CHAPTER 19

2. (a) Number of moles $= \dfrac{10.0\,g}{107.87\,g/mol} = 0.0927\,mol$

Number of atoms = Number of moles × Avogadro's num

$= 0.0927\,mol \times 6.022 \cdot 10^{23}\,mol^{-1} = 5.77 \cdot 10^{22}$

Number of electrons = number of atoms × 47 $= \underline{2.71 \cdot 10^{24}}$

(b) Total number of added electrons $= \dfrac{\text{total charge}}{\text{elem. charge}}$

$= \dfrac{1.00 \cdot 10^{-3}\,C}{1.6 \cdot 10^{-19}\,C} = 6.25 \cdot 10^{15}$

Number of added electrons for every 10^9 electrons already present $= \dfrac{6.25 \cdot 10^{15}}{2.71 \cdot 10^{24}} \cdot 10^{9} = \underline{2.3}$

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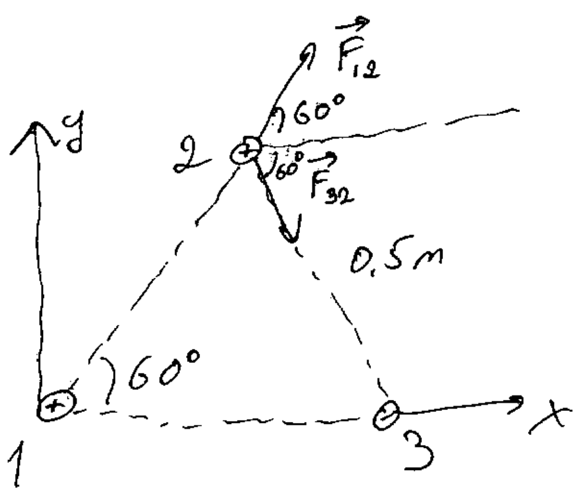


$q_1 = 2.00\,\mu C = 2 \cdot 10^{-6}\,C$

$q_2 = 7.00\,\mu C = 7 \cdot 10^{-6}\,C$

$q_3 = -4.00\,\mu C = 4 \cdot 10^{-6}\,C$

$F_{12} = \dfrac{k|q_1||q_2|}{r^2} = 0.504\,N$

$F_{32} = \dfrac{k|q_3||q_2|}{r^2} = 1.008\,N$

$F_{12x} = F_{12} \cdot \cos 60° = 0.252\,N$

$F_{12y} = F_{12} \cdot \sin 60° = 0.436\,N$

$F_{32x} = F_{32} \cdot \cos(-60°) = 0.504\,N$

$F_{32y} = F_{32} \sin(-60°) = -0.873\,N$

$\vec{F} = \vec{F}_{12} + \vec{F}_{32}$

$F_x = F_{12x} + F_{32x} = 0.756\,N$

$F_y = F_{12y} + F_{32y} = -0.437\,N$

Magnitude of force:

$F = \sqrt{F_x^2 + F_y^2} = \underline{0.873\,N}$

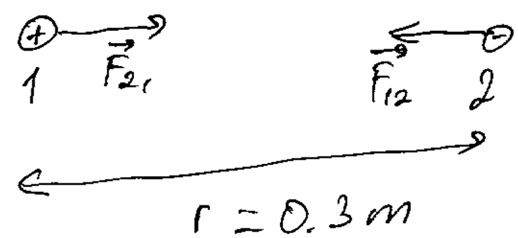
Direction with respect to x-axis:

$\tan\varphi = \frac{F_y}{F_x}$ $\qquad \varphi = \text{atan}\left(\frac{-0.437\,N}{0.756\,N}\right) = \underline{-30°}$

i.e. $\underline{30° \text{ below } x\text{-axis.}}$

7 (a) $q_1 = 12.0\,nC = 12 \cdot 10^{-9}\,C$

$q_2 = -18.0\,nC = -18 \cdot 10^{-9}\,C$



$$|\vec{F}_{21}| = |\vec{F}_{12}| = \frac{k|q_1||q_2|}{r^2} = \underline{2.16 \cdot 10^{-5} N}$$

(B) Since the spheres are conducting and identical, they will have the same charge after being connected by a conducting wire. By conservation of charge, the total charge will be

$$Q = q_1 + q_2 = -6 \cdot 10^{-9} C$$

Each sphere will have

$$q_1' = q_2' = \frac{Q}{2} = -3 \cdot 10^{-9} C$$

$$|\vec{F}_{21}'| = |\vec{F}_{12}'| = \frac{k|q_1'||q_2'|}{r^2} = \underline{9 \cdot 10^{-7} N}$$

