Physics 2a, Nov 23, lecture 27

⋆Reading: sections 12.1, 12.2, 12.3.

• finish ladder example from last time: we found $n_1 = (M_R + M_L)g$, $f_f = \mu n_1 = n_2 = (M_R(d/L) + \frac{1}{2}M_L)g\cot\theta$ and thus need $\mu = (M_R(d/L) + \frac{1}{2}M_L)(M_R + M_L)^{-1}\cot\theta$. Check: for small θ, need more friction, which makes sense if you think about the ladder wanting to slide-out if it isn't vertical enough. Also, need more friction as d/L increases, which also makes sense – as Romeo climbs up the ladder, the sliding-out risk increases.

• A few more things about equilibrium. Stable vs unstable. Tipping over if center of mass is outside region above supports.

• Gravity! Newton's result (1687): any two masses attract each other with a force of magnitude

$$F_{gravity} = \frac{G_N m_1 m_2}{r^2}.$$

G_N is a fundamental constant of Nature, the strength of gravity. If we could communicate to a civilization on the other side of the universe, and agree on our choice of units of mass, length, and time, (e.g. by specifying them in terms of properties of the Hydrogen atom), we could compare with them the value of G_N, and we expect that we should agree.

• Consider the historical situation in 1797. They knew $g = 9.8m/s^2$, they knew the earth's radius, $R_e = 6.38 \times 10^6 m$, they knew Newton's result above, so they knew $G_N M_e = (9.8)(6.38)^2 \times 10^{12} m^3/s^2$, but they didn't yet separately know G_N or the mass of the earth. Then, in 1798, Henry Cavendish directly measured G, by directly measuring the gravitational attraction between a test mass and a mountain. He found

$$G = 6.67 \times 10^{-11} m^3/kgs^2.$$

He called his result "weighing the earth," since using it in the above leads to $M_e = 5.97 \times 10^{24} kg$.

• On the surface of a planet of mass M and radius R, get $F_{gravity} = GMm/R^2 = ma$, so the downward gravitational acceleration felt by a mass m near the surface is $g = G_N M/R^2$. In particular, this applies for the earth.

• Example: planet X has $M_X = 2M_e$ and $R_X = 2R_e$. How much does a 150 lb person weigh on planet X? A: the weight is $w = m(GM_X/R_X^2) = m(G2M_e/4R_e^2) = \frac{1}{2}mg_e$, where $g_e = GM_e/R_e^2$ is the acceleration on earth. So the 150 lb person weighs 75lbs on planet X. Of course their mass is unchanged, regardless of the environment.

• Find the gravitational acceleration of a mass m, due to the earth, as a function of the distance r from the center of the earth, $g(r)$, with $g(r = R_e) \equiv g_e = 9.8 m/s^2$. The result is $\sim 1/r^2$ above the earth. Someone asked the interesting question about what would happen if you dug a hole inside the earth. The result is that only the mass inside the sphere of radius r contributes, that outside cancels out. If we approximate the mass density of the earth as uniform (this is not true, since the center of the earth is a very dense core), then $M(r) \sim r^3$, proportional to the volume of a sphere of radius r. So $F(r) \sim M(r)/r^2 \sim r$: the force grows linearly with r for r inside the earth. The result is thus:

$$g(r) = \begin{cases} g_e \left(\frac{R_e}{r}\right)^2 & \text{for } r \geq R_e \\ g_e \frac{r}{R_e} & \text{for } r \leq R_e. \end{cases}$$

If there is a little hollow cavity in the center of the earth, objects inside would float around, weightless, like they're floating in empty space – the gravity from the earth canceling out.