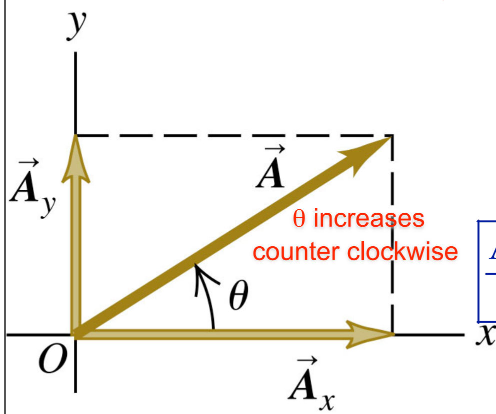
Physics 2A
Lecture 2: Sept 27, 2010

Components Of A Vector

- In Cartesian coordinate system, you can represent any vector lying in x-y plane as sum of a vector parallel to x-axis and a vector parallel to y-axis



$$\vec{A} = \vec{A}_x + \vec{A}_y$$

Magnitudes A_x & A_y are components of $\vec{A}$

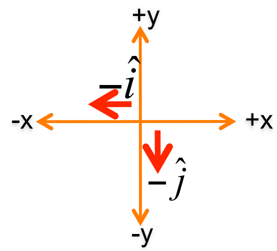
$$\frac{A_x}{A} = \cos\theta \quad \frac{A_y}{A} = \sin\theta \quad \frac{A_y}{A_x} = \tan\theta$$

$$A_x = A\cos\theta \; ; \; A_y = A\sin\theta$$

More On Unit Vectors

$-\hat{i}$ is a unit vector in the direction of $-X$

$-\hat{j}$ is a unit vector in the direction of $-Y$



Pop Quiz

If $\hat{i}, \hat{j}$ and $\hat{k}$ are unit vectors then is the vector

$\vec{r} = \hat{i} + \hat{j} + \hat{k}$ also a unit vector ?

Hint #1 : What is the magnitude of the vector $\vec{r}$?

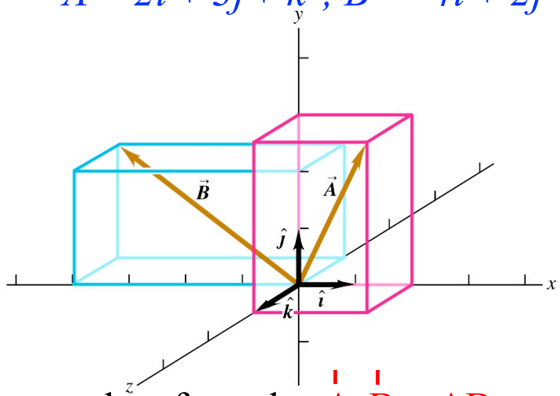
Hint #2: Can a unit vector have magnitude > 1 ?

Multiplying Vectors: Vector Product

- In mechanics, can express many physical relationships by using vector product
- Vector product is not like multiplying #s
- Two different kind of vector products
  - scalar product yields a value that's scalar
  - vector product yields <u>another</u> vector !

Finding Angle Between Two Vectors

$\vec{A} = 2\hat{i} + 3\hat{j} + \hat{k}$; $\vec{B} = -4\hat{i} + 2\hat{j} - \hat{k}$, What's angle ϕ between them?



Dot product formula : $\vec{A} \cdot \vec{B} = AB\cos\phi = A_x B_x + A_y B_y + A_z B_z$

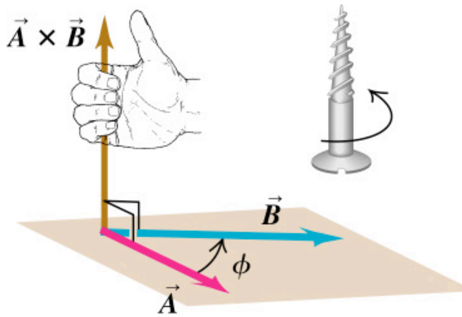
$$\Rightarrow \cos\phi = \frac{A_x B_x + A_y B_y + A_z B_z}{AB} = \frac{-3}{\sqrt{14}\sqrt{21}} = -0.175$$

$$\Rightarrow \boxed{\phi = \cos^{-1}(-0.175) = 100^{o}}$$

Vector Product Of $\vec{A}$ & $\vec{B}$: Definition

$\vec{A} \times \vec{B} = \vec{C}$ = Vector $\perp$ to plane containing $\vec{A}$ & $\vec{B}$

with magnitude $|\vec{C}| = AB\sin\phi$



Always two directions perpendicular to a plane, which one to choose ?

