4-6. (a) $f = \pi b^2 nt$ (Equation 4-5)

For Au, $n = 5.90 \times 10^{28}\, atoms / m^3$ (see Example 4-2) and for this foil $t = 2.0\mu m = 2.0 \times 10^{-6} m$.

$$b = \frac{kq_\alpha Q}{m_\alpha v^2}\cot\frac{\theta}{2} = \frac{(2)(79)ke^2}{2K_\alpha}\cot\frac{90}{2} = \frac{(2)(79)(1.44 eV\cdot nm)}{2(7.0\times 10^6 eV)}$$

$$= 1.63\times 10^{-5} nm = 1.63\times 10^{-14} m$$

$$f = \pi\left(1.63\times 10^{-14} m\right)^2\left(5.90\times 10^{28} / m^3\right)\left(2.0\times 10^{-6} m\right) = 9.8\times 10^{-5}$$

(b) For $\theta = 45°$, $b(45°) = b(90°)(\cot 45°/2)/(\cot 90°/2)$

$$= b(90°)(\tan 90°/2)/(\tan 45°/2)$$

$$= 3.92\times 10^{-5} nm = 3.92\times 10^{-14} m$$

and $f(45°) = 5.7\times 10^{-4}$

For $\theta = 75°$, $b(75°) = b(90°)(\tan 90°/2)/(\tan 75°/2)$

$$= 2.12\times 10^{-5} nm = 2.12\times 10^{-14} m$$

and $f(75°) = 1.66\times 10^{-4}$

Therefore, $\Delta f(45° - 75°) = 5.7\times 10^{-4} - 1.66\times 10^{-4} = 4.05\times 10^{-4}$

(Problem 4-6 continued)

(c) Assuming the Au atom to be a sphere of radius *r*,

$$\frac{4}{3}\pi r^3 = \frac{M}{N_A \rho} = \frac{197 g / mole}{\left(6.02\times 10^{23} atoms / mole\right)\left(19.3 g / cm^3\right)}$$

$$r = \left[\frac{3}{4\pi}\frac{197 g / mole}{\left(6.02\times 10^{23} atoms / mole\right)\left(19.3 g / cm^3\right)}\right]^{1/3}$$

$$r = 1.62\times 10^{-3} cm = 1.62\times 10^{-10} m = 16.2 nm$$

4-7. $\Delta N \propto \frac{1}{\sin^4(\theta/2)} = \frac{A}{\sin^4(\theta/2)}$ (From Equation 4-6), where *A* is the product of the two quantities in parentheses in Equation 4-6.

(a) $$\frac{\Delta N(10°)}{\Delta N(1°)} = \frac{A/\sin^4(10°/2)}{A/\sin^4(1°/2)} = \frac{\sin^4(0.5°)}{\sin^4(5°)} = 1.01\times10^{-4}$$

(b) $$\frac{\Delta N(30°)}{\Delta N(1°)} = \frac{\sin^4(0.5°)}{\sin^4(15°)} = 1.29\times10^{-6}$$

4-9. $$r_d = \frac{kq_\alpha Q}{(1/2)m_\alpha v^2} = \frac{ke^2\cdot2\cdot79}{E_{k\alpha}} \qquad \text{(Equation 4-11)}$$

For $E_{k\alpha} = 5.0MeV$: $$r_d = \frac{(1.44MeV\cdot fm)(2)(79)}{5.0MeV} = 45.5fm$$

For $E_{k\alpha} = 7.7MeV$: $r_d = 29.5fm$

For $E_{k\alpha} = 12MeV$: $r_d = 19.0fm$

4-11. $x_{rms} = \sqrt{N}(\delta) \qquad 10° = \sqrt{N}(0.01°) \rightarrow N = (10°/0.01°)^2 = 10^6$ collisions

$$n = \frac{t}{\Delta t} = \frac{10^{-6}m}{10^{-10}m} = 10^4 \text{ layers}$$

10^4 atomic layers is not enough to produce a deflection of $10°$, assuming 1 collision/layer.

