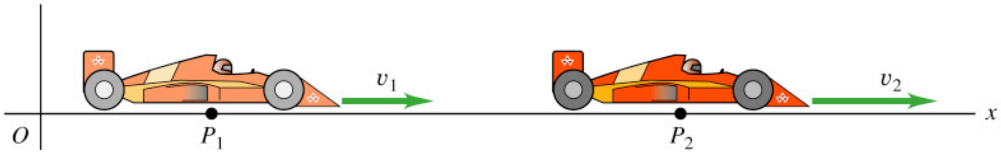
Physics 2A
Lecture 3: Sept 28, 2010

Average & Instant. Acceleration



$$\text{Average Acceleration } a_{av\text{-}x} = \frac{v_{2x} - v_{1x}}{t_2 - t_1} = \frac{\Delta v_x}{\Delta t}$$

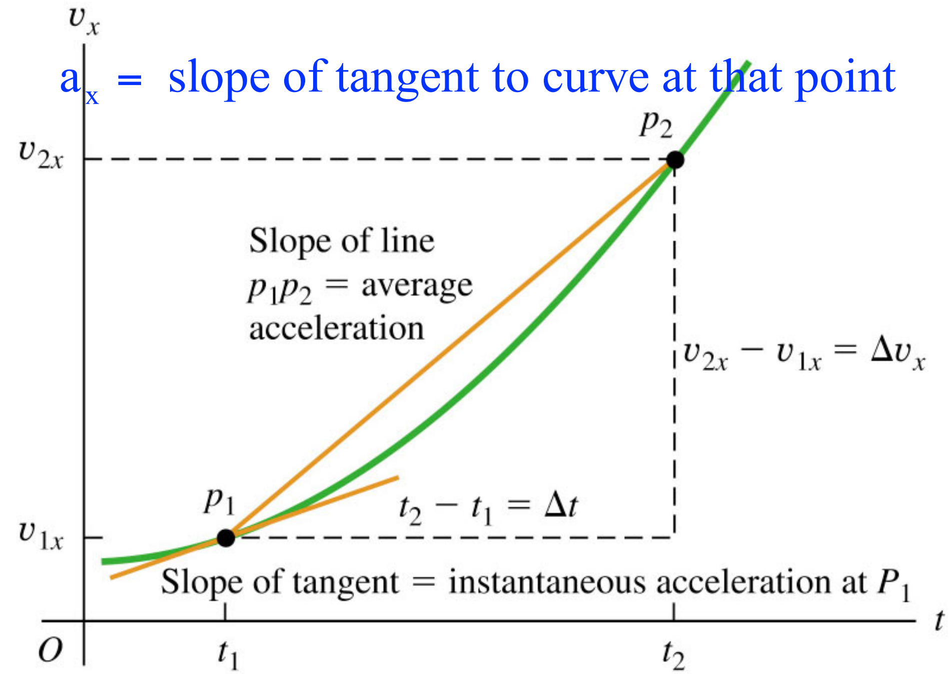
Instant acceleration = limit of average accelaration

when time $\Delta t \rightarrow 0$. $$a_x = \lim_{\Delta t \rightarrow 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt}$$

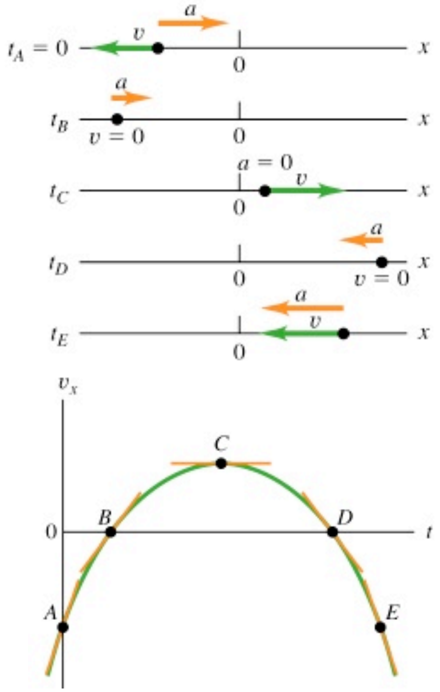
Acceleration has units of (m/s^2)

