points A and B. That is, at point A, $I_1 = I_2 + I_{34}$ is a valid application of the Junction Rule. At point B, I_{34} splits up into I_3 and I_4. So at point B, $I_{34} = I_3 + I_4$ also follows from the Junction rule. But choice b is not a valid application of the Junction rule at any junction and is thus rejected.

If one starts at the lower-right corner (point D) and travels in a loop clockwise, and along the top-most branch between points A and C, the loop rule gives you $+9\text{V} - (5\Omega)(I_1) + 9\text{V} - (4\Omega)(I_2) - 2\text{V} = 0$. You're traveling with the current at all locations, so ΔV is negative when traversing resistors, ΔV is positive when traversing each 9V battery (neg. to pos.) and ΔV is negative when traversing the 2V battery (pos. to neg.). While this expression is similar to choice c, some of the signs in choice c are wrong, so choice c is eliminated.

If one starts at point D and travels counter-clockwise, going against the flow of current, and going along the middle branch to get from point C to point A, the loop rule yields $+2\text{V} + (4\Omega)(I_3) + (5\Omega)(I_1) - 9\text{V} = 0$. While this expression is similar to choice e, one of the signs in choice e is wrong, so choice e is eliminated.

Choice d is the correct answer: If one starts at point B and travels clockwise around the smallest loop only, the loop rule yields $-(4\Omega)(I_3) + (3\Omega)(I_4) = 0$ (the latter term is positive because you're going against the current while in the lower branch and you will experience a voltage increase). This expression can be re-written as $(4\Omega)(I_3) = (3\Omega)(I_4)$.

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3. A capacitor (C = 40μF) is initially uncharged, but is connected to a battery (ϵ = 10V), a resistor (R = 50Ω), and a switch in series. The switch is closed at time t=0. Charge begins to accumulate on the capacitor. At what time t after the switch is closed will the voltage difference between the plates of the capacitor, ΔV_{C}, be equal to one-half of the battery's EMF?
a. 2.00 milliseconds
b. 1.00 milliseconds
c. 1.39 milliseconds
d. 2.89 milliseconds
e. 0.60 milliseconds

Solution: The time constant is τ = RC = (40 $\times 10^{-6}$F)(50Ω) = 0.002s = 2 ms.
The expression for the voltage drop across the capacitor is $\Delta V_{\mathrm{C}} = \Delta V_{\mathrm{C,max}}(1 - \mathrm{e}^{-t/\tau})$ where $\Delta V_{\mathrm{C,max}} = \epsilon$ (Reminder: ΔV_{C} is initially zero, but increases towards its maximum value, ϵ, as charge accumulates).

ΔV_{C} / $\Delta V_{\mathrm{C,max}} = (1 - \mathrm{e}^{-t/\tau})$
$0.5 = (1 - \mathrm{e}^{-t/\tau})$
$-0.5 = -\,\mathrm{e}^{-t/\tau}$
$0.5 = \mathrm{e}^{-t/\tau}$
Take the natural logarithm of both sides:
$\ln(0.5) = -t/\tau$
$-0.693 = -t/\tau$
$t = (0.693)(\tau)$

At one time constant, t=τ, ΔV_{C} reaches (1 – 1/e) = 0.63 of the maximum value. So it makes sense that ΔV_{C} reaches 0.50 of its maximum value at an earlier time.
t = (0.693)(2 ms) = **1.39 ms.**

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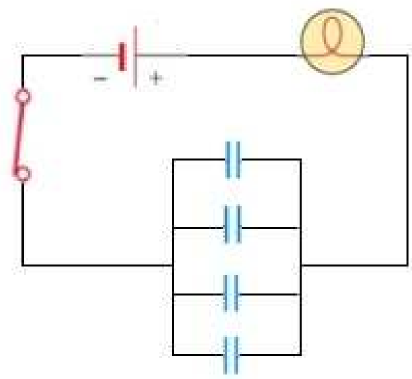
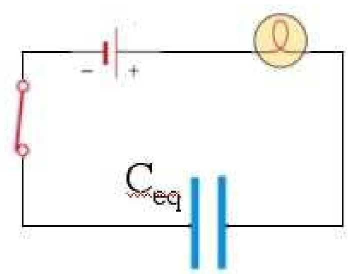


4. At lecture last Tuesday, we saw the RC circuit demonstration which contained a light bulb as the resistor, and a bank of four capacitors connected in parallel (circuit diagram shown above). Suppose you had a similar circuit wherein the equivalent capacitance C_{eq} was 0.25 F, the resistance of the light bulb filament was 4Ω (ignore any temperature dependence), and the battery supplied an EMF of 72 Volts. Recall that the power dissipated in a resistor at any given instant in time will be proportional to the square of the current ($P = I^2R$). What is the instantaneous power output of the light bulb at a time $t = 2.5$ seconds after the switch is closed?
a. 0.33 W
b. 0.87 W
c. 1.48 W
d. 4.38 W
e. 8.76 W

Solution: Because we are already given the equivalent capacitance for the bank of four capacitors in parallel, we could re-draw the circuit diagram to be a bit more simple:



Now the circuit digram shows all the circuit elements in series, a familiar configuration.
The time constant $\tau = \text{R}C_{\text{eq}} = (4\Omega)(0.250 \text{ F}) = 1.0$ sec.
For either charging or discharging the capacitor, the current as a function of time will follow an exponential decay:
$\text{I(t)} = \text{I}_{\text{max}} e^{-t/\tau}$
where $\text{I}_{\text{max}} = \epsilon/\text{R} = 72\text{V}/4\Omega = 18$ A.
So $\text{I}(t\text{=2.5s}) = (18\text{A})e^{-2.5s/1.0s} = (18\text{A})e^{-2.5} = (18\text{A})(0.082) = 1.48$ A.
$\text{P} = \text{I}^2\text{R} =$ **8.76 W**

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5. A soft magnetic material has which property?
a. It attracts only slowly moving charges.
b. It cannot be magnetized in the presence of an external magnetic field.
c. It is easy to magnetize in the presence of an external magnetic field.
d. It is hard to magnetize in the presence of an external magnetic field.
e. It is comprised of relatively low-density metal.

Solution: Easy to magnetize in the presence of an external magnetic field.

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6. A stray electron, part of the solar wind, is moving at 3 km/s parallel to the Earth's surface, in an easterly direction, crossing the Earth's magnetic equator at a point in space where the magnetic field points due north, is parallel to the surface, and has a magnitude of 0.3 Gauss. What is the force (magnitude and direction) acting on the electron at this instant?
a. 1.4×10^{-20} N, downward (towards the Earth's surface)
b. 4.3×10^{-17} N, upward (away from the Earth's surface)
c. 1.4×10^{-20} N, upward (away from the Earth's surface)
d. 1.4×10^{-17} N, in a westerly direction
e. 4.3×10^{-20} N, downward (towards the Earth's surface)

Solution: Use $\vec{F_{\mathrm{B}}} = \mathrm{q}\vec{v} \times \vec{B} = \mathrm{q\ v\ B}\sin(\theta)$.
For an ion/proton with a charge of PLUS 1e, we can calculate the magnitude of $\mathrm{F_B}$ as $(1.6 \times 10^{-19}\ \mathrm{C})(3000\ \mathrm{m/s})(0.3 \times 10^{-4}\ \mathrm{T})\sin(90^\circ) = \mathbf{1.4} \times 10^{-20}$ **N.**
For a POSITIVELY-charged ion/proton, we use either right-hand rule: $\vec{v}$ points E (thumb); $\vec{B}$ points north (fingers); so the palm of your hand points upward, so $\vec{F_{\mathrm{B}}}$ for a positively charged particle points upward, away from the Earth's surface.
Using the right hand rule where you curl your fingers from E (first vector) towards N (second vector), your thumb also points upward, defining the direction of $\vec{F_{\mathrm{B}}}$ for a positively charged particle as upward.
The magnitude of the magnetic force on a charge of −1e will be the same as this, but the direction will be reversed: in this case, the magnetic force points **downwards, towards the Earth's surface.**

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7. At the Fermilab accelerator in Illinois, a stream of protons has been accelerated to a velocity of 1.0×10^{7} m/s and held in a circular orbit of radius 15.0 m by a vertical magnetic field. What magnetic field strength must be used to maintain this orbit?
a. 7×10^{-7} G
b. 0.0070 G
c. 12 G
d. 70 G
e. 1200 G

Solution: We can use $r = (m_p v)/(qB)$.
Rearranging to solve for B, we get $B = (m_p v)/(rq) = (1.67\times10^{-27}\ \mathrm{kg})(1.0\times10^{7}\ \mathrm{m/s}) / (15.0\ \mathrm{m})(+1.6\times10^{-19}) = 0.007$ T
The final step is to convert to Gauss (since all the choices are in Gauss): 0.007T = **70 Gauss**.

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8. You wish to design a velocity selector such that the velocity of each electron passing through it undeflected will be 1.0×10^{6} m/s. You have a magnet which produces a uniform magnetic field of 0.01 T. You want to produce just the right electric field (oriented perpendicular to the magnetic field) using a parallel-plate capacitor whose plates are separated by 1.0 cm. What voltage difference across the plates of the capacitor is required?
a. 10 V
b. 100 V
c. 1000 V
d. 10^{4} V
e. 10^{5} V

Solution: For a velocity selector, $v = E/B$.
$E = Bv = (0.01\mathrm{T})(1.0\times10^{6}\ \mathrm{m/s}) = 1.0\times10^{4}$ V/m.
For parallel-plate capacitors, $\Delta V = Ed = (1.0\times10^{4}\ \mathrm{V/m})(0.01\mathrm{m}) =$ **100 V**.

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9. A charged particle moves in a circular path in a plane perpendicular to a uniform magnetic field. Which statement below best describes the work the field does on the particle?
a. Because the particle always moves counterclockwise, the work done is always positive.
b. The field does not do any work on the particle.
c. Work is done at an ever-increasing rate.
d. Work done on the particle is negative.
e. Work done on a positively-charged particle is positive; work done on a negatively-charged particle is negative.

Correct answer = B. The magnetic field changes the direction of the particle's motion (the direction of $\vec{v}$ changes) but the field has no effect on the magnitude of $\vec{v}$ because $\vec{F}_B$ is always perpendicular to $\vec{v}$. The kinetic energy of the particle thus does not change.

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Please double-check that your intended choices are bubbled in correctly on the scantron form. Double-check that you've bubbled in the test version correctly – this is version A ! And please double-check to make sure you have bubbled in your Exam Code Number on the scantron form correctly!