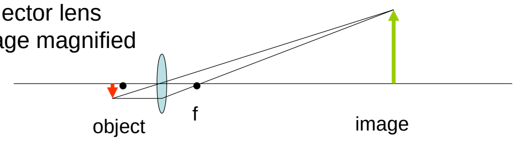
5.1 Simple Optical Systems

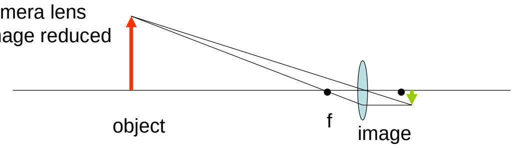
Projector
Camera
The Eye
Combinations of lenses
Correcting defects in vision

Properties of a converging lens

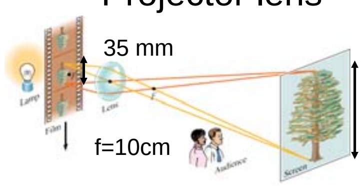
Projector lens
image magnified



Camera lens
Image reduced



Projector lens



Suppose you want to project the image of a transparency 35 mm high on to a screen that is 1.5 m high using a lens with a focal length of 10 cm. Where would you position the film? How far from the lens would you place the screen?

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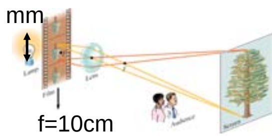
Projector lens



A real image is needed
Magnification is negative.

$$M = \frac{h'}{h} = -\frac{q}{p} \quad \Rightarrow \quad \frac{h'}{h} = -\frac{1.5}{35x10^{-3}} = -42.8 \quad = -\frac{q}{p} \qquad q = 42.8p$$

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} \quad \Rightarrow \quad \frac{1}{p} + \frac{1}{42.8p} = \frac{1}{f}$$

Solve for p

$$p = f\left(\frac{42.8+1}{42.8}\right) = 10\left(\frac{43.8}{42.8}\right) = 10.23\text{cm}$$

slightly larger than f

.

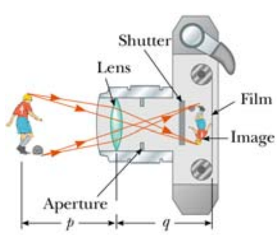
$$q = 42.8p = 42.8(10.23) = 438\text{cm}$$

Camera

Camera lens produces a reduced image near focal point of the lens

Focused by movement of the lens detector distance.

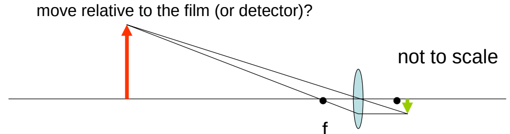
Aperture changes the light intensity level of the image.



Not to scale

Example

A camera lens with a focal length of 50 mm is focused on the mountains very far away (at infinity). Now the photographer refocuses the camera on a person 2.0 m away. How far does the lens have to move relative to the film (or detector)?



Focused at infinity – image is formed when q = f=0.05 m

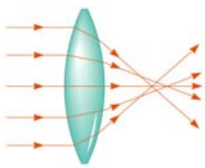
Focused at 2 m – image is formed when q= 0.0513 m

$$q = \frac{fp}{p-f} = \frac{(0.050)(2.0)}{(2.0-0.050)} = 0.0513$$

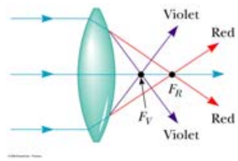
lens moves 1.3 mm

Lens aberrations

- Aberrations prevent the formation of a perfect image and limit the magnification of a lens or mirror.
  - Spherical Aberration- due to deviation of spherical surface from the ideal parabolic shape.
  - Chromatic aberration – due to the difference in refractive index and thus the focal length for different wavelengths of light.



Spherical aberration



Chromatic aberration

Camera lens

Good camera lenses are a combination of several lenses to correct for spherical and chromatic aberration.

May have either a fixed focal length or variable focal length (zoom lens).