Follow right hand rule→ direction of thumb or advance of a right hand screw when vector **A** sweeps towards vector **B**

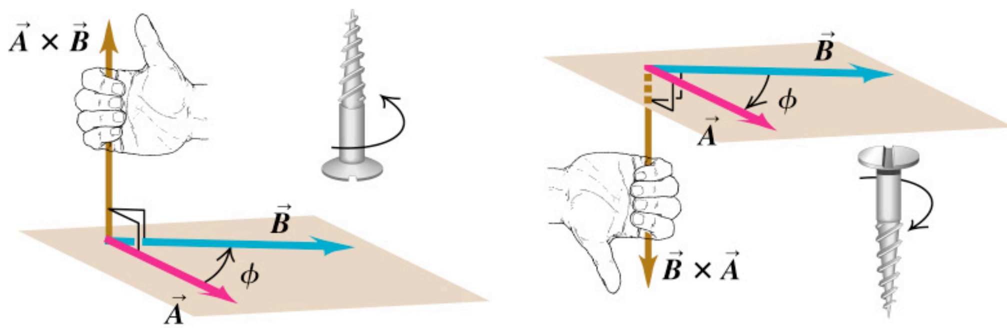
Right Hand Rule

Need to practice this until obvious !

Vector Product Of A & B

As a Result of the Definition

$$\vec{A} \times \vec{B} = \vec{C} = -(\vec{B} \times \vec{A})$$



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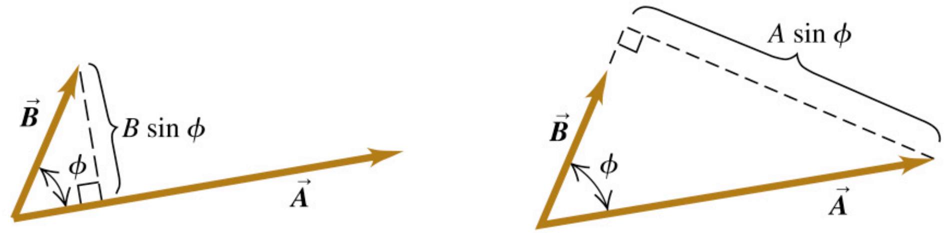


Angle φ measured positive when from A turns towards B

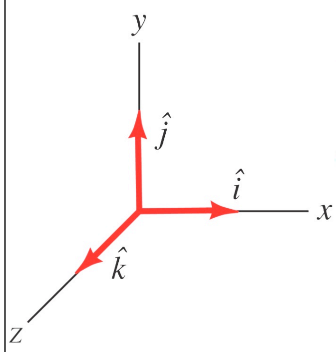
When $\vec{A} \perp \vec{B}$, angle $\varphi = 90^o$, magnitude maximum

When $\vec{A} \parallel \vec{B}$, angle $\varphi = 0^o$, magnitude = 0

Vector Product of Unit Vectors

$$\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$$

verify now !



$$\hat{i} \times \hat{j} = -\hat{j} \times \hat{i} = \hat{k}$$

$$\hat{j} \times \hat{k} = -\hat{k} \times \hat{j} = \hat{i}$$

$$\hat{k} \times \hat{i} = -\hat{i} \times \hat{k} = \hat{j}$$

Vector Product of Two 3-D Vectors

$$\vec{A} \times \vec{B} = (A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}) \times (B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k})$$

$$= A_x\hat{\imath} \times B_x\hat{\imath} + A_x\hat{\imath} \times B_y\hat{\jmath} + A_x\hat{\imath} \times B_z\hat{k}$$

$$+ A_y\hat{\jmath} \times B_x\hat{\imath} + A_y\hat{\jmath} \times B_y\hat{\jmath} + A_y\hat{\jmath} \times B_z\hat{k}$$

$$+ A_z\hat{k} \times B_x\hat{\imath} + A_z\hat{k} \times B_y\hat{\jmath} + A_z\hat{k} \times B_z\hat{k}$$

rewrite the individual terms as $A_x\hat{\imath} \times B_y\hat{\jmath} = (A_xB_y)\hat{\imath} \times \hat{\jmath}$, and so on.