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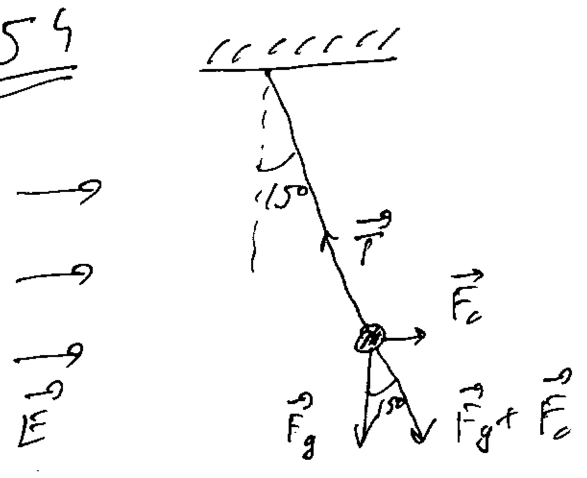


$m = 2g = 2 \cdot 10^{-3} kg$

$\ell = 20 cm = 0.2 m$

$E = 1 \cdot 10^3 N/C$

Since the ball is in equilibrium, the sum of all the forces on it (gravity, Coulomb force, string tension) must be 0, which means that the sum of gravitational and Coulomb forces must be opposite (and equal) to the string tension. Then we have

$$\tan 15^{\circ} = \frac{F_c}{F_g}$$

$$F_c = F_g \tan 15^{\circ} = mg \tan 15^{\circ} = 5.25 \cdot 10^{-3} N$$

On the other hand

$$F_c = qE$$

$$q = \frac{F_c}{E} = \underline{5.25 \cdot 10^{-6} C}$$

It must be <u>positive</u> since $\vec{F_c}$ and $\vec{E}$ point in the same direction.

<u>10 (a)</u>

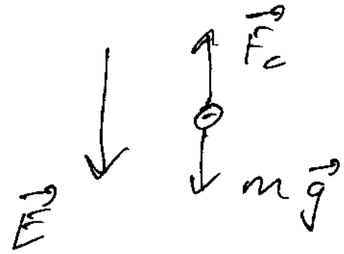
$$F_c = mg$$

$$e \cdot E = mg$$

$$E = \frac{mg}{e}$$

For electron:

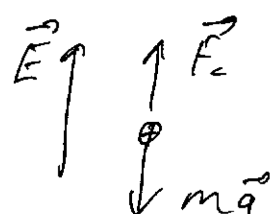
$$E = \frac{9.11 \cdot 10^{-31} kg \cdot 9.8 m/s^2}{1.6 \cdot 10^{-19} C} = 5.58 \cdot 10^{-11} \frac{N}{C}$$



Since the electron is negatively charged, the coulomb force is opposite to the electric field, which implies that the electric field points in the direction of gravity (downwards).

(B) For proton

$$E = \frac{1.67 \cdot 10^{-27}\,kg \cdot 9.8\,m/s^2}{1.6 \cdot 10^{-19}\,C} = 1.02 \cdot 10^{-7}\,\frac{N}{C}$$



The proton is positively charged, so the Coulomb force points in the direction of the electric field, implying that the electric field is in the opposite direction of gravity (upwards).

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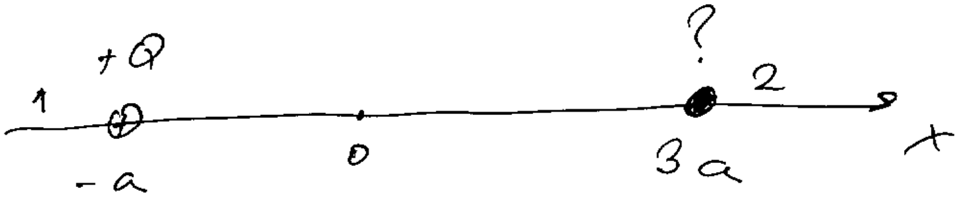


The electric field by charge 1 points in the positive x-direction with magnitude

$E_1 = \frac{kQ}{a^2}$ so $E_{1x} = +\frac{kQ}{a^2}$

There are two possible directions of the total electric field at the origin:

possibility 1 positive x-direction, i.e.

$$E_x = +2\frac{kQ}{a^2}$$

$E_x = E_{1x} + E_{2x}$ $\qquad E_{2x} = E_x - E_{1x} = +\frac{kQ}{a^2}$

which means charge 2 is negative.

