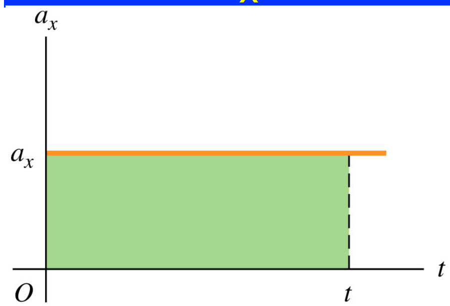
The a_x– t Graph For Constant a_x



$$a_x = \frac{v_{2x} - v_{1x}}{t_2 - t_1} = \text{constant}$$

Start clock at $t_1 = 0$, observe again at time $t_2 = t$

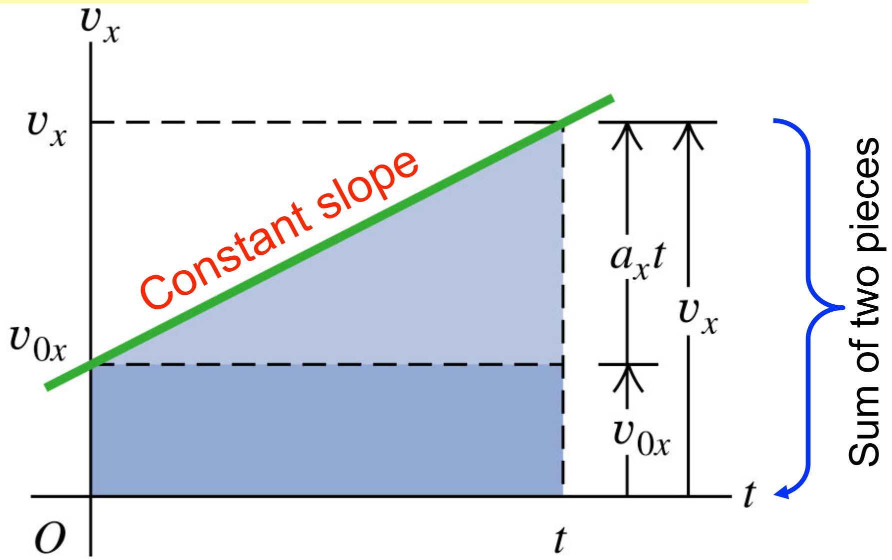
Call v_{0x} the velocity at $t_1 = 0$, v_x the velocity at $t_2 = t$

$$a_x = \frac{v_x - v_{0x}}{t - 0}$$

$$\Rightarrow \quad \boxed{v_x = v_{0x} + a_x t} \quad ; \; a_x = \text{constant}$$

The v_x-t Graph For Constant a_x

$$v_x = v_{0x} + a_x t \quad ; \; a_x = \text{constant}$$



Evolution of x vs t when a_x=Constant

At time $t_1 = 0$, object at $x = x_0$, has $v_{x1} = v_{0x}$

At time $t_2 = t$, object at $x = x$, has $v_{x2} = v_x$

then $\boxed{v_{av\text{-}x} = \frac{x - x_0}{t}}$

When a_x = constant, velocity changes at const rate

so for time interval 0 ® t, $v_{av\text{-}x} = \frac{v_{0x} + v_x}{2}$

But since $v_x = v_{0x} + a_x t \Rightarrow v_{av\text{-}x} = \frac{1}{2}\left(v_{0x} + v_{0x} + a_x t\right)$

$$= \boxed{v_{0x} + \frac{1}{2} a_x t}$$

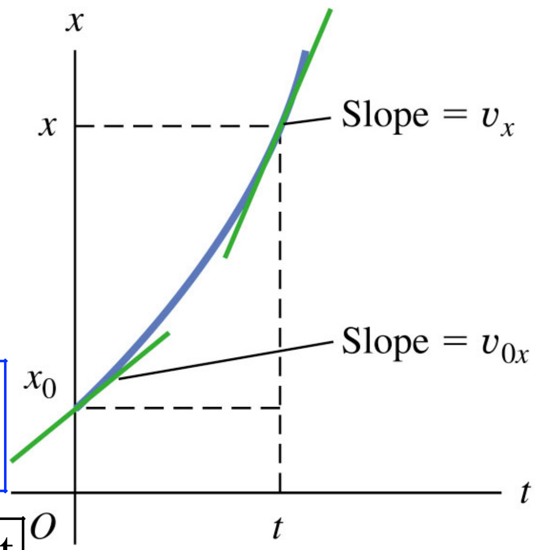
Evolution of x vs t when a_x=Constant

$$v_{av\text{-}x} = \frac{x - x_0}{t}$$

$$= v_{0x} + \frac{1}{2}a_x t$$

$$\Rightarrow x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$$

$\Rightarrow$ Parabolic curve in x-t



Relating x, v_x & a_x (without time t)

