CHAPTER 20

1. (a) Let the proton start from rest at potential V_i and be accelerated to kinetic energy K_f at potential V_f. By conservation of energy:

$$0 + e \cdot V_i = K_f + eV_f$$

$$K_f = e(V_i - V_f)$$

Since the proton is positively charged we need $V_i > V_f$ for it to be accelerated, so we need $V_i - V_f = 120\,V$

$$\frac{m_p v^2}{2} = K_f = e \cdot 120\,V$$

$$v = \sqrt{\frac{2e \cdot 120\,V}{m_p}} = \underline{1.5 \cdot 10^5\,m/s}$$

(b) The electron is negatively charged, so the conservation of energy gives

$$0 - eV_i = K_f - eV_f$$

$$K_f = e(V_f - V_i)$$

Now we need $V_f > V_i$, i.e. $V_f - V_i = 120V$.

$$\frac{m_e v^2}{2} = K_f = e \cdot 120V$$

$$v = \sqrt{\frac{2e \cdot 120V}{m_e}} = \underline{6.5 \cdot 10^6 \, m/s}$$

2. The initial and final energies of one electron are

$U_i = -eV_i$
$U_f = -eV_f$
respectively, so the work done is

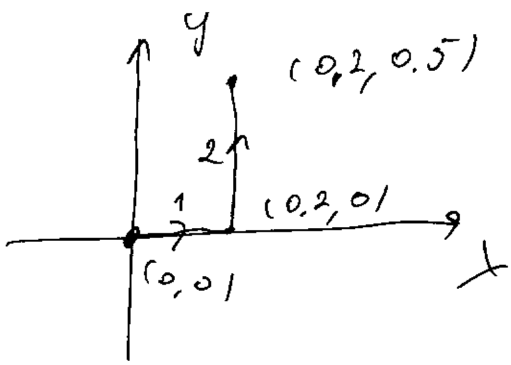
$$W = U_f - U_i = e(V_i - V_f)$$

For Avogadro's number of electrons;

$$W_{total} = N_A \cdot W = N_A \, e(V_i - V_f)$$

$$W_{total} = \underline{1.35 \cdot 10^6 \, J}$$

3 (a) $E_x = 250 V/m$
$E_y = 0$
$q = 12 \mu C = 1.2 \cdot 10^{-5} C$



We choose the path as shown. Through path 1 we have

$\Delta U_1 = -q E_x \cdot \Delta x = -q \cdot E_x \cdot 0{,}2m$

Through path 2:

$\Delta U_2 = -q E_y \cdot \Delta y = 0$

So $\Delta U = \Delta U_1 + \Delta U_2 = -q \cdot E_x \cdot 0{,}2m$

$\Delta U = \underline{-6 \cdot 10^{-4}\ J}$

(B) $\Delta V = \dfrac{\Delta U}{q} = \underline{-50\ V}$

4. $|\Delta V| = |E| \cdot d$

$|E| = \dfrac{|\Delta V|}{d} = \dfrac{25000V}{0{,}015m} = \underline{1.67 \cdot 10^6 \frac{V}{m}}$

7. (a) $V = \dfrac{k \cdot e}{r} = \underline{1.44 \cdot 10^{-7} V}$

(b) $\Delta V = V_2 - V_1 = \dfrac{ke}{r_2} - \dfrac{ke}{r_1} = \underline{-7.2 \cdot 10^{-8} V}$

(c) Charge of the electron is $-e$, so

$V = -\dfrac{ke}{r} = \underline{-1.44 \cdot 10^{-7} V}$ for (a) and

$$\Delta V = -\frac{ke}{r_2} - \left(-\frac{ke}{r_1}\right) = \underline{7.2 \cdot 10^{-8} V} \quad \text{for (B)}$$

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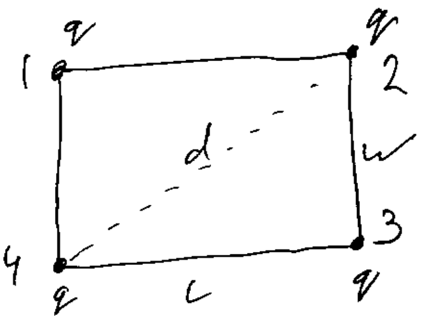