$|\vec{E}_2| = \frac{k|Q_2|}{(3a)^2} \Rightarrow |Q_2| = 9Q$

$$Q_2 = -9Q$$

possibility 2 negative x-direction, i.e.

$$E_x = -2\frac{kQ}{a^2}$$

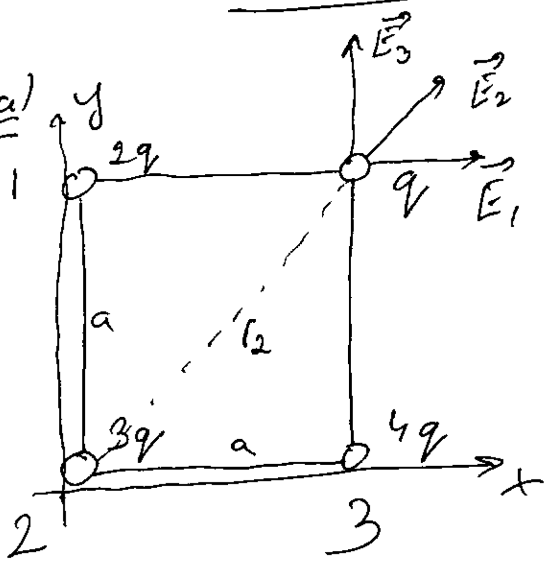
$E_x = E_{1x} + E_{2x}$ $\qquad E_{2x} = E_x - E_{1x} = -3\frac{kQ}{a^2}$

which means charge 2 is positive.

$$|\vec{E}_2| = \frac{k|Q_2|}{(3a)^2} \Rightarrow |Q_2| = (3a)^2 \cdot |\vec{E}_2| \cdot \frac{1}{k} = 27Q$$

$$\underline{Q_2 = +27Q}$$

15(a)



$$E_1 = \frac{k \cdot 2q}{a^2} = 2\frac{kq}{a^2}$$

$$E_{1x} = E_1 = 2\frac{kq}{a^2}$$

$$E_{1y} = 0$$

$$E_2 = \frac{k \cdot 3q}{r_2^2} = 3\frac{kq}{a^2+a^2} = \frac{3}{2}\frac{kq}{a^2}$$

$$E_{2x} = E_{2y} = E_2 \cdot \cos 45^\circ = E_2 \sin 45^\circ =$$

$$= \frac{3}{2}\frac{kq}{a^2} \cdot \frac{\sqrt{2}}{2} = \frac{3\sqrt{2}}{4}\frac{kq}{a^2}$$

$$E_3 = \frac{k \cdot 4q}{a^2} = 4\frac{kq}{a^2}$$

$$E_{3x} = 0 \qquad E_{3y} = E_3 = 4\frac{kq}{a^2}$$

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$$

$$E_x = E_{1x} + E_{2x} + E_{3x} = \frac{kq}{a^2}\left(2 + \frac{3\sqrt{2}}{4} + 0\right) = 3.06\frac{kq}{a^2}$$

$$E_y = E_{1y} + E_{2y} + E_{3y} = \frac{kq}{a^2}\left(0 + \frac{3\sqrt{2}}{4} + 4\right) = 5.06\frac{kq}{a^2}$$

Magnitude:

$$E = \sqrt{E_x^2 + E_y^2} = 5.91 \frac{kq}{a^2}$$

Direction with respect to x-axis:

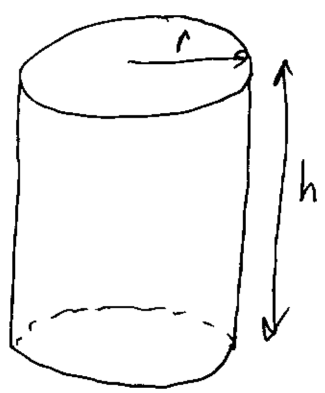
$$\tan\varphi = \frac{E_y}{E_x} = 1.65 \qquad \varphi = 58.8^\circ \text{ (above x-axis)}$$

(b) $\vec{F} = q \cdot \vec{E}$

Magnitude: $F = 5.91 \frac{kq^2}{a^2}$

Direction: same as $\vec{E}$, i.e. 58.8° above x-axis

23 (a)



$r = 2.50\text{ cm} = 0.025\text{ m}$

$h = 6.00\text{ cm} = 0.06\text{ m}$

$\sigma = 15.0\text{ nC/m}^2 = 1.5 \cdot 10^{-8}\text{ C/m}^2$

Curved lateral surface area:

$$A_{curved} = h \cdot 2\pi r = 9.42 \cdot 10^{-3}\text{ m}^2$$

Base area:

$$A_{Base} = \pi r^2 = 1.96 \cdot 10^{-3}\text{ m}^2$$

Total surface area:

$A_{total} = A_{curved} + 2A_{base} = 1.334 \cdot 10^{-2} m^2$

$q = \sigma \cdot A_{total} = \underline{2 \cdot 10^{-10} C}$

(b) $q = \sigma \cdot A_{curved} = \underline{1.41 \cdot 10^{-10} C}$

(c) $\rho = 500 nC/m^3 = 5 \cdot 10^{-7} C/m^3$

Volume: $V = h \cdot \pi r^2 = 1.18 \cdot 10^{-4} m^3$

$q = \rho \cdot V = \underline{5.88 \cdot 10^{-11} C}$

24 (a) Number of lines is proportional to charge.