$$\vec{A} \times \vec{B} = (A_yB_z - A_zB_y)\hat{\imath} + (A_zB_x - A_xB_z)\hat{\jmath} + (A_xB_y - A_yB_x)\hat{k}$$

the components of $\vec{C} = \vec{A} \times \vec{B}$ are given by

$$C_x = A_yB_z - A_zB_y \quad C_y = A_zB_x - A_xB_z \quad C_z = A_xB_y - A_yB_x$$

Vector Product of Two 3-D Vectors

$$\text{If } \vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

and

$$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$$

then

In the Determinant form

$$\vec{C} = \vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$C_x = A_yB_z - A_zB_y$, $C_y = A_zB_x - A_xB_z$, $C_z = A_xB_y - A_yB_x$

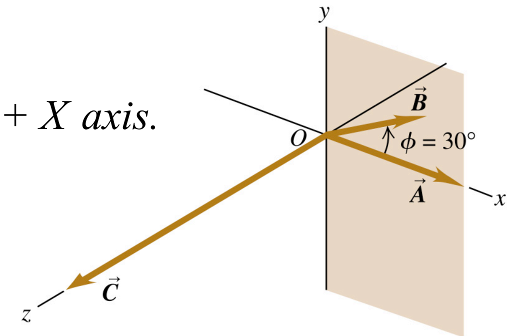
Calculating A Vector Product

$\vec{A} = 6\hat{i}$, $\vec{B}$ *has magnitude 4 units*

and lies in x - y *plane*

making an angle of 30^o *w.r.t* $+X$ *axis.*

Find $\vec{A} \times \vec{B}$



$\vec{C} = \vec{A} \times \vec{B};$

$|\vec{C}| = AB \sin\phi = (6)(4)(\sin 30^o) = 12$

Use right hand rule

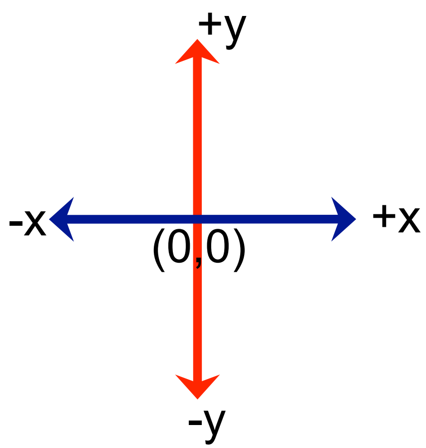
Direction of $\vec{A} \times \vec{B}$ is along +Z axis

$$\Rightarrow \boxed{\vec{A} \times \vec{B} = 12\hat{k}}$$

Chapter 2: Motion In a Straight Line

- Study of Mechanics: Study of relationships among force, matter and motion
  - *Kinematics*: study of motion of objects
  - *Dynamics*: relation of motion to its causes
- Simplest form of motion: object moving along a straight line.
  - Describe object's in motion by a ***point*** particle
  - Measure its displacement in direction x Vs time t, describe motion in terms of
    - → its speed, velocity & acceleration

Need to Define A Reference Frame First

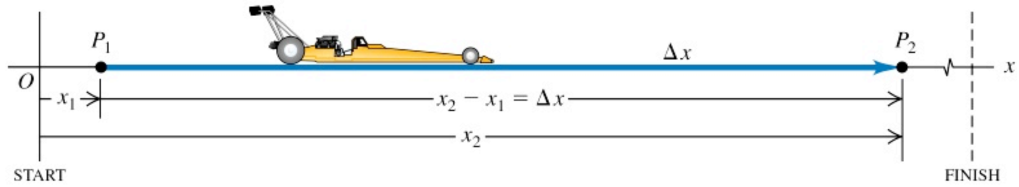


Defines for a displacement vector
which way is positive and which way is negative

Displacement Vector $\vec{x}$

Describe race car's motion by the that of a representative point on car→middle

Need a coordinate system to describe car's change in position
Choose x axis of coord. system to lie along car's straight line path



Displacement $\Delta x = x_2 - x_1$

Velocity: Average and Instantaneous

Define x-component of *average* velocity

$$v_{av-x} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$$

(unit= m/s)

