PHYSICS 4E FINAL EXAM SPRING QUARTER 2010
PROF. HIRSCH JUNE 11

<u>Formulas and constants:</u>

$hc = 12{,}400\ eV\,A$; $k_B = 1/11{,}600\ eV/K$; $ke^2 = 14.4 eVA$; $m_e c^2 = 0.511\times 10^6 eV$; $m_p/m_e = 1836$

Relativistic energy - momentum relation $E = \sqrt{m^2c^4 + p^2c^2}$; $c = 3\times 10^8 m/s$

Photons: $E = hf$; $p = E/c$; $f = c/\lambda$ Lorentz force: $\vec{F} = q\vec{E} + q\vec{v}\times\vec{B}$

Photoelectric effect : $eV_0 = (\frac{1}{2}mv^2)_{\max} = hf - \phi$, $\phi \equiv$ work function

Integrals : $I_n \equiv \int_0^\infty x^n e^{-\lambda x^2} dx$; $\frac{dI_n}{d\lambda} = -I_{n+2}$; $I_0 = \frac{1}{2}\sqrt{\frac{\pi}{\lambda}}$; $I_1 = \frac{1}{2\lambda}$; $\int_0^\infty \frac{x^3}{e^x - 1}dx = \frac{\pi^4}{15}$

Planck's law : $u(\lambda) = n(\lambda)\bar{E}(\lambda)$; $n(\lambda) = \frac{8\pi}{\lambda^4}$; $\bar{E}(\lambda) = \frac{hc}{\lambda}\frac{1}{e^{hc/\lambda k_B T} - 1}$

Energy in a mode/oscillator : $E_f = nhf$; probability $P(E) \propto e^{-E/k_B T}$

Stefan's law : $R = \sigma T^4$; $\sigma = 5.67\times 10^{-8} W/m^2K^4$; $R = cU/4$, $U = \int_0^\infty u(\lambda)d\lambda$

Wien's displacement law : $\lambda_m T = hc/4.96k_B$

Compton scattering : $\lambda' \text{-} \lambda = \frac{\mathrm{h}}{\mathrm{m_e}c}(1 - \cos\theta)$; $\lambda_c \equiv \frac{\mathrm{h}}{\mathrm{m_e}c} = 0.0243A$

Rutherford scattering: $b = \frac{kq_\alpha Q}{m_\alpha v^2}\cot(\theta/2)$; $\Delta N \propto \frac{1}{\sin^4(\theta/2)}$

Electrostatics : $F = \frac{kq_1q_2}{r^2}$ (force) ; $U = q_0 V$ (potential energy) ; $V = \frac{kq}{r}$ (potential)

Hydrogen spectrum : $\frac{1}{\lambda} = R(\frac{1}{m^2} - \frac{1}{n^2})$; $R = 1.097\times 10^7\ m^{-1} = \frac{1}{911.3A}$

Bohr atom : $r_n = r_0 n^2$; $r_0 = \frac{a_0}{Z}$; $E_n = -E_0\frac{Z^2}{n^2}$; $a_0 = \frac{\hbar^2}{mke^2} = 0.529A$; $E_0 = \frac{ke^2}{2a_0} = 13.6eV$; $L = mvr = n\hbar$

$E_k = \frac{1}{2}mv^2$; $E_p = -\frac{ke^2 Z}{r}$; $E = E_k + E_p$; $F = \frac{ke^2Z}{r^2} = m\frac{v^2}{r}$; $hf = hc/\lambda = E_n - E_m$

Reduced mass : $\mu = \frac{mM}{m+M}$; X - ray spectra : $f^{1/2} = A_n(Z - b)$; K : $b = 1$, L : $b = 7.4$

de Broglie : $\lambda = \frac{h}{p}$; $f = \frac{E}{h}$; $\omega = 2\pi f$; $k = \frac{2\pi}{\lambda}$; $E = \hbar\omega$; $p = \hbar k$; $E = \frac{p^2}{2m}$; $\hbar c = 1973\ eV\,A$

group and phase velocity : $v_g = \frac{d\omega}{dk}$; $v_p = \frac{\omega}{k}$; Heisenberg : $\Delta x \Delta p \sim \hbar$; $\Delta t \Delta E \sim \hbar$

Wave function $\Psi(x,t) = |\Psi(x,t)|\mathrm{e}^{\mathrm{i}\theta(x,t)}$; $P(x,t)\,dx = |\Psi(x,t)|^2\,dx$ = probability

Schrodinger equation : $-\frac{\hbar^2}{2\mathrm{m}}\frac{\partial^2\Psi}{\partial x^2} + \mathrm{V(x)\Psi(x,t)} = \mathrm{i}\hbar\frac{\partial\Psi}{\partial t}$; $\Psi(\mathrm{x,t}) = \psi(\mathrm{x})\mathrm{e}^{-\mathrm{i}\frac{\mathrm{E}}{\hbar}t}$

Time – independent Schrodinger equation : $-\frac{\hbar^2}{2\mathrm{m}}\frac{\partial^2\psi}{\partial x^2} + \mathrm{V(x)}\psi(\mathrm{x}) = \mathrm{E}\psi(\mathrm{x})$; $\int_{-\infty}^{\infty} dx\ \psi^*\psi = 1$

