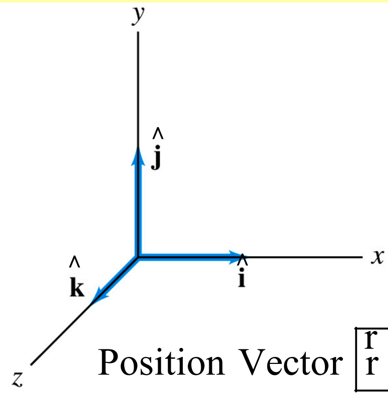
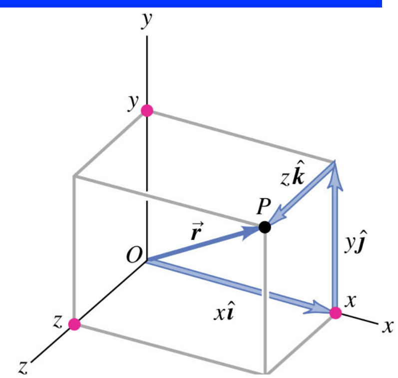
Generalizing Motion From 1D → 3D

Cartesian coordinate system in 3D





Position Vector $\vec{r} = x\,\hat{i} + y\,\hat{j} + z\,\hat{k}$

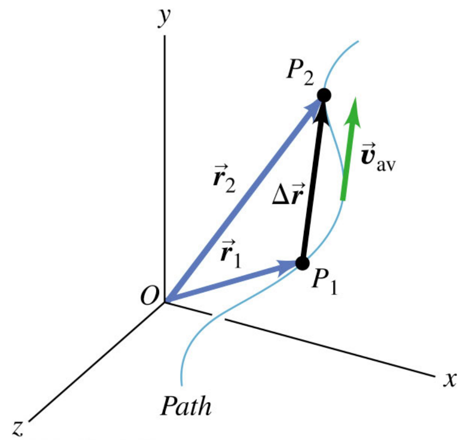
Path of particle moving in 3D space is a curve

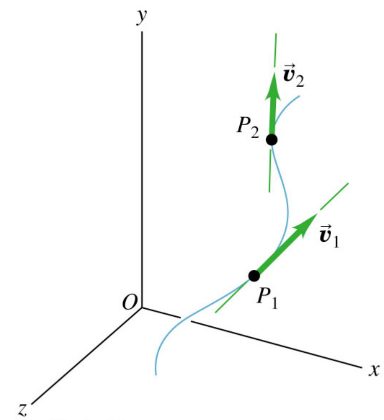
When particle moves from $P_1(x_1,y_1,z_1)$ ® $P_2(x_2,y_2,z_2)$

displacement $\Delta\vec{r} = (x_2-x_1)\,\hat{i} + (y_2-y_1)\,\hat{j} + (z_2-z_1)\,\hat{k}$

$= \Delta x\,\hat{i} + \Delta y\,\hat{j} + \Delta z\,\hat{k}$

Velocity in 3 Dimensions





Average Velocity Vector

$$\vec{V}_{av} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1} = \frac{\Delta\vec{r}}{\Delta t}$$

Instant. Velocity Vector

$$\vec{V} = \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

at every point along the path, vector $\vec{V}$ is tangent to the path at that point

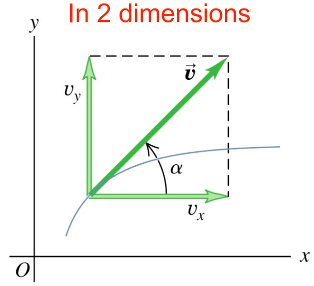
Components of Velocity Vector

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\,\hat{i} + \frac{dy}{dt}\,\hat{j} + \frac{dz}{dt}\,\hat{k}$$

$$= v_x\,\hat{i} + v_y\,\hat{j} + v_z\,\hat{k}$$

$$\text{Speed} = v = |\vec{v}| = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