$q = +10\,\mu C = 10^{-5} C$

$L = 60\text{ cm} = 0{,}6\text{ m}$

$w = 15\text{ cm} = 0{,}15\text{ m}$

The potential created by charge 1 at point 4 is

$$V_1 = \frac{kq}{w} = 6 \cdot 10^5 V$$

By charge 2:

$$V_2 = \frac{kq}{d} = \frac{kq}{\sqrt{w^2 + L^2}} = 1.46 \cdot 10^5 V$$

By charge 3:

$$V_3 = \frac{kq}{L} = 1.5 \cdot 10^{5} V$$

Total:

$$V = V_1 + V_2 + V_3 = 8.96 \cdot 10^5 V$$

Potential energy of charge q at point 4 is

$$U_4 = V \cdot q = 8.96\ J$$

while at infinity: $U_\infty = 0$

So $\Delta U = U_4 - U_\infty = \underline{8.96\ J}$

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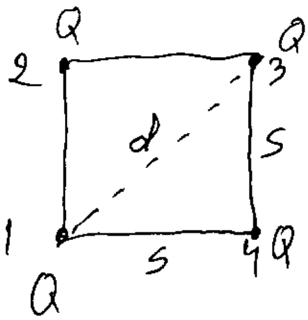


We bring the charges one by one. First we bring charge 1, since there are no other charges the work is 0 : $W_1 = 0$. Then bring charge 2, The potential energy in the presence of charge 1 is

$$U_2 = \frac{k \cdot Q \cdot Q}{s} = \frac{kQ^2}{s}$$

so the work required is

$$W_2 = U_2 - U_\infty = U_2 - 0 = U_2 = \frac{kQ^2}{s}$$

Then bring charge 3. Now 1 and 2 are present, so

$$U_3 = \frac{k \cdot Q \cdot Q}{s} + \frac{k \cdot Q \cdot Q}{d} = \frac{kQ^2}{s} + \frac{kQ^2}{\sqrt{s^2+s^2}} = \frac{kQ^2}{s} + \frac{kQ^2}{\sqrt{2}s} =$$

$$= 1.707 \frac{kQ^2}{s}$$

$$W_3 = U_3 - U_\infty = U_3 - 0 = 1.707 \frac{kQ^2}{s}$$

Finally, bring charge 4. 1, 2, 3 are present, so

$$U_4 = \frac{k \cdot Q \cdot Q}{s} + \frac{k \cdot Q \cdot Q}{d} + \frac{k \cdot Q \cdot Q}{s} = 2.707 \frac{kQ^2}{s}$$

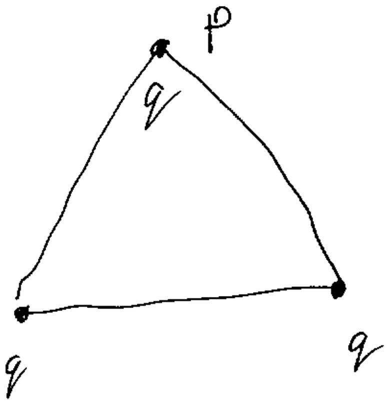
$$W_4 = U_4 - U_\infty = 2.707 \frac{kQ^2}{s}$$

Total work

$$W = W_1 + W_2 + W_3 + W_4 = \underline{5.41 \frac{kQ^2}{s}}$$

17 (a) Each charge creates a potential $\frac{kq}{r}$ at distance r which is always positive, so there are no cancellations at any point since all charges are positive, so the potential is 0 only infinitely far away.