write $t = \dfrac{v_x - v_{0x}}{a_x}$

substitute in $x = x_0 + v_{0x} t + \dfrac{1}{2} a_x t^2$

$$\Rightarrow x = x_0 + v_{0x}\left(\frac{v_x - v_{0x}}{a_x}\right) + \frac{1}{2} a_x \left(\frac{v_x - v_{0x}}{a_x}\right)^2$$

$$\Rightarrow (x - x_0)2a_x = \boxed{2v_{0x}v_x} - 2v_{0x}^2 + v_x^2 \boxed{-2v_{0x}v_x} + v_{0x}^2$$

$$\Rightarrow \boxed{v_x^2 = v_{0x}^2 + 2a_x(x - x_0)}$$

An Expression Without a_x

Since $v_{av\text{-}x} = \frac{x - x_0}{t}$ and $v_{av\text{-}x} = \frac{v_{0x} + v_x}{2}$

$$\Rightarrow \frac{x - x_0}{t} = \frac{v_{0x} + v_x}{2}$$

$$\Rightarrow \quad x - x_0 = \left(\frac{v_{0x} + v_x}{2}\right)t$$

This is a useful expression to have

when a_x = constant but unknown

Equations For a=constant

Master equations relating x, v, a and t

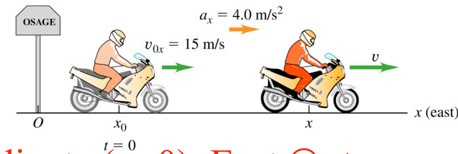
$$v_x = v_{0x} + a_x t$$

$$x = x_0 + v_{0x} t + \frac{1}{2} a_x t^2$$

$$v_x^2 = v_{0x}^2 + 2a_x (x - x_0)$$

$$x - x_0 = \left(\frac{v_{0x} + v_x}{2} \right) t$$

Motorcyclist going east, accelerates after passing signpost. He accelerates at $4.0 m/s^2$. At t=0, he is 5.0m east of signpost, moving east 15 m/s. (a) find his position and velocity at t=2.0s. Where is motorcyclist when his velocity is 25 m/s ?



Take signpost as origin of coordinate (x=0), East ® +x

At $t=0, x_0=5.0m, v_{0x}=15m/s;\ a_x=4.0m/s^2$

(a) what is x, v_x at t=2.0s, (b) x when $v_x=25m/s$

(a) Use $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$

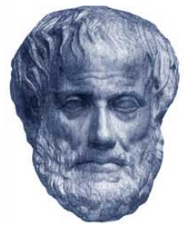
$$= 5.0m + (15m/s)(2.0s) + \frac{1}{2}(4.0m/s^2)(2.0s)^2 = 43m$$

Velocity at x=43m: $v_x = v_{0x} + a_x t = 23m/s$

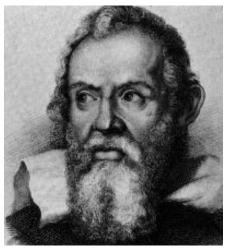
(b)no t given ! so use $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$

$$\Rightarrow x = x_0 + \frac{v_x^2 - v_{0x}^2}{2a_x} = 5.0m + \frac{(25m/s)^2 - (15m/s)^2}{2a_x} = 55m$$

Motion With Constant Acceleration: Freely Falling Bodies



Aristote (4 BC) believed (didn't check!) that heavier objects fall faster through a medium than lighter ones



