Physics 4E **Quiz 1 Solutions** Spring 2010

Problem 1

Power emitted is $R = \frac{c}{4} . U . A$, A = surface area. This is total power.

Power emitted within $\Delta\lambda$ of λ: $R(\lambda)\Delta\lambda = \frac{c}{4} A\, u(\lambda)\Delta\lambda$

with $$u(\lambda) = \frac{8\pi}{\lambda^5} \frac{hc}{e^{hc/\lambda kT}-1}$$

Maximum power at $\lambda_m = 2900 Å$. From $\lambda_m T = \frac{hc}{4.96 k_B} \Rightarrow$

$$T = \frac{hc}{4.96 k_B \lambda_m} = \frac{12,400 \cdot 11,600}{4.96 \cdot 2900} K = \boxed{10,000 K = T} \text{ (a)}$$

(b) It emits 2W in range $\Delta\lambda$ around $\lambda_m = 2900$ A $\Rightarrow$

$$2W = R(\lambda_m)\Delta\lambda = \frac{8\pi}{\lambda_m^5} \frac{hc\,\Delta\lambda}{e^{hc/\lambda_m kT}-1} . A$$

At $\lambda = 5800 A = 2\lambda_m$, same $\Delta\lambda$, it emits

$$R(2\lambda_m)\Delta\lambda = \frac{8\pi}{2^5\lambda_m^5} \frac{hc\Delta\lambda}{e^{hc/2\lambda_m kT}-1} . A = \frac{8\pi}{\lambda_m^5} \frac{hc\Delta\lambda}{e^{hc/\lambda_m kT}-1} . A . \frac{1}{2^5} . \frac{e^{hc/\lambda_m kT}-1}{e^{hc/2\lambda_m kT}-1} =$$

$$= 2W \times \frac{1}{2^5} \times \frac{e^{4.96}-1}{e^{2.48}-1} = 2W \times 0.404 = \boxed{0.809 W}$$

(c) At 50 A, $e^{hc/\lambda kT} = e^{288}$, $\frac{1}{e^{hc/\lambda kT}-1} = \frac{1}{e^{288}} \sim 0$

At 10,000 A, $e^{hc/\lambda kT} = e^{1.44}$, $\frac{1}{e^{hc/\lambda kT}-1} = 0.3$

Clearly, it emits <u>much</u> more power for $\lambda \sim 10,000 Å$ than for $\lambda \leq 50 Å$.

Problem 2

By energy conservation,

$$E_e = \frac{hc}{\lambda} - \frac{hc}{\lambda'} \Rightarrow \frac{hc}{\lambda'} = \frac{hc}{\lambda} - E_e = 12{,}400\,eV - 200\,eV = 12{,}200\,eV$$

$$\Rightarrow \lambda' = \frac{hc}{12{,}200\,eV} = 1.0164\,\mathring{A}$$

$$\lambda' - \lambda = \frac{h}{m_e c}(1-\cos\theta) \Rightarrow 1-\cos\theta = \frac{m_e c}{h}(\lambda'-\lambda) \Rightarrow$$

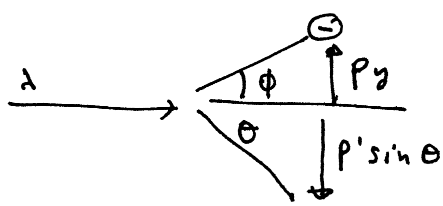
$$\cos\theta = 1 - \frac{m_e c}{h}(\lambda'-\lambda) = 1 - \frac{1}{0.0243} \times 0.0164 = 0.325 \Rightarrow$$

$$\Rightarrow \boxed{\theta = 71^\circ}$$

(b) Maximum energy for electrons is for $\theta = \pi$, $\phi = 0$, $\lambda' = \lambda + 2\lambda_c$

$$\Rightarrow E_e^{max} = \frac{hc}{\lambda} - \frac{hc}{\lambda'} = 12{,}400\left(1 - \frac{1}{1.0486}\right)eV = \boxed{575\,eV}$$

(c)



$$p_y = p'\sin\theta, \quad p' = \frac{h}{\lambda'}$$

For $\theta = 1^\circ$, $\lambda' \sim \lambda$, $\sin\theta \sim \theta = \frac{2\pi \times 1^\circ}{360^\circ}$

$$p_y = \frac{hc}{\lambda'}\sin\theta \cdot \frac{1}{c} = \frac{12{,}400\,eV\mathring{A}}{1\,\mathring{A}} \times \frac{2\pi}{360} \cdot \frac{1}{c} \Rightarrow$$

$$\Rightarrow \boxed{p_y = 216.4\,\frac{eV}{c}}$$

Problem 3

When particle is closest to nucleus, potential energy is

$$U = \frac{k e^2 . 2Z}{R} = \frac{14.4 \, eV.Å . 2.79}{3 . 10^{-4} Å} = 7.584 \, MeV$$

So the kinetic energy, by energy conservation, is

$$E_\alpha' = E_\alpha - U = 10 \, MeV - 7.584 \, MeV = \boxed{2.416 \, MeV}$$

(b) Angular momentum initially: $L = m_\alpha V b$

Angular momentum at $r = R$: $L = m_\alpha V' . R$

(note that at that point $\vec{V}'$ is perpendicular to $\vec{R}$, so $|\vec{R} \times \vec{V}'| = RV'$)

Conservation of $L \Rightarrow$ $m_\alpha V b = m_\alpha V' R \Rightarrow$

$$b = \frac{V'}{V} . R$$

Kinetic energy is $E_\alpha = \frac{1}{2} m_\alpha V^2 \Rightarrow \frac{V'}{V} = \left(\frac{E_\alpha'}{E_\alpha}\right)^{1/2}$

$$\Rightarrow \boxed{b = \left(\frac{E_\alpha'}{E_\alpha}\right)^{1/2} . R}$$

For $E_\alpha = 10 \, MeV$, $E_\alpha' = 2.416 \, MeV$,

$$\boxed{b = 0.49 \, R = 1.47 \times 10^{-4} Å}$$