Now on, use acceleration to mean instant acceleration

Acceleration A_x On A v_x- t Graph



Examining a $v_x – t$ Graph



	v_x-t graph	Motion of particle
A	$v_x < 0$; positive slope, so $a_x > 0$	moving in $-x$-direction, slowing down
B	$v_x = 0$; positive slope, so $a_x > 0$	instantaneously at rest, about to move in $+x$-direction
C	$v_x > 0$; zero slope, so $a_x = 0$	moving in $+x$-direction at maximum speed
D	$v_x = 0$; negative slope, so $a_x < 0$	instantaneously at rest, about to move in $-x$-direction
E	$v_x < 0$; negative slope, so $a_x < 0$	moving in $-x$-direction, speeding up

Acceleration & The x-t Graph

Remember Calculus ?

Can rewrite definition of (instant.) acceleration a_x

$$a_x = \frac{dv_x}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$

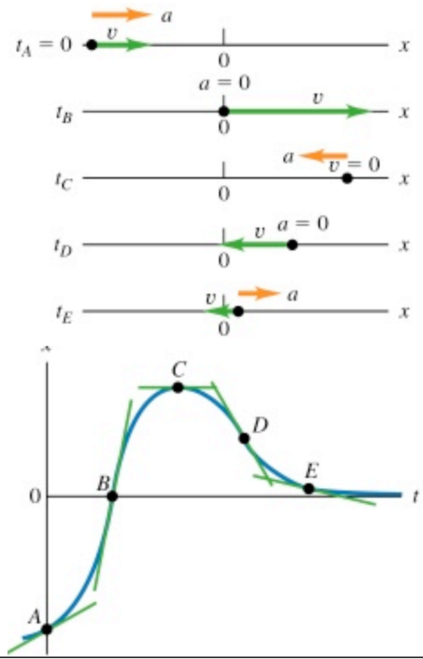
When

$\frac{d^2x}{dt^2} > 0$, x – t curve is concave (curves ↑), v_x increasing

$\frac{d^2x}{dt^2} < 0$, x – t curve is convex (curves ↓), v_x decreasing

$\frac{d^2x}{dt^2} = 0$, x – t curve has no curvature (inflexion pt), v_x = const