4-13. (a) $$r_n = \frac{n^2 a_0}{Z} \qquad \text{(Equation 4-18)}$$

$$r_6 = \frac{6^2(0.053nm)}{1} = 1.91nm$$

(b) $$r_6(He^+) = \frac{6^2(0.053nm)}{2} = 0.95nm$$

4-15. $$\frac{1}{\lambda} = Z^2 R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) \qquad \text{(Equation 4-22)}$$

(Problem 4-15 continued)

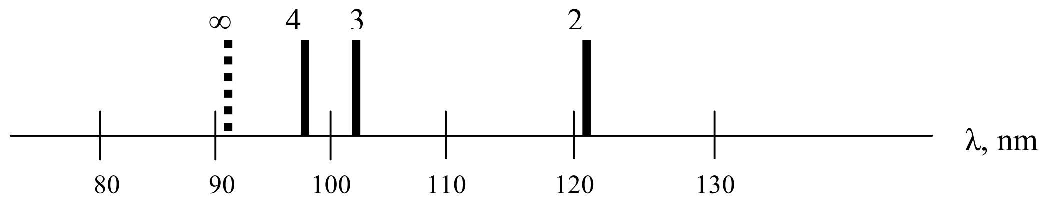
$$\frac{1}{\lambda_{ni}} = R\left(\frac{1}{1^2} - \frac{1}{n_i^2}\right) = R\left(\frac{n_i^2 - 1}{n_i^2}\right)$$

$$\lambda_{ni} = \frac{n_i^2}{R\left(n_i^2 - 1\right)} = \frac{n_i^2}{\left(1.0968\times10^7 m\right)\left(n_i^2 - 1\right)} = \left(91.17nm\right)\left(\frac{n_i^2}{n_i^2 - 1}\right)$$

$$\lambda_2 = \frac{4}{3}\left(91.17nm\right) = 121.57nm \qquad \lambda_3 = \frac{9}{8}\left(91.17nm\right) = 102.57nm$$

$$\lambda_4 = \frac{16}{15}\left(91.17nm\right) = 97.25nm \qquad \lambda_\infty = 91.17nm$$

None of these are in the visible; all are in the ultraviolet.



4-19. (a)

$$a_u = \frac{\hbar^2}{\mu_\mu k e^2} = \frac{\mu_e}{\mu_\mu} \cdot \frac{\hbar^2}{\mu_e k e^2} = \frac{\mu_e}{\mu_\mu} a_0 = \frac{9.11\times10^{-31}kg}{1.69\times10^{-28}kg}\left(0.0529nm\right) = 2.56\times10^{-4}nm$$

(b) $$E_\mu = \frac{\mu_\mu k^2 e^4}{2\hbar^2} = \frac{\mu_\mu}{\mu_e} \cdot \frac{\mu_e k^2 e^4}{2\hbar^2} = \frac{\mu_\mu}{\mu_e} \cdot E_0 = \frac{1.69\times10^{-28}kg}{9.11\times10^{-31}kg}\left(13.6eV\right) = 2520eV$$

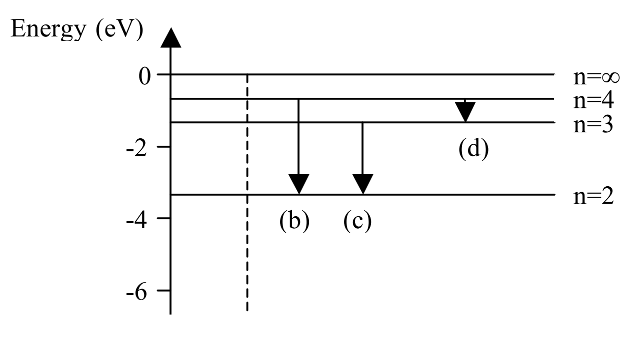
(Problem 4-19 continued)

(c) The shortest wavelength in the Lyman series is the series limit ($n_i = \infty,\ n_f = 1$). The photon energy is equal in magnitude to the ground state energy $-E_\mu$.

$$\lambda_\infty = \frac{hc}{E_\mu} = \frac{1240eV \cdot nm}{2520eV} = 0.492nm$$

(The reduced masses have been used in this solution.)

4-21.