The “speed” of the lens is determined by the f-number = focal length/ diameter

$$\text{f-number} = \frac{f}{D}$$

The lower the f-number the higher the intensity of light at the film (detector)

Zoom Lens
Variable focal length

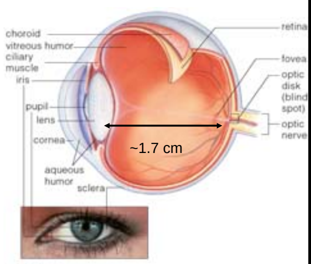
Eye

Cornea, aqueous humor and lens refract light

Detection at the retina. Principally at the fovea.

The amount of light entering the eye is regulated by the size of the pupil which is the aperture in the iris.

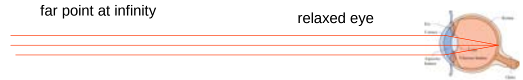
The eye is focused by changing the flatness of the lens, in a process called accommodation.



Range of Normal Vision

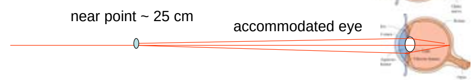
far point at infinity

relaxed eye



near point ~ 25 cm

accommodated eye



The relaxed eye can focus on an object at infinity (far point)

The accommodated eye can focus on an object at 25 cm (near point)

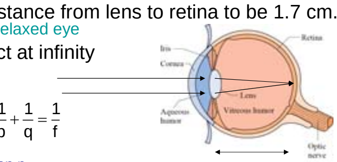
The near point and the far point can vary in different individuals and can change with age.

Question

For the normal eye what is the focal length of the relaxed eye. What is the focal length of the accommodated eye focusing on an object at the near point? Take the distance from lens to retina to be 1.7 cm.

Relaxed eye

object at infinity



$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$

1.7 cm

When $p = \infty$

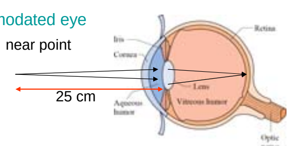
$q = f = 1.70$ cm

Question

For the normal eye what is the focal length of the relaxed eye. What is the focal length of the accommodated eye focusing on an object at the near point (25 cm) ? Take the distance from lens to retina to be q=1.7 cm.

Accommodated eye

near point



25 cm

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$

When p =25 cm

$$f = \frac{pq}{p+q} = \frac{(25)(1.7)}{25+1.7} = 1.6\text{cm}$$

f decreases by about a millimeter

Combinations of lenses

- When two lenses are used in combination, the image of the first lens is the object for the second lens.
- The total magnification is the product of the magnifications of the first and second lens.

Find the image formed by two lenses in combination

Image formed by lens 1

$$q_1 = \frac{p_1 f_1}{p_1 - f_1} = \frac{(30)(10)}{30-10} = 15\text{cm}$$

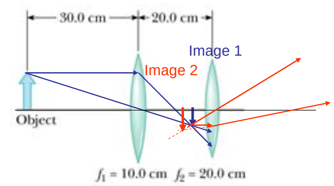
from lens 1

Image formed by lens 2

$p_2 = 20 - q_1 = 20 - 15 = 5$ cm

$$q_2 = \frac{p_2 f_2}{p_2 - f_2} = \frac{(5)(20)}{5-20} = -6.7\text{cm}$$

from lens 2



Magnification

$$M = M_1 M_2 = \left(-\frac{q_1}{p_1}\right)\left(-\frac{q_2}{p_2}\right) = \left(-\frac{15}{30}\right)\left(-\frac{-6.7}{5}\right) = -0.67$$

Inverted
Reduced

Lens Power

The power of a lens is defined as

$$P = \frac{1}{f}$$

Units of diopter (m^{-1})

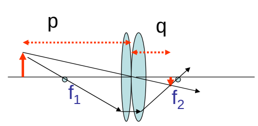
where f is the focal length of the lens in meters.