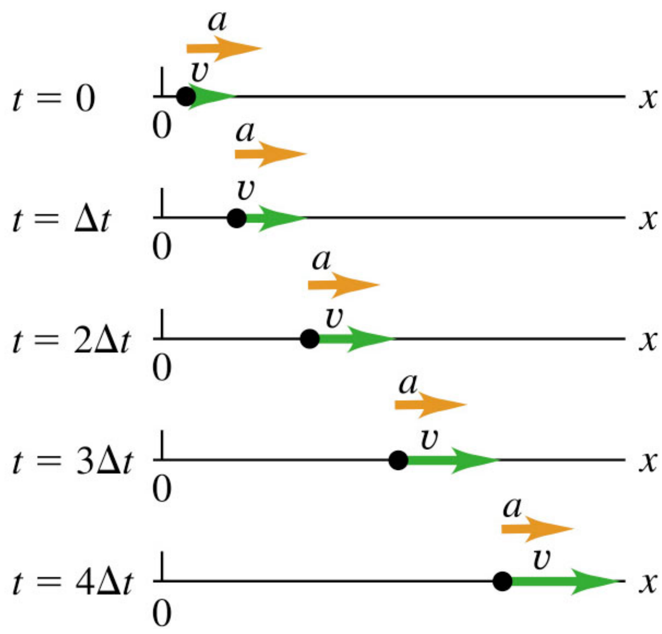
The x-t Graph For Same Motion



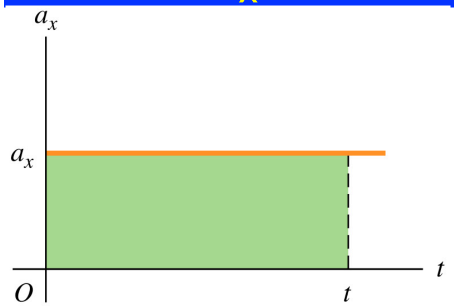
	x-t graph	Motion of particle
A	positive slope, upward curvature, so $v_x > 0$, $a_x > 0$	moving in $+x$-direction, speeding up
B	positive slope, zero curvature, so $v_x > 0$, $a_x = 0$	moving in $+x$-direction, speed not changing
C	zero slope, downward curvature, so $v_x = 0$, $a_x < 0$	instantaneously at rest, velocity changing from + to −
D	negative slope, zero curvature, so $v_x < 0$, $a_x = 0$	moving in $-x$-direction, speed not changing
E	negative slope, upward curvature, so $v_x < 0$, $a_x > 0$	moving in $-x$-direction, slowing down

Motion With Constant Acceleration

Motion diagram showing position, velocity & acceleration of an object



The a_x– t Graph For Constant a_x



$$a_x = \frac{v_{2x} - v_{1x}}{t_2 - t_1} = \text{constant}$$

Start clock at $t_1 = 0$, observe again at time $t_2 = t$

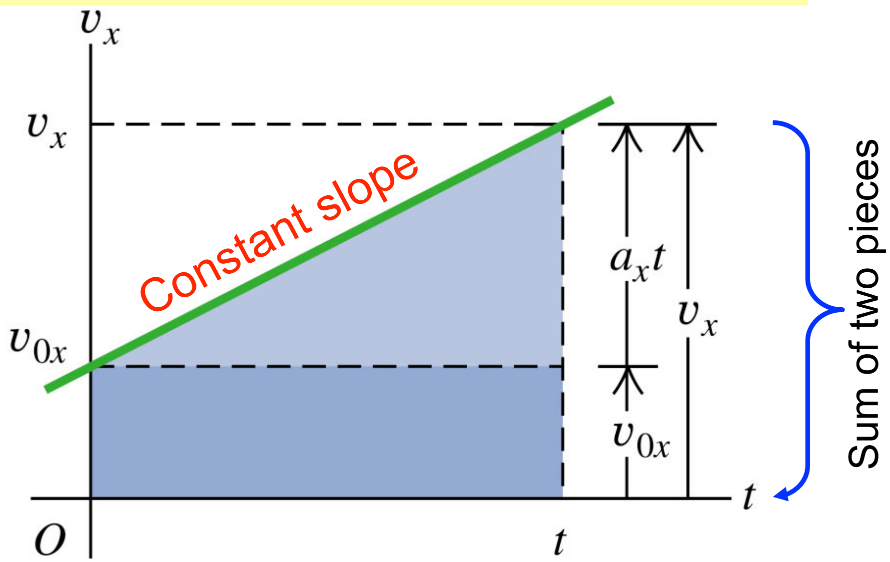
Call v_{0x} the velocity at $t_1 = 0$, v_x the velocity at $t_2 = t$

$$a_x = \frac{v_x - v_{0x}}{t - 0}$$

$$\Rightarrow \quad \boxed{v_x = v_{0x} + a_x t} \quad ; \; a_x = \text{constant}$$

The v_x-t Graph For Constant a_x

$$v_x = v_{0x} + a_x t \quad ; \; a_x = \text{constant}$$



Evolution of x vs t when a_x=Constant

At time $t_1 = 0$, object at $x = x_0$, has $v_{x1} = v_{0x}$

At time $t_2 = t$, object at $x = x$, has $v_{x2} = v_x$

then $$v_{av\text{-}x} = \frac{x - x_0}{t}$$

When a_x = constant, velocity changes at const rate

so for time interval 0 ® t, $v_{av\text{-}x} = \dfrac{v_{0x} + v_x}{2}$

But since $v_x = v_{0x} + a_x t \Rightarrow v_{av\text{-}x} = \dfrac{1}{2}\left(v_{0x} + v_{0x} + a_x t\right)$

$$= v_{0x} + \frac{1}{2} a_x t$$

Evolution of x vs t when a_x=Constant

$$v_{av\text{-}x} = \frac{x - x_0}{t}$$

$$= v_{0x} + \frac{1}{2} a_x t$$

$$\Rightarrow x = x_0 + v_{0x} t + \frac{1}{2} a_x t^2$$

$\Rightarrow$ Parabolic curve in x-t