(a) Lyman limit, (b) H_β line, (c) H_α line, (d) longest wavelength line of Paschen series

4-24. (a) The reduced mass correction to the Rydberg constant is important in this case.

$$R = R_\infty\left(\frac{1}{1+m/M}\right) = R_\infty\left(\frac{1}{2}\right) = 5.4869\times10^{6}m^{-1} \quad \text{(from Equation 4-26)}$$

$$E_n = -hcR/n^2 \quad \text{(from Equations 4-23 and 4-24)}$$

$$E_1 = -\left(1240eV\cdot nm\right)\left(5.4869\times10^{6}m^{-1}\right)\left(10^{-9}m/nm\right)/\left(1\right)^2 = -6.804eV$$

Similarly, $E_2 = -1.701eV$ and $E_3 = -0.756eV$

(b) Lyman α is the $n = 2 \rightarrow n = 1$ transition.

$$\frac{hc}{\lambda} = E_2 - E_1 \quad \rightarrow \quad \lambda_\alpha = \frac{hc}{E_2 - E_1} = \frac{1240eV\cdot nm}{-1.701eV - \left(-6.804eV\right)} = 243nm$$

Lyman β is the $n = 3 \rightarrow n = 1$ transition.

$$\lambda_\beta = \frac{hc}{E_3 - E_1} = \frac{1240eV\cdot nm}{-0.756eV - \left(-6.804eV\right)} = 205nm$$

4-26. (a) $\dfrac{1}{\lambda} = R\left(Z-1\right)^2\left(\dfrac{1}{n_1^2} - \dfrac{1}{n_2^2}\right)$

$$\lambda_3 = \left[\left(1.097\times10^{7}m^{-1}\right)\left(42-1\right)^2\left(\frac{1}{1^2} - \frac{1}{3^2}\right)\right]^{-1} = 6.10\times10^{-11}m = 0.0610nm$$

$$\lambda_4 = \left[\left(1.097\times10^7 m^{-1}\right)\left(42-1\right)^2\left(\frac{1}{1^2}-\frac{1}{4^2}\right)\right]^{-1} = 5.78\times10^{-11}m = 0.0578nm$$

(b) $$\lambda_{limit} = \left[\left(1.097\times10^7 m^{-1}\right)\left(42-1\right)^2\left(\frac{1}{1^2}-0\right)\right]^{-1} = 5.42\times10^{-11}m = 0.0542nm$$

4-27. $\frac{1}{\lambda} = R\left(Z-1\right)^2\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right) = R\left(Z-1\right)^2\left(\frac{1}{1^2}-\frac{1}{2^2}\right)$ for K_α

$$Z-1 = \left[\frac{1}{\lambda R\left(1-\frac{1}{4}\right)}\right]^{1/2} = \left[\frac{1}{\left(0.0794nm\right)\left(1.097\times10^{-2}/nm\right)\left(3/4\right)}\right]^{1/2}$$

$Z = 1+39.1 \approx 40$ Zirconium

4-29. $r_n = \frac{n^2 a_0}{Z}$ (Equation 4-18)

The n =1 electrons "see" a nuclear charge of approximately $Z-1$, or 78 for Au.

$r_1 = 0.0529nm/78 = 6.8\times10^{-4}nm\left(10^{-9}m/nm\right)\left(10^{15}fm/m\right) = 680fm$, or about 100

times

the radius of the Au nucleus.

4-36. $\Delta E = \frac{hc}{\lambda} = \frac{1240eV\cdot nm}{790nm} = 1.610eV$. The first decrease in current will occur when

the

voltage reaches $1.61V$.