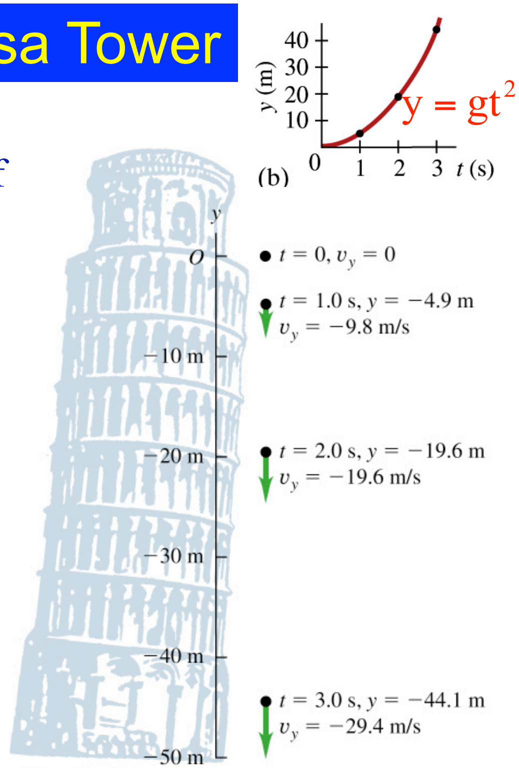
19 centuries later, Galileo did some experiments, disproved this by asserting that all objects falling freely experience a downward acceleration that is constant and independent of object's weight

Galileo's Famous Experiments

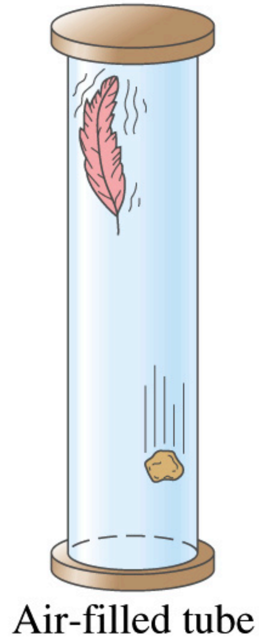
Motion of a Ball on an Inclined plane

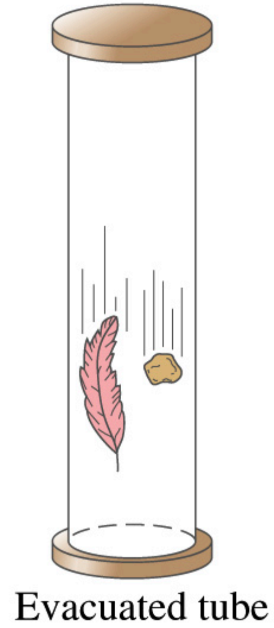
Free Fall From Pisa Tower

- Examine a falling object
- Free fall: An idealization of the motion where one ignores “small” effects like
  - Air
  - Earth’s rotation
  - Altitude at location etc
- Free fall is motion with constant acceleration
  - Down or up
- Acceleration g = -9.8 m/s^2 on earth, -1.6m/s^2 on moon & -270 m/s^2 on the sun