(b)



Each of the two charges at the base creates a potential $\frac{kq}{a}$ at point P, so the total potential is

$$\underline{2 \frac{kq}{a}}$$

22 (a) Inside $V = \frac{kQ}{R} = \text{const}$

$$E_r = -\frac{dV}{dr} = \underline{0}$$

(b) Outside $V = \frac{kq}{r}$

$$E_r = -\frac{dV}{dr} = \underline{\frac{kq}{r^2}}$$

62 The electric potential of proton at distance r is

$$V(r) = \frac{k \cdot e}{r}$$

given $r = n^2 (0{,}0529 \cdot 10^{-9}\, m)$ we get

$$V(r) = \frac{1}{n^2} \cdot \frac{k\, e}{0{,}0529 \cdot 10^{-9}\, m} = \frac{27.2\, V}{n^2}$$

(a) $n = 1 \Rightarrow V = 27.2\, V$

$U(\text{electron}) = -e \cdot V = \underline{-27.2\, eV}$

(b) $n = 2 \Rightarrow V = 6.8\, V$

$U(\text{electron}) = -e \cdot V = \underline{-6{,}8\, eV}$

(c) $r = \infty \Rightarrow V = 0 \Rightarrow U(\text{electron}) = \underline{0}$

28 The electric potential at the surface of a spherical conductor is

$$V = \frac{kq}{R}$$

$$q = \frac{VR}{k} = \frac{7.5 \cdot 10^3 V \cdot 0{,}3 m}{9 \cdot 10^9 \frac{N \cdot m^2}{C^2}} = 2.5 \cdot 10^{-7} C$$

The number of electrons to be removed is then

$$N = \frac{q}{e} = \underline{1.56 \cdot 10^{12}}$$

29 $R = 14 cm = 0{,}14 m$

$q = 26.0 \mu C = 2{,}6 \cdot 10^{-5} C$

(a) $r = 10 cm = 0{,}1 m < R$

Inside the conductor the electric field is always $\underline{0}$. The electric potential is constant, which means it is the same as at its surface:

$$V = \frac{kq}{R} = \underline{1{,}67 \cdot 10^6 V}$$

(b) $r = 20\,cm = 0.2\,m > R$

Outside the conducting sphere we can regard the sphere as a point charge at the center, so

$$E = \frac{kq}{r^2} = \underline{5.85 \cdot 10^6 \frac{V}{m}}$$

$$V = \frac{kq}{r} = \underline{1.17 \cdot 10^6 V}$$

(c) $r = 14\,cm = 0.14\,m = R$

At the surface we can again regard the sphere as a point at the center:

$$E = \frac{kq}{r^2} = \underline{1.19 \cdot 10^7 \frac{V}{m}}$$

$$V = \frac{kq}{r} = \underline{1.67 \cdot 10^6 V}$$

31 (a) $C = 4\,\mu F = 4 \cdot 10^{-6} F$

$V = 12\,V$

$$Q = CV = \underline{48 \cdot 10^{-6} C = 48\,\mu C}$$

(b) $V = 1.5V$ $\quad Q = CV = \underline{6 \cdot 10^{-6} C = 6\,\mu C}$

35

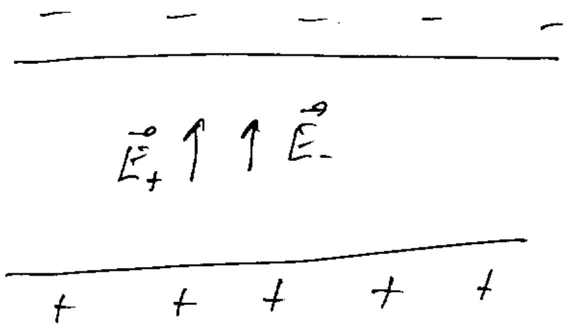
$A = 7.6\ cm^2 = 7.6 \cdot 10^{-4}\ m^2$

$d = 1,8\ mm = 1.8 \cdot 10^{-3}\ m$

$V = 20\ V$

(a) $E = \frac{V}{d} = \underline{1.11 \cdot 10^4\ V/m}$

(B) Each plate creates an electric field of $E_1 = \frac{\sigma}{2\varepsilon_0}$ in the same direction :



so over all $E = \frac{\sigma}{\varepsilon_0}$

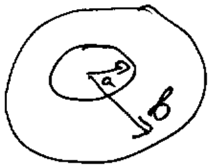
$\sigma = E \cdot \varepsilon_0 = \underline{9.83 \cdot 10^{-8}\ C/m^2}$