q_1 — 6 lines

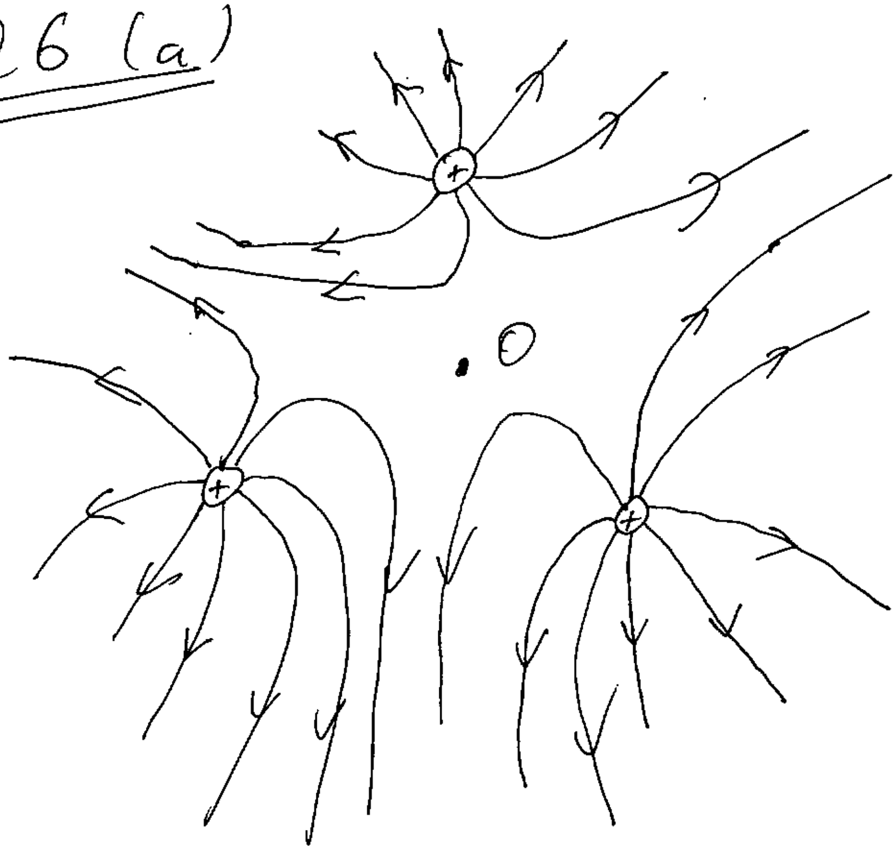
q_2 — 18 lines

$$\frac{|q_1|}{|q_2|} = \frac{6}{18} = \underline{\frac{1}{3}}$$

(b) Lines begin on positive and end on negative charges. q_1 is negative, q_2 is positive

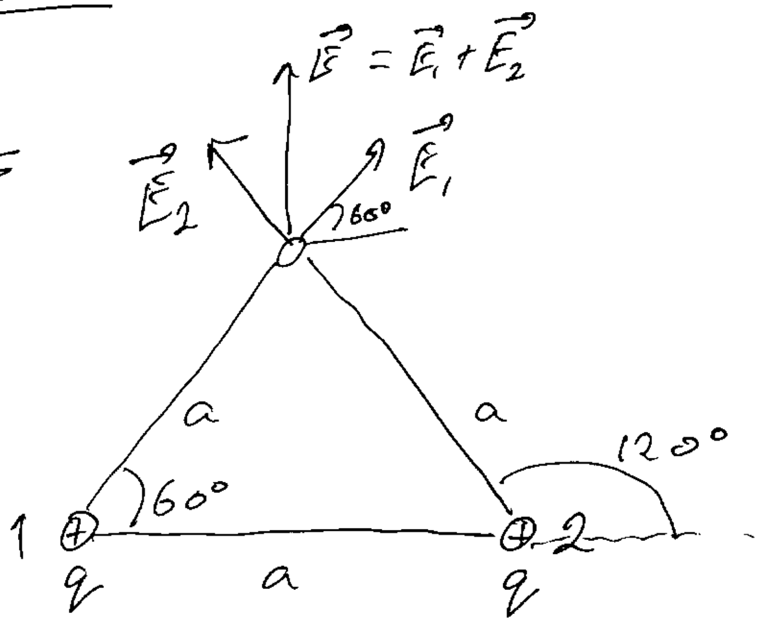
$$\frac{q_1}{q_2} = -\frac{1}{3}$$

26 (a)



The electric field is 0 at the center of the triangle.