Positive $v_{av\text{-}x}$ $\Rightarrow$ x coordinate increases with time t

Negative $v_{av\text{-}x}$ $\Rightarrow$ x coordinate decreases with time t

definition -x ← X=0 → +x

Time always increases

The *x-t* Graph of An Object's Motion

Pictorial representation of object's motion in x as function of time t

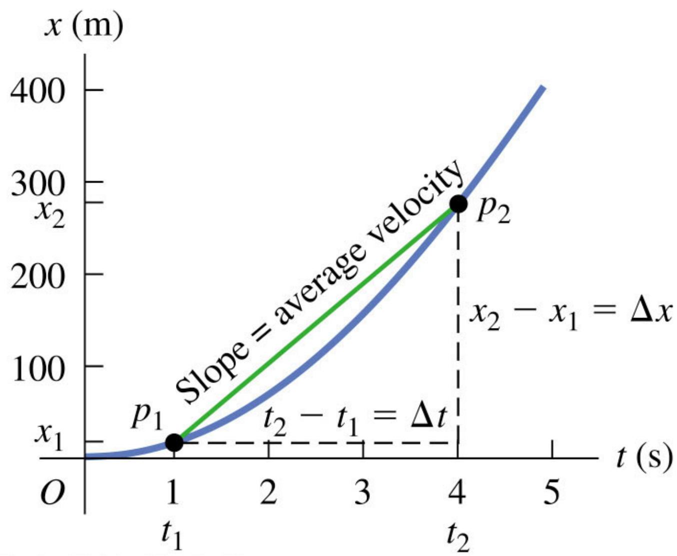
Not the path the object took along x axis !

$$v_{av-x} = \frac{\Delta x}{\Delta t}$$

$=$

slope of line p_1p_2

v_{av-x} depends on Δx & Δt not on details of motion !



Some Velocities

A snail's pace	10^{-3} m/s
A brisk walk	2 m/s
Fastest human	11 m/s
Running cheetah	35 m/s
Fastest car	341 m/s

Random motion of air molecules	500 m/s
Fastest airplane	1000 m/s
Orbiting communications satellite	3000 m/s
Electron orbiting in a hydrogen atom	2×10^6 m/s
Light traveling in a vacuum	3×10^8 m/s

Instantaneous Velocity v_x

- Definition: Velocity of an object at any specific instant of time or location
  - Is the limit of average velocity as the time interval $\Delta t \rightarrow 0$

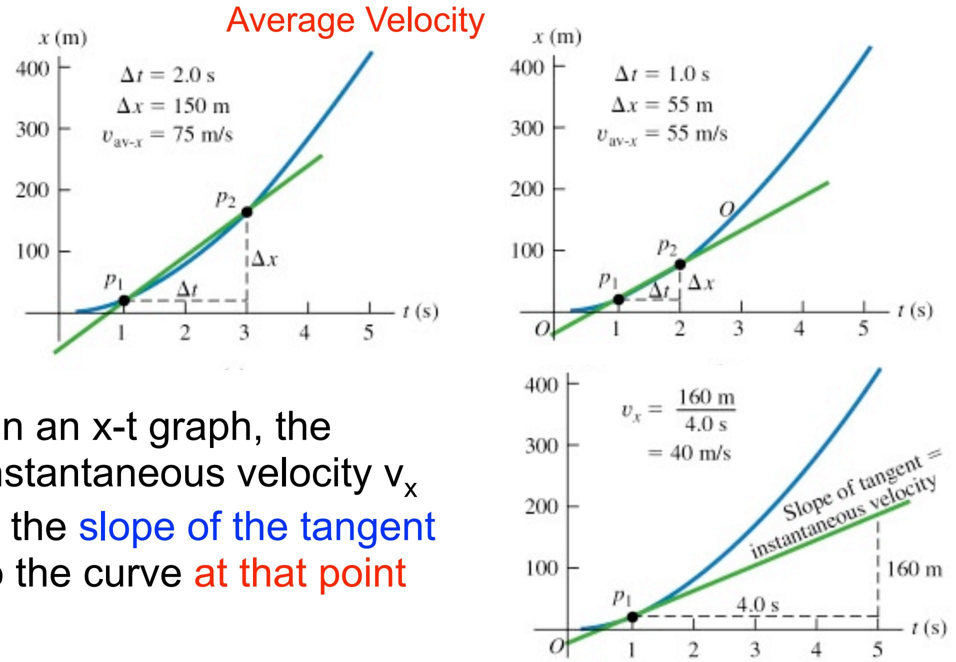
$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

v_{av-x} & v_x are both vectors, can be + or - depending on the change in displacement