(d) $Q = \sigma A = \underline{7.47 \cdot 10^{-11}\ C}$

(c) $C = \frac{Q}{V} = \frac{\sigma A}{V} = \frac{E \cdot \varepsilon_0 \cdot A}{V} = \frac{\frac{\cancel{V}}{d} \cdot \varepsilon_0 \cdot A}{\cancel{V}} = \frac{\varepsilon_0 A}{d}$

$C = \underline{3.74 \cdot 10^{-12}\ F}$

36 (a)



$a = 2.58\,mm = 2.58 \cdot 10^{-3}\,m$

$b = 7.27\,mm = 7.27 \cdot 10^{-3}\,m$

$\ell = 50\,m$

$$C = \frac{\ell}{2k \ln\left(\frac{b}{a}\right)}$$

(see "The Cylindrical Capacitor", page 659-660)

$C = \underline{2.68 \cdot 10^{-9}\,F}$

(b) $Q = 8.10\,\mu C = 8.1 \cdot 10^{-6}\,C$

$V = \frac{Q}{C} = \underline{3.02 \cdot 10^{3}\,V = 3.02\,kV}$

39 $C_1 = 5\,\mu F = 5 \cdot 10^{-6}\,F$

$C_2 = 12\,\mu F = 1.2 \cdot 10^{-5}\,F$

$V = 9\,V$

(a) in parallel: $C = C_1 + C_2$

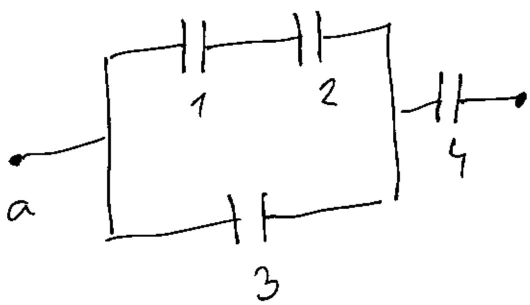
$C = \underline{17 \cdot 10^{-6}\,F = 17\,\mu F}$

(b) $V = \underline{V_1 = V_2 = 9V}$

(c) $Q_1 = V_1 C_1 = \underline{45 \cdot 10^{-6}\,C = 45\,\mu C}$

$$Q_2 = V_2 C_2 = \underline{108 \cdot 10^{-6}\, C = 108\, \mu C}$$

<u>4) (a)</u>



$C_1 = 15\, \mu F = 15 \cdot 10^{-6} F$

$C_2 = 3\, \mu F = 3 \cdot 10^{-6} F$

$C_3 = 6\, \mu F = 6 \cdot 10^{-6} F$

$C_4 = 20\, \mu F = 20 \cdot 10^{-6} F$

1 and 2 are in series:

$$\frac{1}{C_{12}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{C_1 + C_2}{C_1 \cdot C_2}$$

$$C_{12} = \frac{C_1 \cdot C_2}{C_1 + C_2} = 2.5 \cdot 10^{-6} F$$

which is in parallel with 3:

$$C_{123} = C_{12} + C_3 = 8.5 \cdot 10^{-6} F$$

which is in series with 4:

$$\frac{1}{C} = \frac{1}{C_{123}} + \frac{1}{C_4} = \frac{C_{123} + C_4}{C_{123} \cdot C_4}$$

$$C = \frac{C_{123} \cdot C_4}{C_{123} + C_4} = \underline{5.96 \cdot 10^{-6} F = 5.96\, \mu F}$$