(b)



$$E_1 = E_2 = \frac{kq}{a^2}$$

$$E_{1x} = E_1 \cos 60° = \frac{1}{2}\frac{kq}{a^2}$$

$$E_{1y} = E_1 \sin 60° = \frac{\sqrt{3}}{2}\frac{kq}{a^2}$$

$$E_{2x} = E_2 \cos 120° = -\frac{1}{2}\frac{kq}{a^2}$$

$$E_{2y} = E_2 \sin 120° = \frac{\sqrt{3}}{2}\frac{kq}{a^2}$$

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

$$E_x = E_{1x} + E_{2x} = 0$$

$$E_y = E_{1y} + E_{2y} = \sqrt{3}\frac{kq}{a^2}$$

Magnitude: $\underline{E = \sqrt{3}\,\frac{kq}{a^2}}$

Direction: <u>positive y-direction (upwards)</u>

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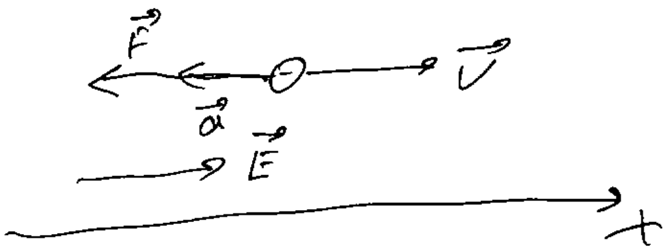

$$K = \frac{m_e v^2}{2} \Rightarrow v^2 = \frac{2K}{m_e}$$

$$d = \frac{v_{final}^2 - v_{initial}^2}{2a_x} = \frac{0 - v^2}{2a_x} = -\frac{v^2}{2a_x}$$

$$a_x = -\frac{v^2}{2d} = -\frac{2K/m_e}{2d} = -\frac{K}{m_e d}$$

$$F_x = m_e \cdot a_x = -\frac{K}{d}$$

$$F_x = -e \cdot E_x$$

$$E_x = -\frac{F_x}{e} = \underline{\frac{K}{de}}$$

Magnitude: $\underline{E = \frac{K}{de}}$

Direction: <u>same as electron's initial velocity (+x)</u>

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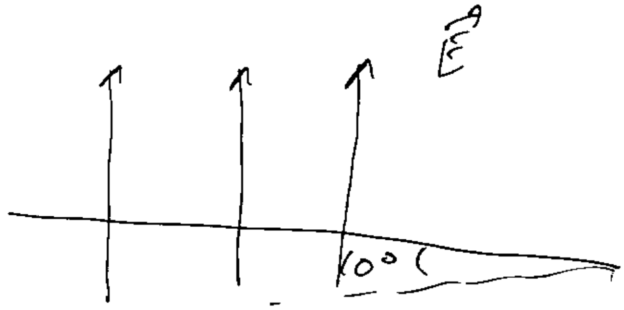

$E = 2 \cdot 10^4 \ N/C$

$a = 6 m$
$b = 3 m$

$\theta = 10°$

$A = a \cdot b = 18 \ m^2$

$\Phi_E = E \cdot A \cdot \cos\theta = \underline{3.55 \cdot 10^5 \ \frac{N \cdot m^2}{C}}$

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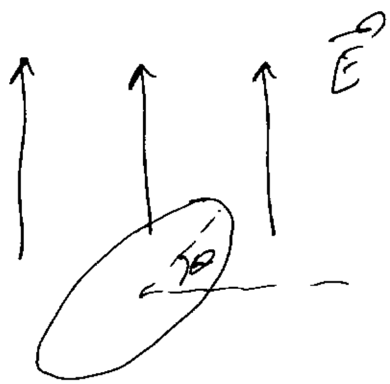


$d = 40.0 \ cm = 0.4 m$

$\Phi_{max} = 5.2 \cdot 10^5 \ \frac{N m^2}{C}$

$A = \pi r^2 = \pi \left(\frac{d}{2}\right)^2 = 0.126 \ m^2$

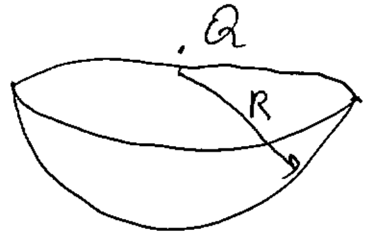
$\Phi_E = E \cdot A \cos\theta$

Φ_E becomes max when $\cos\theta$ is max, which is 1 for $\theta = 0$, i.e.

