

★Reading: chapter 13.

- Last time, mass on a spring, $F = -kx = ma = m \frac{d^2x}{dt^2}$ is solved by

$$x(t) = A \cos(\omega t + \phi), \quad (1)$$

where $\omega = \sqrt{k/m}$. The amplitude A is the maximum displacement from equilibrium. Then

$$v(t) = \frac{dx}{dt} = -\omega A \sin(\omega t + \phi), \quad (2)$$

so the maximum speed is ωA . As expected, the energy is a constant:

$$E = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}kx^2 = \frac{1}{2}m\omega^2 A^2(\sin^2(\omega t + \phi) + \cos^2(\omega t + \phi)) = \frac{1}{2}m\omega^2 A^2.$$

For any potential $U(x)$, expanding (in a Taylor's series) near a point of stable equilibrium will always give something that's approximately quadratic for small displacements (ignoring higher terms in the Taylor's series), so a mass on a spring behaves exactly like ANY small oscillations around a point of stable equilibrium.

Example: vibrations of molecules, $U \approx U_0((\frac{R_0}{r})^{12} - 2(\frac{R_0}{r})^6)$ has a minimum at $r = R_0$, and expanding $r = R_0 + x$ gives a SHO with $k = 72U_0/R_0^2$.

- Example: putty dropped on mass on spring, either when mass is at equilibrium position or when mass is at maximum displacement. In the first case energy is lost and the amplitude is reduced.

- Angular SHM: If $\tau = -\kappa\theta$ (κ is called the torsion constant), then $\tau = I\alpha = I \frac{d^2\theta}{dt^2}$ is solved by $\theta = \Theta(\cos \omega t + \phi)$, where $\omega = \sqrt{\kappa/I}$.

- Pendulum. $F_\theta = -mg \sin \theta \approx -mg\theta$, so $\tau = LF_\theta \approx -mgL\theta$, which gives the angular SHO, with $\kappa = mgL$. Since $I = mL^2$, we have $\omega = \sqrt{g/L}$. (Check: units). This is the approximate motion for small initial angle displacements Θ . If Θ is big, then need to keep full expression $\tau = -mgL \sin \theta$ and then can't solve the differential equation as easily in terms of trig functions, need special functions.

For a rigid body acting as a pendulum, replace L with distance d between the pivot point and the center of mass, and $\omega = \sqrt{mgd/I}$.

Example: meter stick of mass m , suspended by an end. Using $I = \frac{1}{3}mL^2$ and $d = L/2$, get period $T = 2\pi\sqrt{2L/3g}$ Using $L = 1m$ get $T \approx 1.64s$.

• Damped oscillations. Add friction term, $F = -kx - b\dot{x} = m\frac{d^2x}{dt^2}$. The solution of this differential equation is

$$x(t) = Ae^{-bt/2m} \cos(\omega't + \phi),$$

where $\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$.

When $\omega' = 0$, the system is critically damped. When ω' is imaginary, the system is overdamped, and the the solution is a sum of exponentials rather than a cos.

The amplitude decreases exponentially in time, thanks to the energy loss to friction. Indeed, the power loss to friction (heat) is seen by the decrease in the oscillation energy: $\frac{dE}{dt} = -b\dot{x}^2$.

• Forced oscillations. Take $F_x = F_0 \cos \omega_d t$, so $m\frac{d^2x}{dt^2} = -kx - b\dot{x} + F_0 \cos \omega_d t$. Solving the differential equation gives (you'll learn how to do this in a later class) amplitude

$$A = \frac{F_0}{\sqrt{(k - m\omega_d^2)^2 + b^2\omega_d^2}}.$$

This amplitude is peaked when $\omega_d = \sqrt{k/m}$ (resonance), with height $\sim 1/b$. A system can have destructively huge oscillation amplitude if driven at its resonance frequency.

That's all folks! Good luck in your finals! Best Wishes, Ken