4-40. Those scattered at $\theta = 180°$ obeyed the Rutherford formula. This is a head-on collision where the α comes instantaneously to rest before reversing direction. At that point its kinetic energy has been converted entirely to electrostatic potential energy, so

$\frac{1}{2}m_\alpha v^2 = 7.7MeV = \frac{k\left(2e\right)\left(79e\right)}{r}$ where r = upper limit of the nuclear radius.

$$r = \frac{k\left(2\right)\left(79\right)e^2}{7.7MeV} = \frac{2\left(79\right)\left(1.440MeV\cdot fm\right)}{7.7MeV} = 29.5fm$$

4-43. $\lambda = \left[R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right) \right]^{-1}$ $\Delta\lambda = \frac{d\lambda}{d\mu}\Delta\mu = \left(-R^{-2}\right)\left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)^{-1} \frac{dR}{d\mu}\Delta\mu$

Because $R \propto \mu$, $dR/d\mu = R/\mu$. $\Delta\lambda \approx \left(-R^{-2}\right)\left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)^{-1} \left(R/\mu\right)\Delta\mu = -\lambda\left(\Delta\mu/\mu\right)$

$$\mu_H = \frac{m_e m_p}{m_e + m_p} \qquad \mu_D = \frac{m_e m_d}{m_e + m_d}$$

$$\frac{\Delta\mu}{\mu} = \frac{\mu_D - \mu_H}{\mu_H} = \frac{\mu_D}{\mu_H} - 1 = \frac{m_e m_d / \left(m_e + m_d\right)}{m_e m_p / \left(m_e + m_p\right)} - 1 = \frac{m_d / \left(m_e + m_d\right)}{m_p / \left(m_e + m_p\right)} - 1 = \frac{m_e\left(m_d - m_p\right)}{m_p\left(m_e + m_d\right)}$$

If we approximate $m_d = 2m_p$ and $m_e \ll m_d$, then $\frac{\Delta\mu}{\mu} \approx \frac{m_e}{2m_p}$ and

$$\Delta\lambda = -\lambda\left(\Delta\mu/\mu\right) = -\left(656.3nm\right)\frac{0.511MeV}{2\left(938.28MeV\right)} = -0.179nm$$

4-45. (a) $E_n = -E_0 Z^2 / n^2$ (Equation 4-20)

For Li^{++}, $Z = 3$ and $E_n = -13.6eV\left(9\right)/n^2 = -122.4/n^2 eV$

The first three Li^{++} levels that have the same (nearly) energy as H are:

(Problem 4-45 continued)

$n = 3,\ E_3 = -13.6eV \qquad n = 6,\ E_6 = -3.4eV \qquad n = 9,\ E_9 = -1.51eV$

Lyman α corresponds to the $n = 6 \rightarrow n = 3$ Li^{++} transitions. Lyman β corresponds

to the $n = 9 \rightarrow n = 3$ Li^{++} transition.

(b) $R\left(H\right) = R_\infty\left(1/\left(1 + 0.511MeV/938.8MeV\right)\right) = 1.096776 \times 10^7 m^{-1}$

$R\left(Li\right) = R_\infty\left(1/\left(1 + 0.511MeV/6535MeV\right)\right) = 1.097287 \times 10^7 m^{-1}$

For Lyman α:

$$\frac{1}{\lambda} = R\left(H\right)\left(1 - \frac{1}{2^2}\right) = 1.096776 \times 10^7 m^{-1}\left(10^{-9} m/nm\right)\left(3/4\right) \rightarrow 121.568nm$$

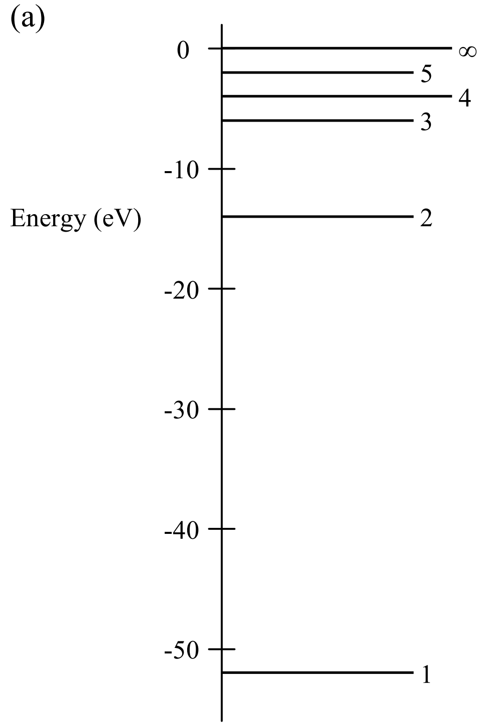
For Li^{++} equivalent:

$$\frac{1}{\lambda} = R(Li)\left(\frac{1}{3^2} - \frac{1}{6^2}\right)Z^2 = 1.097287\times10^7 m^{-1}\left(10^{-9} m/nm\right)\left(\frac{1}{9} - \frac{1}{36}\right)(3)^2$$