A lens with a focal length of -20 cm has a power of

$$P = \frac{1}{-0.2\text{m}} = -5.0 \text{ diopters.}$$

Powers are often used by optometrists to describe eye glass lenses

For two lenses in contact the total power is the sum of powers of the individual lenses



$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$P = P_1 + P_2$$

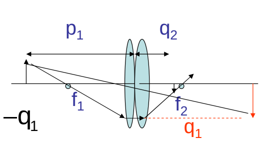
Proof: Two lenses in contact

$$\frac{1}{p_1} + \frac{1}{q_1} = \frac{1}{f_1} \qquad \frac{1}{p_2} + \frac{1}{q_2} = \frac{1}{f_2}$$

$$-\frac{1}{q_1} + \frac{1}{q_2} = \frac{1}{f_2}$$

substitute

$$p_2 = -q_1$$



Eliminate q_1

$$\frac{1}{p_1} + \frac{1}{q_2} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{f}$$

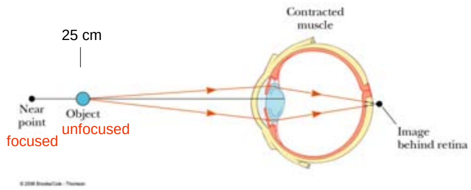
The image of the first lens is the object for the second lens. Virtual object has a negative sign.

Defects in vision

Nearsightedness and farsightedness – due to the mismatch between the focal length of the eye and the distance between lens and retina.

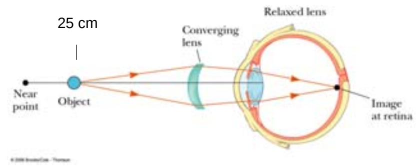
Corrected by using a second lens to either increase or decrease the focal length of the eye.

Farsightedness
Lack of near vision



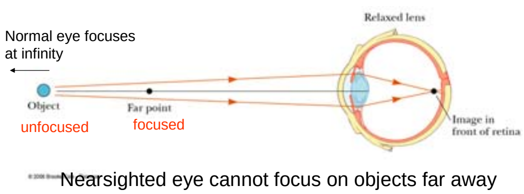
Farsighted eye cannot focus on an object at the near point of a normal eye.
The lens-retina distance is too short and/ or the lens is not convergent enough.

Correcting farsightedness



The light is made to converge more by using a converging lens.

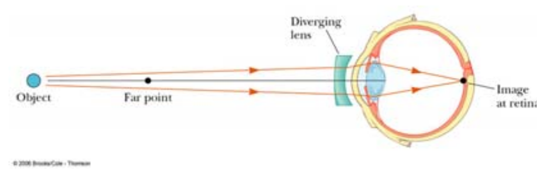
Nearsightedness
lack of far vision



Nearsighted eye cannot focus on objects far away (further than the far point < infinity)

The lens retina distance is too long and/or the lens is too converging.

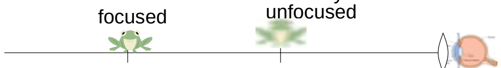
Correcting nearsightedness



Nearsighted vision can be corrected by using a diverging lens (making the light less convergent).

Example

A farsighted person has a near point of 50 cm. What power lens will correct this to normal vision. (ignore the distance between the lens and the eye.



Use a lens that can take an object at 25 cm and form a virtual image at a distance of 50 cm.

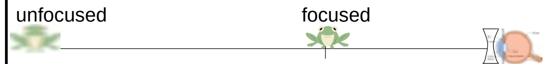
$$f=\frac{pq}{p+q}=\frac{(25)(-50)}{25-50}=+50cm=0.50m$$

P= 1/0.5m =2.0 diopters

A converging lens

Example

A nearsighted person has a far point of 25 cm. What power lens will correct this to normal vision. (ignore the distance between the lens and the eye.



Use a lens that can take an object at infinity and form a virtual image at 25 cm.

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f} \qquad \frac{1}{\infty}+\frac{1}{-25}=\frac{1}{f}$$

A diverging lens

f=-25 cm=-0.25m

P= $\frac{1}{-0.25}=-4.0$diopters