Physics 2a, Nov 16, lecture 24

⋆Reading: chapters 9 and 10.

• Last time, rotational motion and analogy with what we've seen before

$$\begin{array}{cc} m \to I & \text{moment of intertia} \\ \vec{p} \to \vec{L} & \text{angular momentum} \\ \vec{F} \to \vec{\tau} & \text{torque.} \end{array}$$

Definitions:

$$I = \sum_i m_i r_i^2 = \int r^2 dm$$

$$\vec{L} = \sum_i \vec{r}_i \times \vec{p}_i = \vec{r}_{cm} \times M\vec{v}_{cm} + I\vec{\omega}.$$

$$\vec{\tau} = \sum_i \vec{r}_i \times \vec{F}_i = \frac{d\vec{L}}{dt}.$$

For example, rotation of rigid bodies, kinetic energy

$$K = \sum_i \tfrac{1}{2} m_i v_i^2 = \tfrac{1}{2} M v_{cm}^2 + \tfrac{1}{2} I\omega^2.$$

• If no torque, $\vec{\tau} = 0$, angular momentum is conserved, $\vec{L} =$ constant. Newton's 3rd law $\to \vec{\tau}_{total} = \vec{\tau}_{external}$, which vanishes for a closed system. So closed systems have conserved angular momentum. At a fundamental level, angular momentum is always conserved, though it can flow in and out of a system. Conservation of angular momentum is a deep principle, like conservation of energy and conservation of momentum. (They are related to symmetries: energy to time translations, momentum to space translations, and angular momentum to rotational invariance).

• Torque does work, $W = \int \vec{\tau} \cdot d\vec{\theta}$.

• As seen above, I is how much an object resists having its rotation changed. An object with a big I is harder to get started rotating and, once it's going, harder to stop. Objects with more of their weight farther from the axis of rotation have bigger I.

Examples of moments of inertia:

$$I = \begin{cases} \frac{1}{12} ML^2 & \text{rod through center} \\ \frac{1}{3} ML^2 & \text{rod through end} \\ \frac{1}{2} MR^2 & \text{solid cylinder} \\ \frac{2}{5} MR^2 & \text{solid sphere} \\ \frac{2}{3} MR^2 & \text{thin walled hollow sphere.} \end{cases}$$

For cylinder or sphere of radius R, write $I = cMR^2$, and note that $c_{hollow} > c_{solid}$ and $c_{cylinder} > c_{sphere}$ make intuitive sense, since bigger c means more mass farther from the axis of rotation.

Parallel axis result: the moment around an axis parallel to, and at a distance d from, one going through the CM is $I_p = I_{cm} + Md^2$. For example, the I of a rod through an end vs through the center are related this way.

• Example: race of the rolling bodies. Race round rigid bodies down an incline plane, which wins? Use conservation of energy. $E_{initial} = Mgh$. $E_{final} = \frac{1}{2}Mv_{cm}^2 + \frac{1}{2}I\omega^2$, and $\omega = v_{cm}/R$ (rolling without slipping), so $E_{final} = \frac{1}{2}(1+c)Mv_{cm}^2$, so $v_{cm} = \sqrt{2gh/(1+c)}$. Smaller I object wins. Makes sense, less energy taken up with rotation means more going into velocity.

• Unwinding cable example. Mass m on string, wrapped around cylinder with mass M and radius R. Mass drops height h. Find it's speed.

$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I(v/R)^2 = \frac{1}{2}(m + \frac{1}{2}M)v^2$, so $v = \sqrt{2gh/(1 + M/2m)}$.