16/ $V = 15V$

total charge:

$$Q = CV = 89.5 \cdot 10^{-6} C$$

123 and 4 are in series, so

$$\underline{Q_{123} = Q_4 = Q = 89.5 \cdot 10^{-6} C = 89.5 \mu C}$$

$$V_{123} = \frac{Q_{123}}{C_{123}} = 10.5 V$$

12 and 3 are in parallel, so

$$V_{12} = V_3 = V_{123} = 10.5 V$$

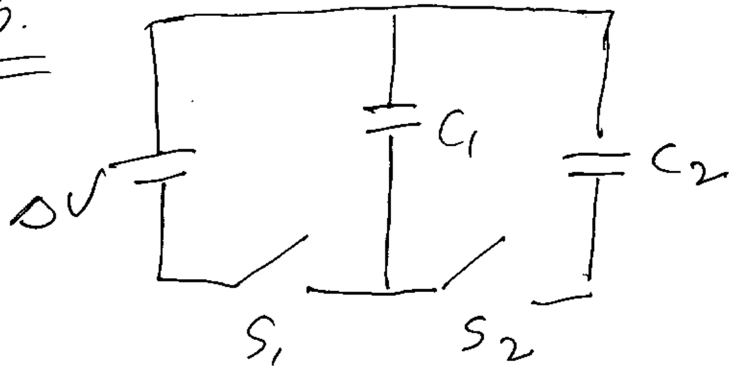
$$\underline{Q_3 = C_3 \cdot V_3 = 63.2 \cdot 10^{-6} C = 63.2 \mu C}$$

$$Q_{12} = C_{12} \cdot V_{12} = 26.3 \cdot 10^{-6} C$$

1 and 2 are in series, so

$$\underline{Q_1 = Q_2 = Q_{12} = 26.3 \cdot 10^{-6} C = 26.3 \mu C}$$

43.



$\Delta V = 20\,V$

$C_1 = 6\,\mu F = 6 \cdot 10^{-6}\,F$

$C_2 = 3\,\mu F = 3 \cdot 10^{-6}\,C$

First S_1 is closed, then

$$Q_1 = C_1 \cdot \Delta V = \underline{120 \cdot 10^{-6}\,C = 120\,\mu C}$$

Then S_1 is opened and S_2 is closed, The total charge Q_1 is redistributed among C_1 and C_2 such that they have the <u>same voltage</u> (in opposite directions), so

$$Q_1 + Q_2 = 120 \cdot 10^{-6}\,C$$

$$\Delta V_1 = \frac{Q_1}{C_1} \qquad \Delta V_2 = \frac{Q_2}{C_2}$$

$$\Delta V_1 = \Delta V_2 \Rightarrow \frac{Q_1}{C_1} = \frac{Q_2}{C_2}$$

$$Q_1 = \frac{C_1}{C_2} \cdot Q_2$$

$$\frac{C_1}{C_2} Q_2 + Q_2 = 120 \cdot 10^{-6}\,C$$

$$3Q_2 = 120 \cdot 10^{-6}\,C$$

$$Q_2 = 40 \cdot 10^{-6}\,C = 40\,\mu C$$

$$Q_1 = 120 \cdot 10^{-6}\,C - Q_2 = 80 \cdot 10^{-6}\,C = 80\,\mu C$$

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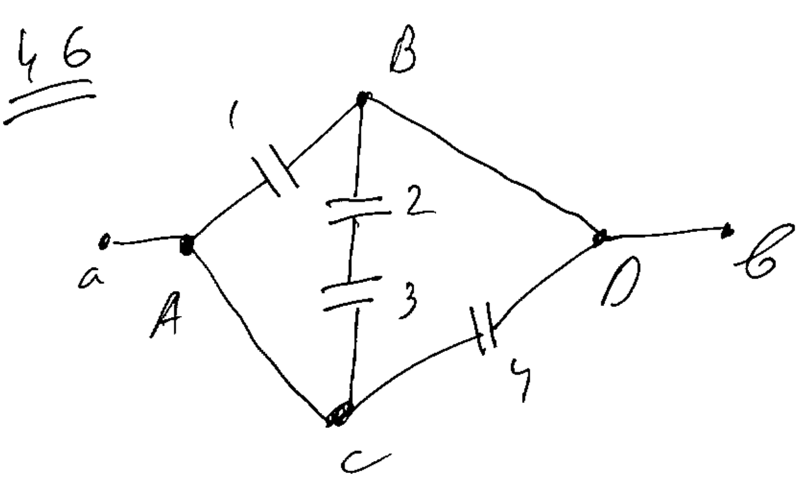


