Physics 2a, Oct 26, lecture 15

$\star$Reading: chapter 6.

• Recall that energy is conserved, can't be created or destroyed, only moved around. The process of moving some energy is to do work. The rate of work is power.

• Examples of work. Pushing down on table and eraser. Pushing eraser across table, first with F vertical. Now pushing with horizontal and vertical components. The horizontal one does work (balancing out the friction). But the vertical one can't be forgotten: it increases the normal force, and thus the friction, requiring a bigger horizontal force, leading to an increase in the work. Emphasize the signs of work.

• $E_{total} = E_{kinetic} + E_{potential} \equiv K + U$. We'll now show that the kinetic energy is

$$K = \frac{1}{2}mv^2.$$

The potential energy is next week's topic, so you can forget about it for now (though we'll see appetizers of potential energy this week).

(Aside: in Einstein's theory of special relativity, replace $K \to \sqrt{(mc^2)^2 + c^2p^2}$, where mc^2 is the rest-mass energy and p^2 is the momentum, which we'll learn about later. These modifications are unimportant for velocities small compared to $c \approx 3 \times 10^8 m/s$, the speed of light.)

• Suppose that A does work on object B, and there are no other forces[1] acting on B. In particular, we'll ignore friction. Then $\vec{F}_{A\to B} = \vec{F}_{total} = m\vec{a}$, where m is the mass of object B and $\vec{a}$ is its acceleration. The work that A does on B is then $dW_{A\to B} = \vec{F} \cdot d\vec{x} = m\frac{d\vec{v}}{dt} \cdot \vec{v}dt = m\vec{v} \cdot d\vec{v} = d(\frac{1}{2}m\vec{v}^2) = dK_B$. This shows that all the work that A does on B has gone into an increase of B's kinetic energy. Integrating, get that the work done is $\Delta W = \int dW = \int dK = K_2 - K_1 \equiv \Delta K = \frac{1}{2}m(v_2^2 - v_1^2)$, the change in kinetic energy.

We can see this in more detail in a special case: acting with a constant force over a distance s gives $W = Fs = K_2 - K_1$, since $v_2^2 - v_1^2 = 2as$. So the work gives the change in the kinetic energy.

[1] As we'll discuss next week, when there are other forces, like gravity (but, as we'll see, unlike friction), we'll include their effect via potential energy.