$\lambda = 121.512nm \qquad \Delta\lambda = 0.056nm$

4-50. For He: $E_n = -13.6eV\,Z^2/n^2 = -54.4eV/n^2$ (Equation 4-20)

(a)



E_1 = -54.4 eV
E_2 = -13.6 eV
E_3 = -6.04 eV
E_4 = -3.04 eV
E_5 = -2.18 eV
E_∞ = 0 eV

(b) Ionization energy is 54.5eV.

(c) H Lyman α: $\lambda = hc/\Delta E = 1240eV\cdot nm/(13.6eV - 3.4eV) = 121.6nm$

H Lyman β: $\lambda = hc/\Delta E = 1240eV\cdot nm/(13.6eV - 1.41eV) = 102.6nm$

He^+ Balmer α: $\lambda = hc/\Delta E = 1240eV\cdot nm/(13.6eV - 6.04eV) = 164.0nm$

He^+ Balmer β: $\lambda = hc/\Delta E = 1240eV\cdot nm/(13.6eV - 3.40eV) = 121.6nm$

$\Delta\alpha = 42.4nm \qquad \Delta\beta = 19.0nm$

(The reduced mass correction factor does not change the energies calculated above
to three significant figures.)

(d) $E_n = -13.6eV\,Z^2/n^2$ because for He^+, $Z = 2$, then $Z^2 = 2^2$. Every time n is an even number a 2^2 can be factored out of n^2 and cancelled with the $Z^2 = 2^2$ in the numerator; e.g., for He^+,

(Problem 4-50 continued)

$$E_2 = -13.6eV \cdot 2^2/2^2 = -13.6eV \quad \text{(H ground state)}$$
$$E_4 = -13.6eV \cdot 2^2/4^2 = -13.6eV/2^2 \quad \text{(H } - 1^{st} \text{ excited state)}$$
$$E_6 = -13.6eV \cdot 2^2/6^2 = -13.6eV/3^2 \quad \text{(H } - 2^{nd} \text{ excited state)}$$
$$\vdots$$
etc.

Thus, all of the H energy level values are to be found within the He^+ energy levels, so

He^+ will have within its spectrum lines that match (nearly) a line in the H spectrum.

4-53. $\dfrac{kZe^2}{r} = \dfrac{mv^2}{r} \quad \rightarrow \quad \dfrac{kZe^2}{r^2} = \dfrac{(\gamma mv)^2}{mr}$ (from Equation 4-12)

$$\gamma v = \left(\frac{kZe^2}{mr}\right)^{1/2} = \frac{v}{\sqrt{1-\beta^2}}$$

$$\frac{c^2\beta^2}{1-\beta^2} = \left(\frac{kZe^2}{mr}\right) \quad \text{Therefore, } \beta^2\left[c^2 + \left(\frac{kZe^2}{mr}\right)\right] = \left(\frac{kZe^2}{mr}\right)$$

(Problem 4-53 continued)

$$\beta^2 \approx \frac{1}{c^2}\left(\frac{kZe^2}{ma_o}\right) \quad \rightarrow \quad \beta = 0.0075Z^{1/2} \quad \rightarrow \quad v = 0.0075cZ^{1/2} = 2.25\times10^6 m/s \times Z^{1/2}$$

$$E = KE - kZe^2/r = mc^2(\gamma - 1) - \frac{kZe^2}{r} = mc^2\left[\frac{1}{\sqrt{1-\beta^2}} - 1\right] - \frac{kZe^2}{r}$$

And substituting $\beta = 0.0075$ and $r = a_o$

$$E = 511\times10^3 eV\left[\frac{1}{\sqrt{1-(0.0075)^2}} - 1\right] - 28.8Z\ eV$$

$$= 14.4eV - 28.8Z\ eV = -14.4Z\ eV$$