∞ square well : $\psi_{\mathrm{n}}(x) = \sqrt{\frac{2}{L}}\sin(\frac{n\pi x}{L})$; $E_n = \frac{\pi^2\hbar^2 n^2}{2mL^2}$; $\mathrm{x_{op}} = x$, $p_{op} = \frac{\hbar}{i}\frac{\partial}{\partial x}$; $<A> = \int_{-\infty}^{\infty} dx\psi^* A_{op}\psi$

Eigenvalues and eigenfunctions : $\mathrm{A_{op}}\ \Psi = a\ \Psi$ (a is a constant) ; uncertainty : $\Delta A = \sqrt{<A^2> - <A>^2}$

Harmonic oscillator : $\Psi_{\mathrm{n}}(x) = C_n H_n(x) e^{-\frac{m\omega}{2\hbar}x^2}$; $E_n = (n + \frac{1}{2})\hbar\omega$; $E = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2 = \frac{1}{2}m\omega^2 A^2$; $\Delta n = \pm 1$

Step potential: $R = \frac{(k_1 - k_2)^2}{(k_1 + k_2)^2}$, $T = 1 - R$; $k = \sqrt{\frac{2m}{\hbar^2}(E - V)}$

Tunneling: $\psi(x) \sim e^{-\alpha x}$; $T \sim e^{-2\alpha\Delta x}$; $T \sim e^{-2\int_a^b \alpha(x)dx}$; $\alpha(x) = \sqrt{\frac{2m[V(x) - E]}{\hbar^2}}$

3D square well: $\Psi(x,y,z) = \Psi_1(x)\Psi_2(y)\Psi_3(z)$; $E = \frac{\pi^2\hbar^2}{2m}(\frac{n_1^2}{L_1^2} + \frac{n_2^2}{L_2^2} + \frac{n_3^2}{L_3^2})$

Spherically symmetric potential: $\Psi_{n,\ell,m}(r,\theta,\phi) = R_{n\ell}(r)Y_{\ell m}(\theta,\phi)$; $Y_{\ell m}(\theta,\phi) = f_{lm}(\theta)e^{im\phi}$

Angular momentum: $\vec{L} = \vec{r} \times \vec{p}$; $L_z = \frac{\hbar}{i}\frac{\partial}{\partial\phi}$; $L^2 Y_{\ell m} = \ell(\ell+1)\hbar^2 Y_{\ell m}$; $L_z = m\hbar$

Radial probability density : $P(r) = r^2 |R_{n,\ell}(r)|^2$; Energy : $E_n = -13.6eV\frac{Z^2}{n^2}$

Ground state of hydrogen and hydrogen-like ions: $\Psi_{1,0,0} = \frac{1}{\pi^{1/2}}(\frac{Z}{a_0})^{3/2}e^{-Zr/a_0}$

Orbital magnetic moment : $\vec{\mu} = \frac{-e}{2m_e}\vec{L}$; $\mu_z = -\mu_B m_l$; $\mu_B = \frac{e\hbar}{2m_e} = 5.79 \times 10^{-5} eV/T$

Spin 1/2: $s = \frac{1}{2}$, $|S| = \sqrt{s(s+1)}\hbar$; $S_z = m_s\hbar$; $m_s = \pm 1/2$; $\vec{\mu}_s = \frac{-e}{2m_e}g\vec{S}$

Total angular momentum: $\vec{J} = \vec{L} + \vec{S}$; $|J| = \sqrt{j(j+1)}\hbar$; $|l - s| \leq j \leq l + s$; $-j \leq m_j \leq j$

Orbital + spin mag moment : $\vec{\mu} = \frac{-e}{2m}(\vec{L} + g\vec{S})$; Energy in mag. field: $U = -\vec{\mu}\cdot\vec{B}$

Two particles : $\Psi(x_1,x_2) = +/- \Psi(x_2,x_1)$; symmetric/antisymmetric

Screening in multielectron atoms: $Z \to Z_{eff}$, $1 < Z_{eff} < Z$

Orbital ordering:
1s < 2s < 2p < 3s < 3p < 4s < 3d < 4p < 5s < 4d < 5p < 6s < 4f < 5d < 6p < 7s < 6d ~ 5f

Boltzmann constant: $k_B = 1/11{,}600\ eV/K$

$f_B(E) = Ce^{-E/kT}$; $f_{BE}(E) = \frac{1}{e^\alpha e^{E/kT} - 1}$; $f_{FD}(E) = \frac{1}{e^\alpha e^{E/kT} + 1}$; $n(E) = g(E)f(E)$

Rotation: $E_R = \frac{L^2}{2I}$, $I = \mu R^2$, vibration: $E_v = \hbar\omega(v + \frac{1}{2})$, $\omega = \sqrt{k/\mu}$, $\mu = m_1 m_2/(m_1 + m_2)$

$g(E) = [2\pi(2m)^{3/2}V/h^3]E^{1/2}$ (translation, per spin) ; Equipartition: $<E> = k_B T/2$ per degree of freedom

Justify all your answers to all problems. Write clearly.