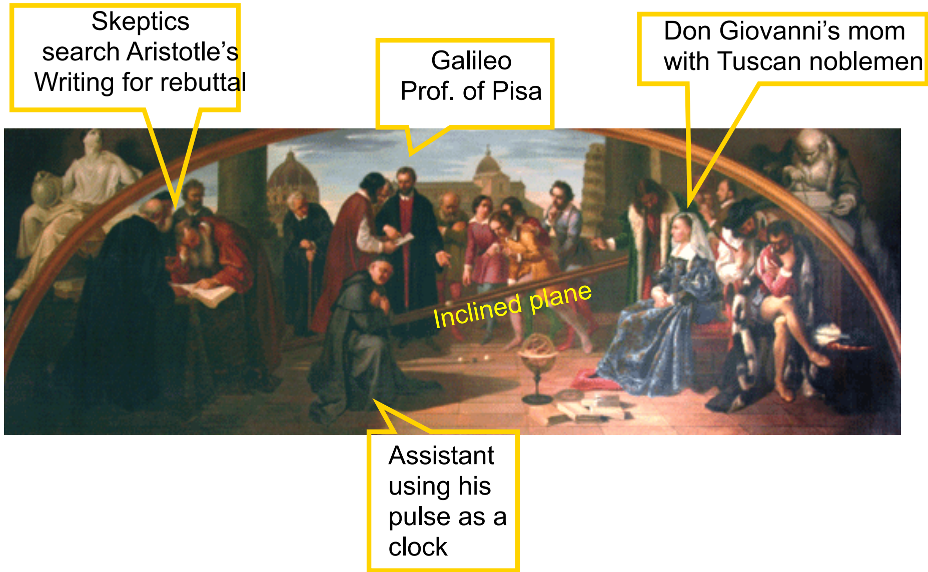


Free Fall: Galileo Chose Well



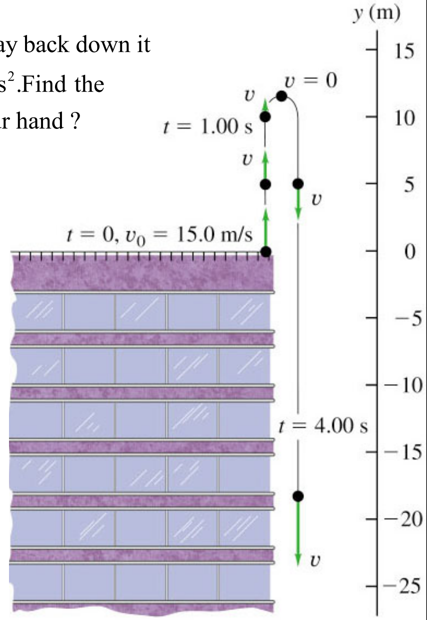


Inclined Plane Demo By Galileo



Giuseppe Bezzuoli, Tribuna di Galileo, Firenze

You throw a ball vertically upwards from roof of a building. Ball leaves your hand at point even with the roof railing with an upward speed of 15.0m/s; ball is then in free fall. On its way back down it just misses the railing. Acceleration due to gravity g =9.80m/s^2.Find the position and velocity of ball 1.00s and 4.00s after leaving your hand ?



Motion is in straight line but vertical (y axis).
y=0 is at the roof and +y direction is upwards

Initial position $y_0 = 0$,

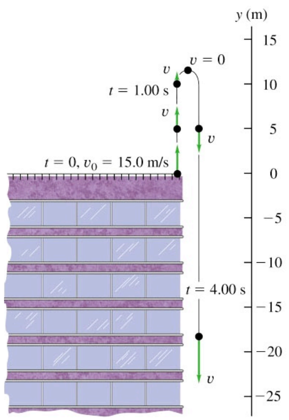
$v_{0y} = +15.0 m/s$,

$a_y = g = -9.80 m/s^2$ down

Find x & v at t=1.00s,4.00s

Position y and velocity v_y after ball leaves hand obtained

from $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2; v_y = v_{0y} + a_y t$ $[a_y = g = -9.80 m/s^2]$



$$y(t = 1s) = 0 + (15.0 m/s)(1s) + \frac{1}{2}(-9.80 m/s^2)(1)^2$$
$$= +10.1m \text{ (above roof)}$$
$$v_y(t = 1s) = 15.0 m/s + (-9.80 m/s^2)(1s)$$
$$= +5.2 m/s \text{ (going upwards)}$$

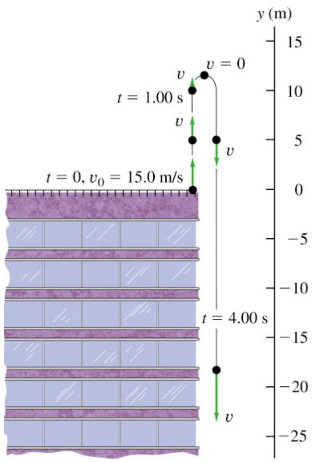
Similarly:

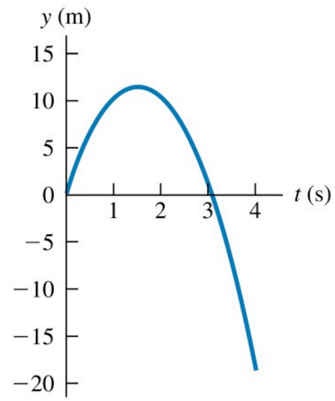
$y(t = 4s) = -18.4m$ (below roof)

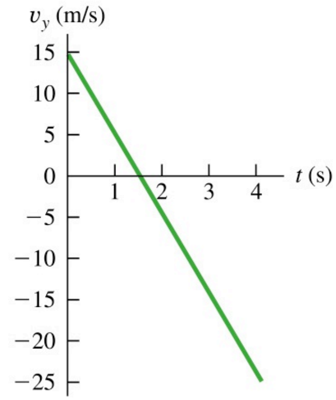
$v_y(t = 4s) = -24.2 m/s$ (going downwards)

This is due to the pull of gravity !