In 2 dimensions



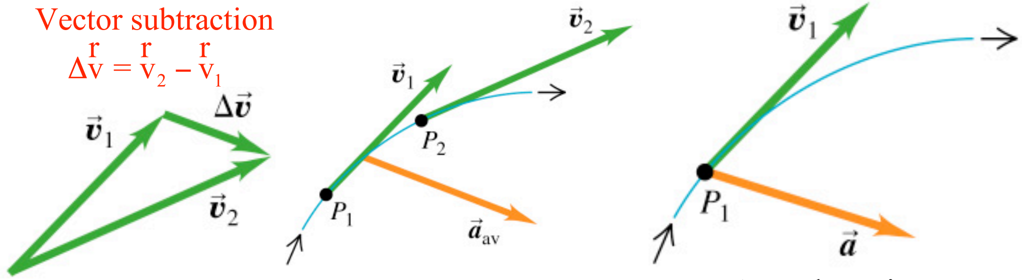
$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\,\hat{i} + \frac{dy}{dt}\,\hat{j} = v_x\,\hat{i} + v_y\,\hat{j}$$

$$\text{Speed} = v = |\vec{v}| = \sqrt{v_x^2 + v_y^2}$$

$$\tan\alpha = \frac{v_y}{v_x}$$

Acceleration In 3 Dimension

Vector subtraction
$\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$



Average Acceleration

$$\vec{a}_{av} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{\Delta\vec{v}}{\Delta t}$$

Instantaneous Acceleration

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$$

$\vec{a}$ always points towards the *concave* side of the curved path

When moving in a curved path

$\vec{a} \neq 0$ even if speed is constant

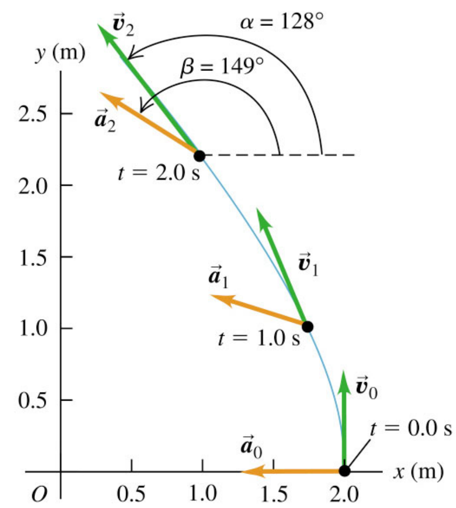
Components of Acceleration Vector

When computing components use the **correct** def. of angle

$$\vec{a} = \frac{dv_x}{dt}\,\hat{i} + \frac{dv_y}{dt}\,\hat{j} + \frac{dv_z}{dt}\,\hat{k}$$

$$= \frac{d^2x}{dt^2}\,\hat{i} + \frac{d^2y}{dt^2}\,\hat{j} + \frac{d^2z}{dt^2}\,\hat{k}$$

$$= a_x\,\hat{i} + a_y\,\hat{j} + a_z\,\hat{k}$$



3D Motion With Constant Acceleration

$$\vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$$

$$\vec{v} = \vec{v}_0 + \vec{a}t$$

$$v^2 = v_0^2 + 2a\,|\,\vec{r} - \vec{r}_0\,|$$

$$\vec{r} - \vec{r}_0 = \left(\frac{\vec{v}_0 + \vec{v}}{2}\right)t$$

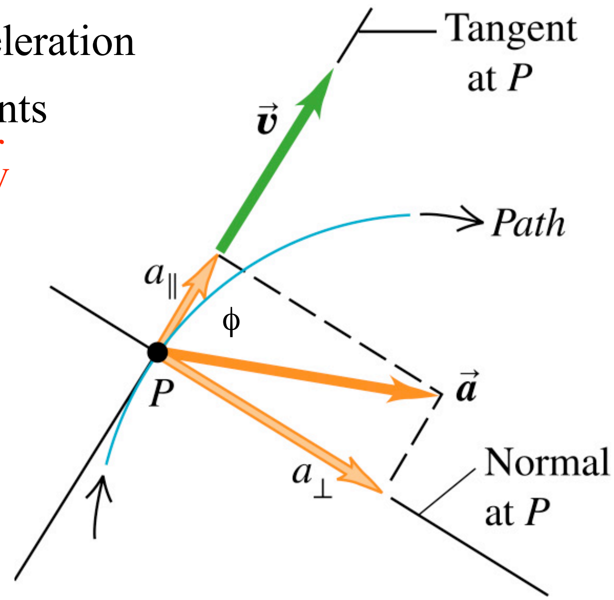
P and ⊥ components of Acceleration $\vec{a}$

