

6.1 Diffraction

Diffraction grating
Single slit diffraction
Circular diffraction

Diffraction and Interference

- Diffraction and interference are similar phenomena.
- Interference is the effect of superposition of 2 coherent waves.
- Diffraction is the superposition of many coherent waves.

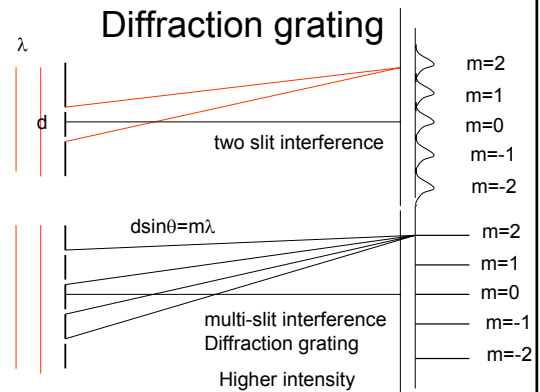
Diffraction grating

- Consists of a flat barrier which contains many parallel slits separated by a short distance d .
- A parallel monochromatic light beam passing through the grating is diffracted by an angle θ

$$d \sin \theta = m \lambda$$

similar to two slit interference.

However, the intensity of the diffracted light is higher and the peaks are much narrower.



Question

A grating in a spectrometer has a length of 2 cm and has contains 10^4 lines. Find the first order diffraction angle for light with a wavelength of 500 nm.

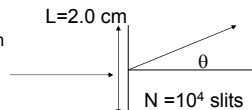
first order $m=1$

$$d = \frac{L}{N} = \frac{2 \times 10^{-2} \text{ m}}{10^4} = 2 \times 10^{-6} \text{ m}$$

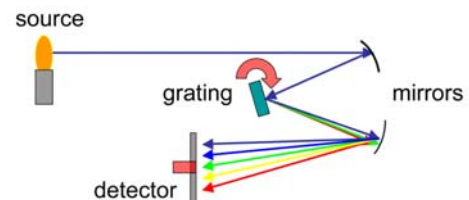
$$d \sin \theta = m \lambda$$

$$\sin \theta = \frac{m \lambda}{d} = \frac{500 \times 10^{-9}}{2 \times 10^{-6}} = 0.25$$

$$\theta = 14.5^\circ$$

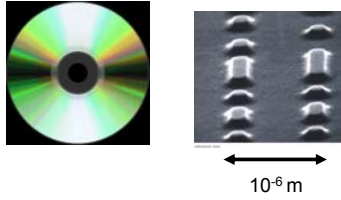


Use of a diffraction grating in a spectrometer



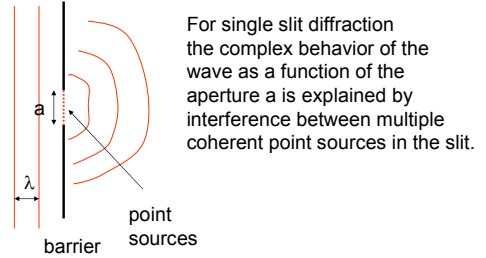
Dispersion of light of different wavelengths

Optical compact disc



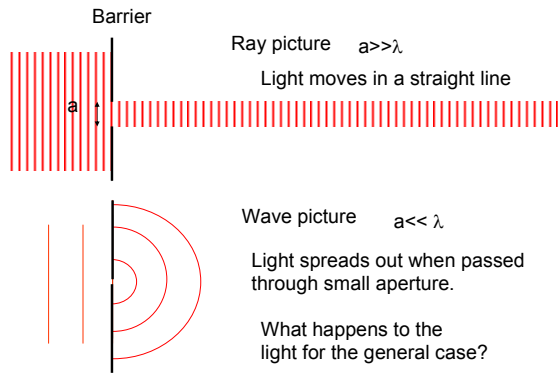
The closely spaced dots act like a diffraction grating.

Single slit diffraction



For single slit diffraction the complex behavior of the wave as a function of the aperture a is explained by interference between multiple coherent point sources in the slit.