Description With y-t and v-t Graphs

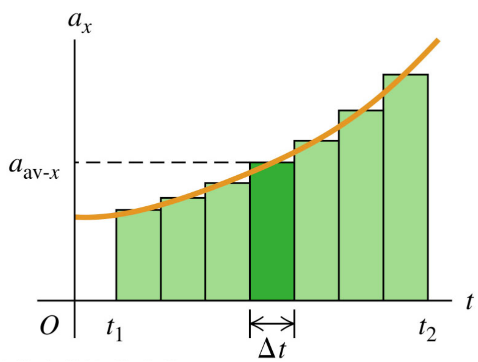






Now: Case when a = a(t) ≠ constant

Graph of acceleration Vs time



Use Calculus, divide interval between t_1 & t_2 in slices of Δt

Change in velocity $\Delta v_x = a_{av-x} \Delta t$

= area of shaded strip with height a_{av-x} & width Δt

Total velocity change from t_1 ® t_2

= total area under $a_x - t$ curve between vertical lines t_1 & t_2

In Calculus parlance, as $\Delta t \to 0$:

Velocity change = integral of a_x with t

$$v_{2x} - v_{1x} = \int_{v_{1x}}^{v_{2x}} dv_x = \int_{t_1}^{t_2} a_x dt$$

Case when $a = a(t) \neq$ constant

Similarly since $v_x = \dfrac{dx}{dt} \Rightarrow dx = v_x dt$

The change in position $$x_2 - x_1 = \int_{x_1}^{x_2} dx = \int_{t_1}^{t_2} v_x dt$$

In Conclusion:

$$v_x = v_{0x} + \int_{t=0}^{t=t} a_x dt$$ and $$x = x_0 + \int_{t=0}^{t=t} v_x dt$$

Sally driving along a straight highway. At t=0, Sally is moving at 10m/s in +x dir when she passes signpost at x=50m. Her acceleration is a= $a(t) = 2.0\,m/s^2 - (0.10\,m/s^3)t$. Find (a) expression for v & x vs t (b) when is v largest & how much is it ? (c) where is car when it reaces this max. v ?

Use $v_x = v_{0x} + \int_{t=0}^{t=t} a_x dt$ & $x = x_0 + \int_{t=0}^{t=t} v_x dt$

At t=0, $x_0 = 50m, v_{0x} = 10m/s$, find $v_x = v_x(t)$

$$v_x = 10m/s + \int [2.0m/s^2 - (0.10m/s^3)t]dt$$

Use $\int t^n dt = \frac{t^{n+1}}{n+1}$ $\Rightarrow v_x = 10m/s + (2.0m/s^2)t - \frac{1}{2}(0.10m/s^3)t^2$

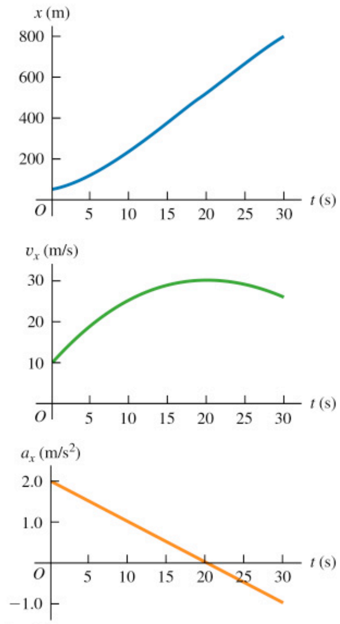
$$\text{and } x = 50m + \int [10m/s + (2.0m/s^2)t - \frac{1}{2}(0.10m/s^3)t^2]dt$$

$$\Rightarrow x = 50m + (10m/s)t + \frac{1}{2}(2.0m/s^2)t^2 - \frac{1}{2\times 3}(0.10m/s^3)t^3$$

Maximum value of v_x when $\frac{dv_x}{dt} = a_x = 0$, Using v_x expression

$$\Rightarrow a_x = 0 = 2.0m/s^2 - (0.10m/s^3)t \Rightarrow \boxed{t = 20s}$$

Graph of x, v_x & a_x



$$v_{x-max} = v_x(t = 20s) = 10m/s + (2.0m/s^2)(20s) + \frac{1}{2}(0.10m/s^3)(20s)^2$$