$\Phi_{max} = E \cdot A$

$\therefore \ E = \frac{\Phi_{max}}{A} = \underline{4.13 \cdot 10^6 \ \frac{N}{C}}$

33 (a)



The electric field at each point of the curved surface is

$$E = \frac{kQ}{R^2}$$

and is perpendicular to the surface, Therefore the flux is

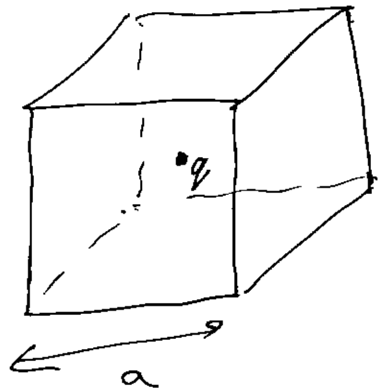
$$\phi = E \cdot A = \frac{kQ}{\cancel{R^2}} \cdot \frac{4\pi \cancel{R^2}}{2} = 2\pi k Q = \frac{Q}{2\varepsilon_0}$$

(b) There are no charges inside the hemispherere, so the total flux through its surface must be 0;

$$\phi_{curved} + \phi_{flat} = 0$$

$$\phi_{flat} = -\phi_{curved} = -\frac{Q}{2\varepsilon_0}$$

34 (a)



$q = 170\,\mu C = 1.7 \cdot 10^{-4} C$

$a = 80.0 cm = 0.8 m$

Total flux:

$$\Phi_{total} = \frac{q}{\varepsilon_0} \quad \text{(Gauss's law)}$$

Because the charge is at center the flux through each face is the same:

$$\Phi_{face} = \frac{\Phi_{total}}{6} = \frac{q}{6\varepsilon_0} = \underline{3.2 \cdot 10^{6} \frac{N \cdot m^2}{C}}$$

(b) $\Phi_{total} = \frac{q}{\varepsilon_0} = \underline{1.92 \cdot 10^{7} \frac{N \cdot m^2}{C}}$

(c) Gauss's law still applies, i.e. the total flux would be the same but the flux through each face would not be the same any more. Answer to part (a) would change, to part (b) would not change.

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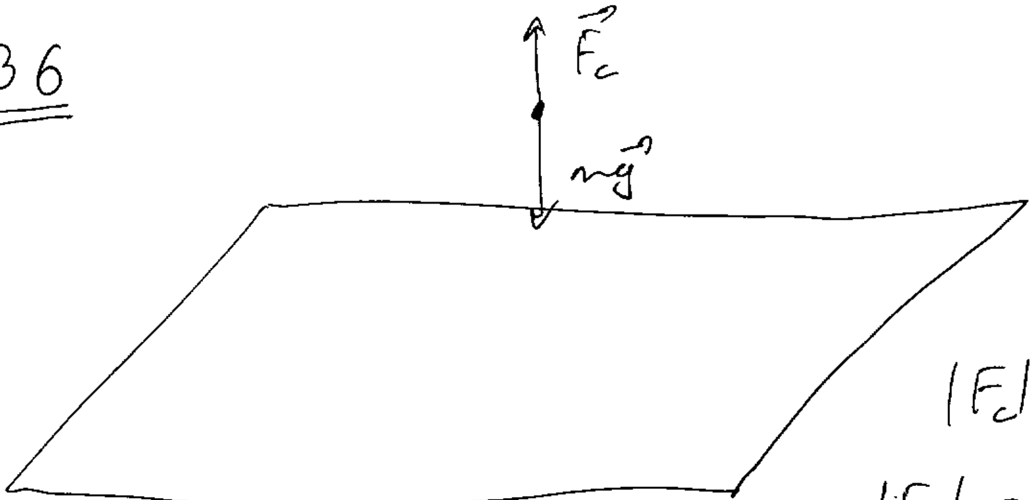

$m = 10.0\,g = 0.01\,kg$

$q = -0.7\,\mu C = -7 \cdot 10^{-7}\,C$

$$|F_c| = mg$$

$$|F_c| = |q| \cdot |E|$$

$$|E| = \frac{|F_c|}{|q|} = \frac{mg}{|q|} = 1.4 \cdot 10^{5}\,\frac{N}{C}$$

On the other hand

$$|E| = \frac{|\sigma|}{2\varepsilon_0}$$

$$|\sigma| = 2\varepsilon_0 \cdot |E| = 2.48 \cdot 10^{-6}\,\frac{C}{m^2}$$

Since q is negative, $\vec{E}$ must point downwards, which implies that the plane is charged negatively, so