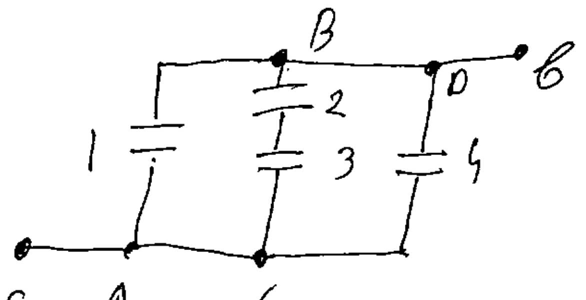
$C_1 = 4\mu F = 4 \cdot 10^{-6} F$

$C_2 = 7\mu F = 7 \cdot 10^{-6} F$

$C_3 = 5\mu F = 5 \cdot 10^{-6} F$

$C_4 = 6\mu F = 6 \cdot 10^{-6} F$

By changing the lengths of wires (without changing the cross points A, B, C, D) can redraw the diagram;



2 and 3 are in series:

$$\frac{1}{C_{23}} = \frac{1}{C_2} + \frac{1}{C_3} = \frac{C_2 + C_3}{C_2 \cdot C_3}$$

$$C_{23} = \frac{C_2 \cdot C_3}{C_2 + C_3} = 2.92 \cdot 10^{-6} F$$

1, 23, and 4 are parallel:

$$C = C_1 + C_{23} + C_4 = \underline{12.92 \cdot 10^{-6} F = 12.92\,\mu F}$$

48 $U = 300\,J$

$C = 30.0\,\mu F = 3\cdot 10^{-5}\,F$

$$U = \frac{1}{2} C (\Delta V)^2$$

$$\Delta V = \sqrt{\frac{2U}{C}} = \underline{4.47\cdot 10^{3}\,V}$$

50 (a) $C = 150\,pF = 150\cdot 10^{-12}\,F$

$V = 10\,kV = 10^{4}\,V$

$$Q = C\cdot V = \underline{1.5\cdot 10^{-6}\,C = 1.5\,\mu C}$$

(b) $U = 250\,\mu J = 2.5\cdot 10^{-4}\,J$

$$U = \frac{1}{2} C (\Delta V)^2$$

$$\Delta = \sqrt{\frac{2U}{C}} = \underline{1.83\cdot 10^{3}\,V}$$

53 $A = 1.75\,cm^2 = 1.75\cdot 10^{-4}\,m^2$

$d = 0.04\,mm = 4\cdot 10^{-5}\,m$

$$\left.\begin{array}{l} K = 2.1 \\ E_{max} = 60\cdot 10^{6}\,V/m \end{array}\right\} \text{Table 20.1, page 669}$$

(a) $C = K\dfrac{\varepsilon_0 A}{d} = \underline{8.13\cdot 10^{-11}\,F}$

(b) $\Delta U_{max} = E_{max} \cdot d \approx \underline{2.4 \cdot 10^3 \ V}$

57 $d = 1.5 \ cm = 1.5 \cdot 10^{-2} m$

$A = 25 \ cm^2 = 2.5 \cdot 10^{-3} m^2$

$\Delta V_i = 250 V$ (initial)

(a) $C_i = \frac{\varepsilon_0 A}{d} = 1.475 \cdot 10^{-12} F$

$Q_i = C_i \cdot \Delta V_i = \underline{3.69 \cdot 10^{-10} C}$

Once disconnected from the source the charge cannot change, so

$Q_f = Q_i = \underline{3.69 \cdot 10^{-10} C}$
(final)

(b) After immersion

$C_f = K \cdot \frac{\varepsilon_0 A}{d} = K \cdot C_i = 80 C_i = \underline{1.18 \cdot 10^{-10} F}$

$V_f = \frac{Q_f}{C_f} = \underline{3.13 V}$

(c) $U_i = \frac{1}{2} C_i (\Delta V_i)^2 = 4.6 \cdot 10^{-8}$ J

$U_f = \frac{1}{2} C_f (\Delta V_f)^2 = 5.78 \cdot 10^{-10}$ J

$\Delta U = U_f - U_i = \underline{-4.54 \cdot 10^{-8} \text{ J}}$