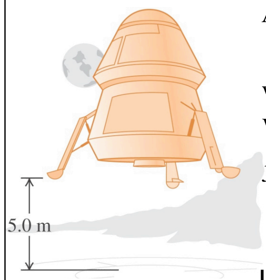
$$= 30m/s$$

To get position x at t=20s when v_x=maximum

Input t = 20s in $x(t) = 50m + (10m/s)t + \frac{1}{2}(2.0m/s^2)t^2 - \frac{1}{6}(0.10m/s^3)t^3$

$$=517m$$

Touchdown On The Moon



A lunar lander is making its descent to moon base. The lander descends slowly under the retro-thrust of its descent engine. The engine is cut off when the lander is 5.0m above surface and has a downward speed of 0.8m/s. With the engine off, the lander is in free fall. What is the speed of the lander just before it touches surface. $g_{moon}=1.6m/s^2$

Apply constant acceleration equations to the motion of the lander

Let downward be positive. Lander is in freefall $\Rightarrow a_y=g_{moon}$

What we know: $v_{0y}=+0.8m/s$, $y-y_0=5.0m$, $a_y = 1.6m/s^2$, no idea about t!

Use $v_y^2 = v_{0y}^2 + 2a_y(y-y_0)$

$$\Rightarrow v_y=\sqrt{v_{0y}^2+2a_y(y-y_0)}=\sqrt{(0.8m/s)^2+2(1.6m/s^2)(5.0m)}$$

$= 4.1m/s$

The same descent on Earth would have led to $v_y=9.9m/s$ due to the stronger acceleration due to gravity g.

Coming to a lecture near you !

Spiderman steps from the top of a tall building. He falls freely from rest to the ground a distance of h. He falls a distance of h/4 in the last 1.0s of his fall. What is the height h of the building?

Which equation to use? depends on what we know?

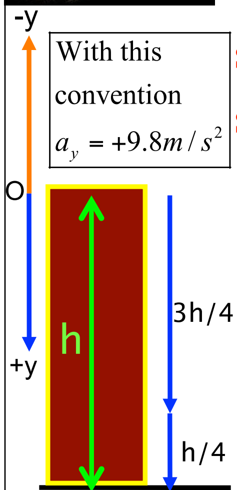
Divide Spidey's motion in 2 segments: y = 0 ® y = 3/4h and y = 3/4h ® h

Motion from roof to h/4 above ground $\Rightarrow y - y_0 = 3/4h, v_0 = 0, a_y = g$

$$\text{Use } v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$$

So we get: $v_y^2 = 0 + a_y(3/4h) \Rightarrow v_y = \sqrt{2\, a_y(3/4h)} = 3.834\sqrt{h}\ \sqrt{\text{m}}/\text{s}$

Spiderman's speed after he has fallen for 3/4h is $\boxed{v_y = 3.83\sqrt{h}\sqrt{\ \text{m}}/\text{s}}$



In the next segment, $y - y_0 = h/4, v_{0y} = 3.83\sqrt{h}\sqrt{\ \text{m}}/\text{s}, a_y = g, t = 1\text{s}$

clearly we should use: $y = y_0 + v_0 t + (1/2)a_y t^2$

$\Rightarrow (h/4) = 3.83\sqrt{h}\sqrt{\ \text{m}} + 4.90m.$ Now solve for h...but how ?

Write as $\frac{1}{4}u^2 - 3.83u\sqrt{m} - 4.90m = 0,$ solve for u (quadratic eq)

$$\text{if } au^2 + bu + c = 0 \Rightarrow u = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Taking the positive root $\Rightarrow$ u=16.52$\sqrt{\text{m}}$ $\Rightarrow$ $\boxed{h = u^2 = 273m}$

A Groovy Crash !

Helicopter carrying Dr. Evil takes off with a constant upward acceleration of $5.0 m/s^2$. Austin Powers jumps on just as the helicopter lifts off the ground. After the two men struggle for 10.0s, Powers shuts off the engine and steps out of the chopper. Assume that the chopper is in free fall after its engine is shut off, and ignore the effects of air-resistence. (a) what is the max. height above ground reached by chopper. (b) Powers deploys a jet pack strapped on his back 7.0s after leaving chopper, and then has a constant downward acceleration with magnitude $2.0 m/s^2$.How far is Powers above the ground when the chopper crashes to the ground?