$$\sigma = -2.48 \cdot 10^{-6}\,\frac{C}{m^2}$$

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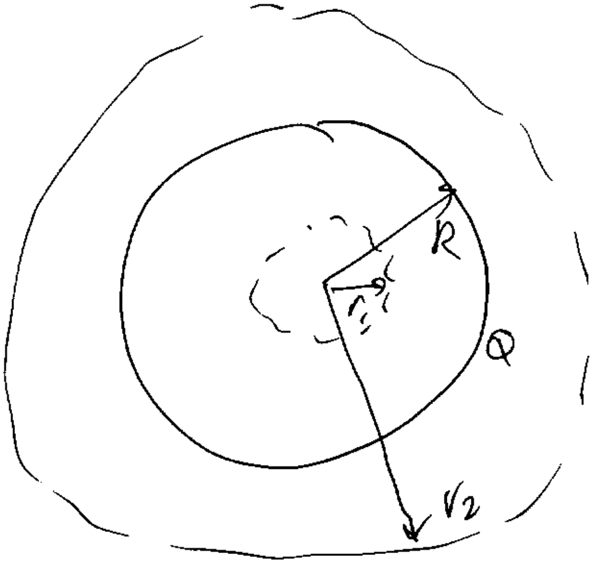

$Q = 32.0\,\mu C = 3.2 \cdot 10^{-5} C$

$R = 14.0 cm = 0.14 m$

$r_1 = 10.0 cm = 0.1 m$

$r_2 = 20.0 cm = 0.2 m$

(a) There is no charge inside surface 1, by Gauss's law total flux is 0, so the electric field is 0.

(b) $\phi = \dfrac{Q}{\varepsilon_0}$

$$E = \frac{\phi}{A} = \frac{\phi}{4\pi r_2^2} = \frac{Q}{4\pi \varepsilon_0 r_2^2}$$

$E = 7.2 \cdot 10^6\ N/C$

in the radial direction everywhere on the surface,

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$q_1 = q_2 = 46e$

$r_1 = r_2 = 5.9 \cdot 10^{-15}\,m$

$d = r_1 + r_2 = 2r_1$

The electric flux through surface S created by charge q_1 is

$$\phi = \frac{q_1}{\varepsilon_0}$$

$$E = \frac{\phi}{A} = \frac{q_1}{4\pi\varepsilon_0 d^2}$$

Force acting on q_2

$$F = E \cdot q_2 = \frac{q_1 q_2}{4\pi\varepsilon_0 d^2} = \frac{q_1^2}{4\pi\varepsilon_0 (2r_1)^2}$$

$$\underline{F = 3500\,N}$$

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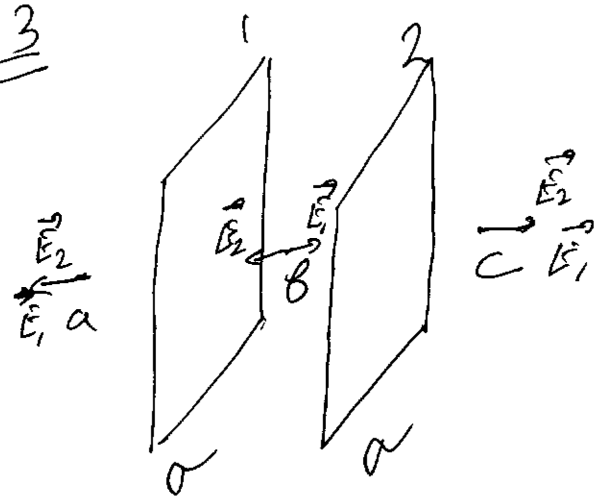


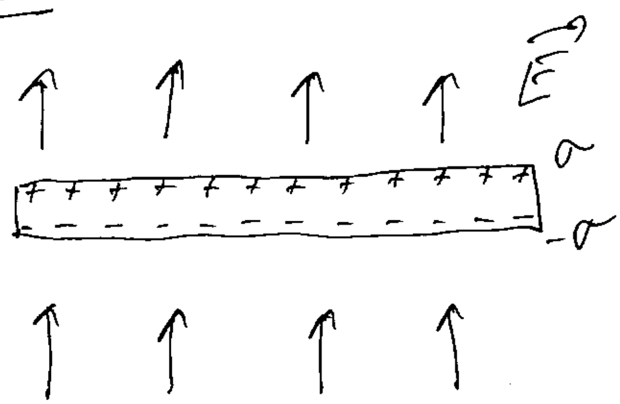
Each sheet creates electric field $E = \frac{\sigma}{2\varepsilon_0}$ away from it.