Two Limiting Cases

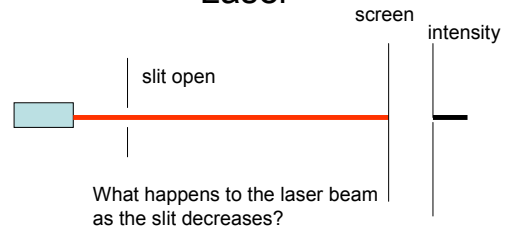


Ray picture $a \gg \lambda$
Light moves in a straight line

Wave picture $a \ll \lambda$
Light spreads out when passed through small aperture.

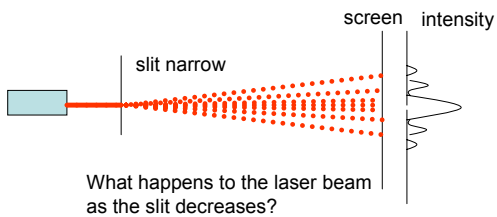
What happens to the light for the general case?

Laser



What happens to the laser beam as the slit decreases?

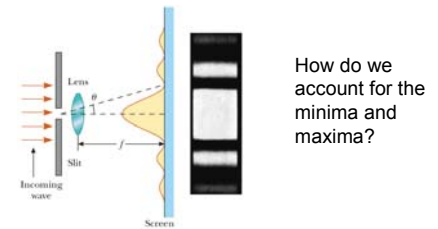
The pattern spreads out as the slit becomes narrower



What happens to the laser beam as the slit decreases?

Single slit diffraction pattern

Single slit diffraction pattern



How do we account for the minima and maxima?

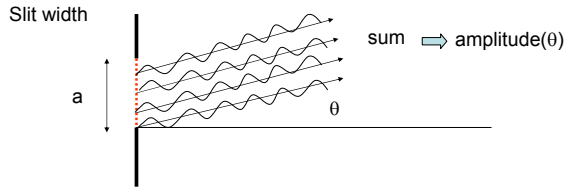
Analyze assuming Fraunhofer diffraction conditions.

- Screen far from the slit or
- Converging lens with screen at the focal length

Single slit diffraction

Huygens principle – Each point in the wave in the slit acts as a source of spherical waves.

sum the waves with different phases



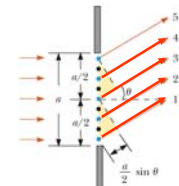
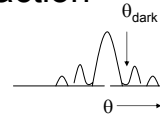
Single slit diffraction

Find the angle at the first minimum
amplitude = 0

Divide the slit into 2 halves

Light from every point in the top half interferes destructively with a light from a point in the bottom half when

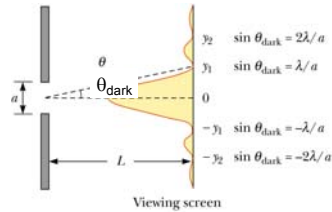
$$\frac{a}{2} \sin \theta_{\text{dark}} = \frac{\lambda}{2} \Rightarrow a \sin \theta_{\text{dark}} = \lambda$$



Condition for a minimum

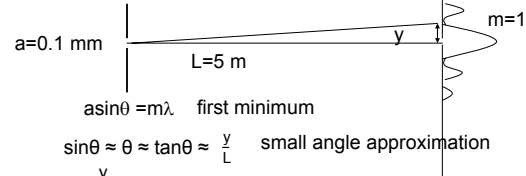
$$a \sin \theta_{\text{dark}} = m\lambda$$

$$m = \pm 1, \pm 2, \dots$$



Example

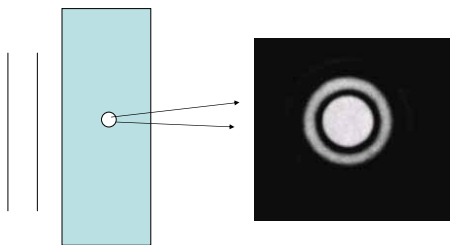
A laser beam with a wavelength of 500 nm is passed through a single slit with a width of 0.1 mm. Find the distance between the central maximum and the first minimum of the diffraction pattern on a screen 5 m away.