- Analyze the action segments in the narrative
  - Chopper/Dr.Evil
    - 10.0s under constant upward acceleration of $5 m/s^2$
    - Followed by free-fall under gravity
      - They continue to go up and then come down
  - Austin Powers
    - 10.0s under constant upward acceleration
    - Followed by free-fall under gravity for 7.0s,
    - Followed by constant downward acceleration $2 m/s^2$
- Set up coordinate system, ground = y =0, up is +y

what is the max. height above ground reached by chopper ?

When engine shuts off, both chopper + Austin + Dr. Evil have :

Same upward velocity $v_y = v_{0y} + a_y t = 0 + (5m/s^2)(10s) = 50m/s$

Same height $y = y_0 + v_{0y}t + (1/2)a_y t^2 = 0 + 0 + 0.5(5.0m/s^2)(10s)^2 = 250m$

How much more do the chopper/Dr.Evil & Austin Powers climb?

When engine shuts off: chopper etc comtinue to climb against gravity, till $v_y=0$

$\Rightarrow v_{0y} = +50m/s,\ y_0=250m, a_y=-9.8m/s^2$ and when $v_y=0m/s$, what is y ?

Use $v_y^2 = v_{0y}^2 + 2a_y(y-y_0) \Rightarrow 0 = v_{0y}^2 + 2a_y(y-y_0) \Rightarrow y = \frac{v_y^2 - v_{0y}^2}{2a_y} + y_0 = \frac{0-(50m/s)^2}{2(-9.8m/s^2)} + 250m$

$\Rightarrow$ y=max height attained=378m. From this height every thing starts to fall down

Time t for chopper to crash to y=0 under free-fall from height of $y_0=+250m$ and $v_{0y}=+50m/s$?

Use $(y-y_0) = v_{0y}t + 0.5a_y t^2 \Rightarrow -250m = (50m/s)t - 0.5(9.8m/s^2)t^2$...solve this quadratic eqn for t !

+ solution of $0.5(9.8m/s^2)t^2 - (50m/s)t - 250m = 0$ is $t = (1/9.8)(50 \pm \sqrt{(50)^2 - 4(4.9)(250)})s$

Time is always positive so t = 13.9s after Powers shuts down Dr. Evil's engine!

Now lets look at Mr. Powers trajectory: when he steps out of the chopper, he retains the initial velocity of the chopper. But his acceleration changes from $+5m/s^2$ to $-9.8m/s^2$. Without the jet pack he would have crashed at same time as chopper+Dr. Evil. But when he turns on the jetpack after 7s of free fall, his acceleration changes to $-2m/s^2$ instead of $-9.8m/s^2$ $\Rightarrow$ he descends to ground will smaller velocity and travels less vertically $\Rightarrow$ he is still above ground when chopper crashes How high above ground is he when the chopper crashes ? $\Rightarrow$ solution in 2 steps!

After 7s of free fall, he is at $y = y_0 + v_{0y}t + 0.5a_y t^2 = (250m) + (50m/s)(7s) - (0.5)(9.8m/s^2)(7s)$

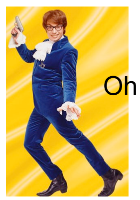
$\Rightarrow$ $y - y_0 = 360m \Rightarrow$ 360m above ground, but he is not out of danger yet!

His velocity at that height is $v_y = v_{0y} + a_y t = 50m/s + (-9.8m/s^2)(7s) = -18.6m/s$

(13.9-7.0=6.9)s before Dr. Evil crashes, Austin fires jet pack, forcing him to accelerate at $a_y = -2m/s^2$

After that 6.9s, he is at $y = y_0 + v_{0y}t + 0.5a_y t^2 = 360m + (-18.6m/s)(6.9s) + 0.5(-2m/s^2)(6.9m/s^2) = 184m$

$\Rightarrow$ Austin Powers is safe at a height above ground of 184m when the chopper with Dr. Evil crashes !



Oh, behave !...I am safe !

But I may show up in the quiz


Relative Velocity makes mid-air refueling possible !

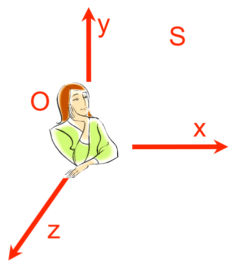
Relative Velocity

Blue Angel pilots must keep track of their velocity w.r.t air so as to maintain enough airflow over their wings to sustain the “lift” & not crash

They must also be aware of relative velocity of their aircraft w.r.t another !