Refer to v_x as velocity

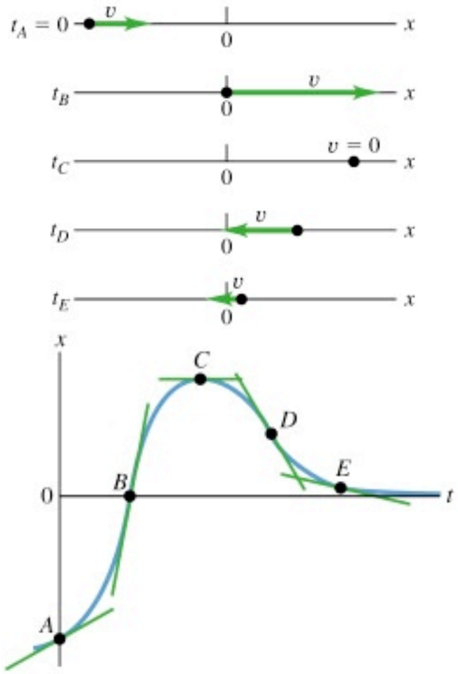
Speed, the magnitude of the velocity vector, is a scalar

Instant. Velocity v_x on x-t Graph



On an x-t graph, the Instantaneous velocity v_x is the slope of the tangent to the curve at that point

Interpreting An x-t Graph



Examine motion of object along x axis with when its velocity is changing vs time

	x-t graph	Motion of particle
A	positive slope, so $v_x > 0$	moving in $+x$-direction
B	larger positive slope, so $v_x > 0$	moving in $+x$-direction faster than at A
C	zero slope, so $v_x = 0$	instantaneously at rest
D	negative slope, so $v_x < 0$	moving in $-x$-direction
E	smaller negative slope, so $v_x < 0$	moving in $-x$-direction more slowly than at D

Indiana Jones and The Temple of Estimates !

What makes Indiana Jones so invincible ? ….the ability to calculate stuff !

Scale Of Things: Universe By Orders of Magnitude

Size or Distance	(m)	Mass	(kg)	Time Interval	(s)
Proton	10^{-15}	Electron	10^{-30}	Time for light to cross nucleus	10^{-23}
Atom	10^{-10}	Proton	10^{-27}	Period of visible light radiation	10^{-15}
Virus	10^{-7}	Amino acid	10^{-25}	Period of microwaves	10^{-10}
Giant amoeba	10^{-4}	Hemoglobin	10^{-22}	Half-life of muon	10^{-6}
Walnut	10^{-2}	Flu virus	10^{-19}	Period of highest audible sound	10^{-4}
Human being	10^{0}	Giant amoeba	10^{-8}	Period of human heartbeat	10^{0}
Highest mountain	10^{4}	Raindrop	10^{-6}	Half-life of free neutron	10^{3}
Earth	10^{7}	Ant	10^{-4}	Period of Earth's rotation	10^{3}
Sun	10^{9}	Human being	10^{2}	Period of Earth's revolution around the Sun	10^{7}
Distance from Earth to the Sun	10^{11}	Saturn V rocket	10^{6}		
		Pyramid	10^{10}	Lifetime of human being	10^{9}
Solar system	10^{13}	Earth	10^{24}	Half-life of plutonium-239	10^{12}
Distance to nearest star	10^{16}	Sun	10^{30}	Lifetime of mountain range	10^{15}
Milky Way galaxy	10^{21}	Milky Way galaxy	10^{41}	Age of Earth	10^{17}
Visible universe	10^{26}	Universe	10^{52}	Age of universe	10^{18}

The Scale of Things

TABLE 1.5 Some approximate lengths

	Length (m)
Circumference of the earth	4×10^7
New York to Los Angeles	5×10^6
Distance you can drive in 1 hour	1×10^5
Altitude of jet planes	1×10^4
Distance across a college campus	1000
Length of a football field	100
Length of a classroom	10
Length of your arm	1
Width of a textbook	0.1
Length of your little fingernail	0.01
Diameter of a pencil lead	1×10^{-3}
Thickness of a sheet of paper	1×10^{-4}
Diameter of a dust particle	1×10^{-5}

TABLE 1.6 Some approximate masses

	Mass (kg)
Large airliner	1×10^5
Small car	1000
Large human	100
Medium-size dog	10
Science textbook	1
Apple	0.1
Pencil	0.01
Raisin	1×10^{-3}
Fly	1×10^{-4}