$$a \frac{y}{L} = \lambda$$

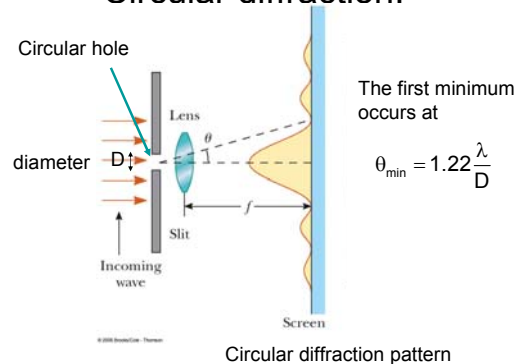
$$y = \frac{\lambda}{a} L = \left[\frac{500 \times 10^{-9} \text{ m}}{0.1 \times 10^{-3} \text{ m}} \right] (5 \text{ m}) = 2.5 \times 10^{-2} \text{ m} = 2.5 \text{ cm}$$

Circular diffraction



Waves passing through a circular hole forms a circular diffraction pattern.

Circular diffraction.



Circular diffraction limits the minimum size do the spot that can be formed by a lens.

According to ray optics.

parallel rays from a point at infinity

Can focus on a point at the focal point of the lens

Circular diffraction limits the minimum size do the spot that can be formed by a lens.

According to wave optics.

diameter D

parallel rays from a point at infinity

has a diffraction pattern with a width of θ_{min}

Example

A camera lens with an f- number (f/D) equal to 1.4 is used to focus light from a distant source. What is the diffraction limited radius of the spot that can be formed for 500 nm light?

$$\theta_{min} = 1.22 \frac{\lambda}{D} = \frac{r}{f}$$

$$r = (1.22 \frac{\lambda f}{D}) = (1.22)(500 \times 10^{-9})(1.4) = 8.5 \times 10^{-7} \text{ m}$$

850 nm
Close to the wavelength of light

Optical CD

focused laser beam

10^{-6} m

The amount of information that can be encoded is limited by the diameter of the diffraction-limited spot.

Blu-ray Disc

Higher density of spot can be obtained using a blue laser with shorter wavelength because of the smaller size of the diffraction limited spot.

Diffraction limits the resolution of light microscopes (200 nm)

Resolution of two images by a lens

The resolution of images is limited by the diffraction pattern.

Rayleigh criterion

For resolution of two object by a circular lens of diameter D the diffraction limit of resolution occurs when the image of the second object is at position of the first minimum of the diffraction pattern of the first object.

$$\theta_{\min} = 1.22 \frac{\lambda}{D}$$

GeoEye 1 Imaging Satellite

1.1 m diameter mirror
Resolution 0.41 m
Altitude of 684 km

Diffraction limit ($\lambda=550\text{nm}$)

$$\theta = \frac{y}{L} = 1.22 \frac{\lambda}{D}$$

$$y = \frac{1.22\lambda L}{D} = \frac{1.22(550 \times 10^{-9} \text{m})(684 \times 10^3 \text{m})}{1.1 \text{m}} = 0.41 \text{m}$$

Satellite picture of Sea World

Resolution limit of the eye

The eye differs from a simple lens in air.

1. The light from the lens travels through a refractive media.
2. The light passing through the middle of the lens changes direction due to refraction.
3. These two effects cancel so the Rayleigh criterion is the same as for a lens in air.

$$\theta_{\min} = \frac{1.22\lambda}{D}$$

Resolution limit of the eye

Two light sources ($\lambda=500\text{nm}$) are separated vertically by 2.0 mm. How far away can these objects be resolved by the eye
Assume a diameter of the pupil of 2.0 mm. $n_{\text{eye}}=1.33$

$$\theta_2 = 1.22 \frac{\lambda_{\text{water}}}{D} = 1.22 \frac{\lambda}{D n_{\text{water}}}$$

$$\theta_{\min} = n_{\text{eye}} \theta_2 = n_{\text{eye}} \left(1.22 \frac{\lambda}{D n_{\text{water}}} \right) = 1.22 \frac{\lambda}{D} = \frac{y}{L}$$

$$L = \frac{yD}{1.22\lambda} = \frac{(2 \times 10^{-3})(2 \times 10^{-3})}{1.22(500 \times 10^{-9})} = 6.6 \text{m}$$

Diffraction limit

- Diffraction limits the resolution of objects viewed through an optical system
- Resolution depends on the size of the aperture and the wavelength of light.
- Consequences
 - atoms cannot be seen with a light microscope (shorter wavelengths are required)
 - Satellite cameras have a limited resolution.
- To attain higher resolution.
 - Larger diameter lenses
 - Shorter wavelengths.