Frames of Reference, Observers & Motion

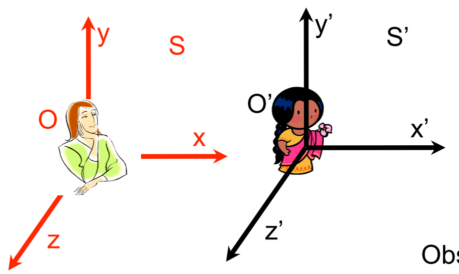
Event: Some thing happening, some where at some time



Frame of reference S = a coordinate system + clock

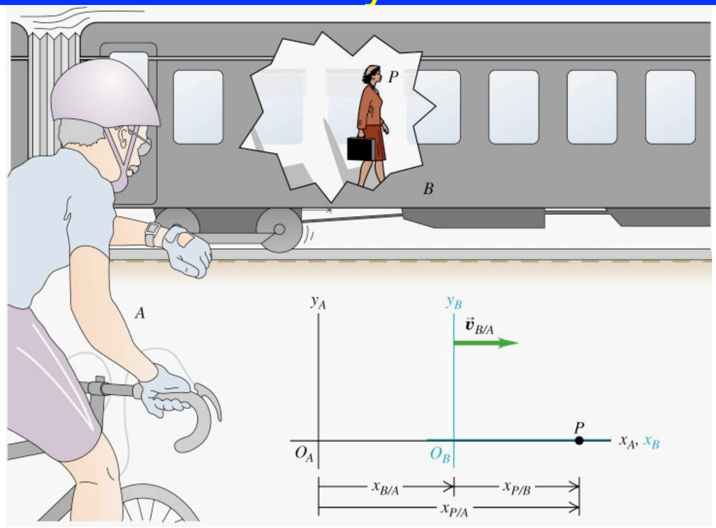
Observer O : sits in S, measures events with ruler, clock

Observers in diff frames of refs, depending on relative location may measure different positions for an event but measure same time. Their clocks are synchronized !



Observers can move w.r.t each other

Relative Velocity in 1 Dimension



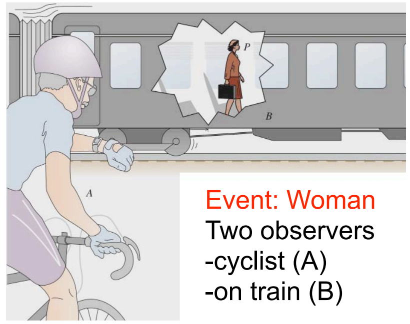
Woman walks with velocity of 1.0 m/s along train's aisle

Train is moving with velocity of 3.0 m/s to the right

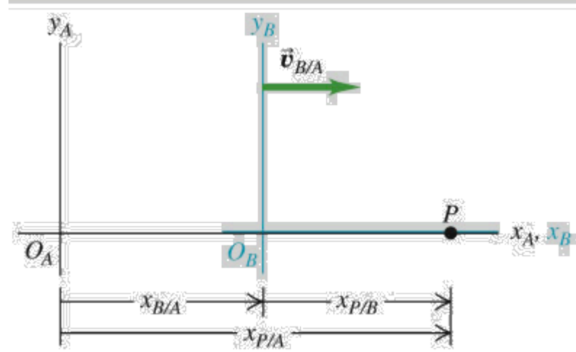
What is the woman's velocity ?

According to which Observer ???

Relative Velocity in 1 Dimension



Event: Woman
Two observers
-cyclist (A)
-on train (B)



At any instant (take a snapshot)

$x_{P/A}$ = pos. of P rel. to frame A ; $x_{P/B}$ = pos. of P rel. to frame B (train)

$x_{B/A}$ = distance from origin of A to origin of B

Clearly $\boxed{x_{P/A} = x_{P/B} + x_{B/A}}$ & $\frac{dx_{P/A}}{dt} = \frac{dx_{P/B}}{dt} + \frac{dx_{B/A}}{dt} \Rightarrow \boxed{v_{P/A} = v_{P/B} + v_{B/A}}$

So woman's velocity as seen by cyclist (A)

$v_{P/A} = 1.0\,m/s + 3.0\,m/s = 4.0\,m/s$ but If woman was walking in opp. dir. in train

$v_{P/A} = -1.0\,m/s + 3.0\,m/s = 2.0\,m/s$