When moving in curved path

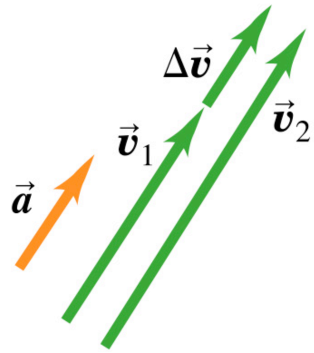
useful to describe acceleration

$\vec{a}$ in terms of components

which are P & ⊥ to $\vec{v}$



P and ⊥ components of Vector $\vec{a}$

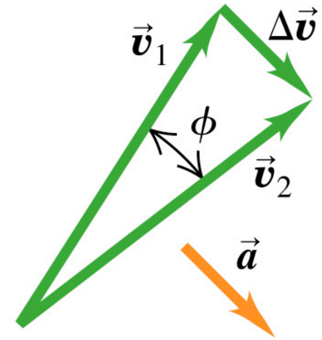


When $\vec{a}$ P $\vec{v}$ or anti-P

vector addition ⇒

change in magnitude of $\vec{v}$

but not its direction



When $\vec{a}$ ⊥ $\vec{v}$

vector addition ⇒

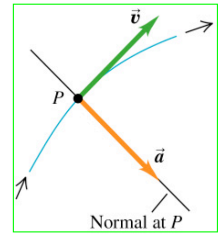
change the direction of $\vec{v}$

but not its magnitude

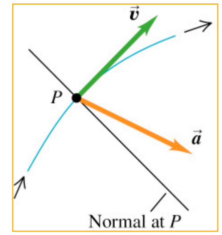
(speed remains unchanged!)

Some Scenarios

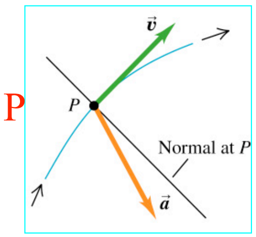
When particle travels along curved path with constant speed, $\vec{a}$ is $\perp$ to the path & $\perp$ to $\vec{v}$



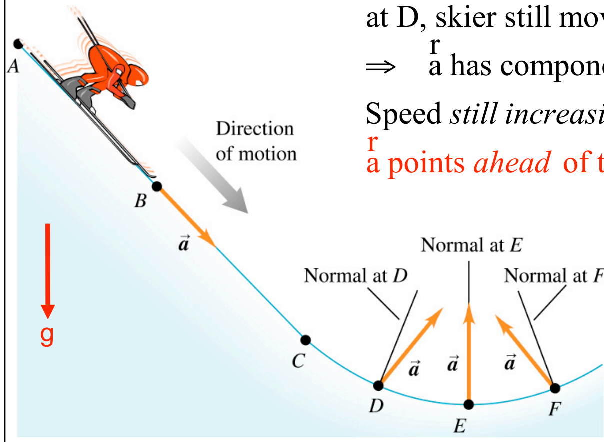
When particle travels along curved path with increasing speed, $\vec{a}$ has components $\perp$ & P to $\vec{v}$ & points ahead of the *normal* to the path



When particle travels along curved path with decreasing speed, $\vec{a}$ has components $\perp$ & anti−P to $\vec{v}$ & points behind the *normal* to the path



Skiing: Curved Path In Snow !



at D, skier still moving along curved path:

$\Rightarrow$ $\vec{a}$ has component $\perp$ to path

Speed *still increasing* $\Rightarrow$ $\vec{a}$ has component P to path

$\vec{a}$ points *ahead* of the normal to her path at point D

at point E, velocity is max.

$$\vec{a}_P = \frac{d\vec{v}_P}{dt} = 0$$

$\vec{a} = \vec{a}_\perp$ and points towards normal

at point F, skier moving along curved path:

$\Rightarrow$ $\vec{a}$ has component $\perp$ to path

Speed now *decreasing* $\Rightarrow$ $\vec{a}$ has component anti-P to path

$\vec{a}$ points *behind* the normal to her path at point F


Relative Velocity makes mid-air refueling possible !