(a) $\vec{E}_1 + \vec{E}_2 = 2\vec{E}_1$ $\quad E_{total} = \frac{\sigma}{\varepsilon_0}$ to the left

(b) $\vec{E}_1 + \vec{E}_2 = 0$

(c) $\vec{E}_1 + \vec{E}_2 = 2\vec{E}_1$ $\quad E_{total} = \frac{\sigma}{\varepsilon_0}$ to the right

44 (a)



$E = 80.0\,\frac{kN}{C} = 8 \cdot 10^4\,\frac{N}{C}$

$a = 50.0\,cm = 0.5\,m$

Since the plate is neutral the total charge on upper and lower faces must cancel, i.e. if the charge density is $+\sigma$ on the upper face, it must be $-\sigma$ on the lower face. Consider a point inside the plate. Since copper is conductor the electric field at that point must be 0. The electric field created by the upper plate is

$$E_{upper} = \frac{\sigma}{2\varepsilon_0} \quad \text{downwards}$$

by the lower plate

$$E_{lower} = \frac{|-\sigma|}{2\varepsilon_0} = \frac{\sigma}{2\varepsilon_0} \quad \text{again downwards}$$

Total:

$$E_{total} = \frac{\sigma}{\varepsilon_0} \quad \text{downwards}$$

which must cancel the external field E, so

$$E = \frac{\sigma}{\varepsilon_0} \qquad \sigma = E \cdot \varepsilon_0 = 7.08 \cdot 10^{-7} \frac{C}{m^2}$$

$$\underline{\sigma_{upper} = +7.08 \cdot 10^{-7} \frac{C}{m^2}}$$

$$\underline{\sigma_{lower} = -7.08 \cdot 10^{-7} \frac{C}{m^2}}$$

(b) $q = \sigma \cdot A$

$A = (0.5\,m)^2 = 0.25\,m^2$

$\underline{q_{upper} = +1.77 \cdot 10^{-7}\,C}$

$\underline{q_{lower} = -1.77 \cdot 10^{-7}\,C}$

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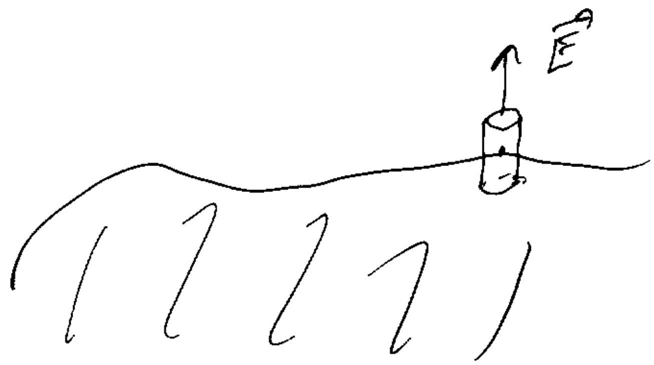


Consider an imaginary cylinder containing a given point of the surface of the conductor, normal to the surface. Let the base area be A. The electric field inside the conductor is 0, and outside it is parallel to the lateral surface of the cylinder, so the flux through the lower face, as well as the lateral surface is 0. The total flux is then

$$\phi = E \cdot A \underset{\text{Gauss's law}}{=} \frac{q}{\varepsilon_0} = \frac{\sigma A}{\varepsilon_0}$$

$\sigma = E \cdot \varepsilon_0$

The charge density is the biggest at the sharpest parts of the surface, i.e. with the smallest radius of curvature.

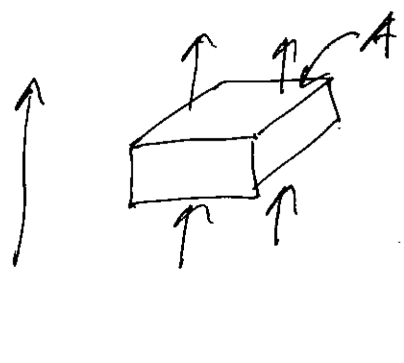
(a) $E = E_{min} = 28.0\ kN/C = 2.8 \cdot 10^4 \frac{N}{C}$

$\sigma = 2.48 \cdot 10^{-7} \frac{C}{m^2}$

(b) $E = E_{max} = 56.0\ kN/C = 5.6 \cdot 10^4 \frac{N}{C}$

$\sigma = 4.96 \cdot 10^{-7} \frac{C}{m^2}$

48 (a) Consider a small parallelepiped inside the volume, two faces of which are perpendicular to the direction of the electric field. The total flux through its surface is



$\phi = (E_{upper} - E_{lower}) \cdot A = \frac{q}{\varepsilon_0}$

where q is the charge inside, so if the charge is 0 then $E_{upper} = E_{lower}$ and the electric field can be uniform in magnitude, but if $q \neq 0$ (which is a possibility for an insulator carrying charge) then $E_{upper} \neq E_{lower}$ so it does not have to be uniform in magnitude.

(b) In empty space $q = 0$ always so $E_{upper} = E_{lower}$ and it implies everywhere (i.e. it does not matter where we put our parallelepiped) so the electric field must be uniform in magnitude.