Physics 4E — Quiz 2 solutions — Spring 2010

Problem 1

Electron 1: $Z=1,\ n=1$ — Electron 2: $Z=2,\ n=2$

$$r_1 = \frac{a_0}{Z} = a_0 \quad , \quad r_2 = \frac{a_0}{Z}\cdot 2^2 = 2a_0$$

(i) $E_1 = -E_0$ , $E_2 = -E_0\dfrac{Z^2}{n^2} = -E_0 \Rightarrow \boxed{E_2/E_1 = 1}$ (i)

(ii) $L_1 = m_1 v_1 r_1 = \hbar$ — $L_2 = m v_2 r_2 = 2\hbar$

$\Rightarrow v_1 = \dfrac{\hbar}{m a_0}$ — $\Rightarrow v_2 = \dfrac{2\hbar}{m r_2} = \dfrac{\hbar}{m a_0} = v_1 \Rightarrow \boxed{v_2/v_1 = 1}$ (ii)

(iii) $\boxed{L_2/L_1 = 2}$ (iii) — (iv) $\boxed{r_2/r_1 = 2}$ (iv)

(v) $E_{k1} = \dfrac{m v_1^2}{2}$ , $E_{k2} = \dfrac{m v_2^2}{2} \Rightarrow \boxed{E_{k2}/E_{k1} = 1}$ (v)

(vi) $E_{p1} = -\dfrac{ke^2}{a_0}$ , $E_{p2} = -\dfrac{ke^2\cdot 2}{2a_0} \Rightarrow \boxed{E_{p2}/E_{p1} = 1}$ (vi)

(vii) $I_1 = -E_1$ , $I_2 = -E_2 \Rightarrow \boxed{I_2/I_1 = 1}$ (vii)

(b) $\mu_1 = \dfrac{m}{1+m/M_H}$ , $\mu_2 = \dfrac{m}{1+m/M_{He}}$ ; $\dfrac{\mu_2}{\mu_1} = \dfrac{1+m/M_H}{1+m/M_{He}} = \dfrac{1+1/1836}{1+1/(4\times 1836)} = 1.0004$

$a_{0i} = \dfrac{\hbar^2}{ke^2\mu_i} \Rightarrow \dfrac{a_{01}}{a_{02}} = \dfrac{\mu_2}{\mu_1} = 1.0004$

$E_1 = -E_0 = -\dfrac{ke^2}{2a_0} \Rightarrow \boxed{E_2/E_1 = \dfrac{a_{01}}{a_{02}} = 1.0004}$ (i)

$L_1 = \mu_1 v_1 r_1 = \hbar$ , $L_2 = \mu_2 v_2 r_2 = 2\hbar \Rightarrow \boxed{v_2/v_1 = 1}$ (ii)

(iii) $\boxed{L_2/L_1 = 2}$ — (iv) $r_2/r_1 = 2a_{02}/a_{01} = 2\mu_1/\mu_2 = \boxed{1.9992}$ (iv)

(v) $E_{ki} = \frac{1}{2}\mu_i v_i^2$ ; $E_{k2}/E_{k1} = \mu_2/\mu_1 = \boxed{1.0004}$ (v)

(vi) $E_{p2}/E_{p1} = a_{01}/a_{02} = \mu_2/\mu_1 = \boxed{1.0004}$ , (vii) $I_2/I_1 = \boxed{1.0004}$

Problem 2

$$p = \frac{h}{\lambda} \quad , \quad E_a = \frac{p^2}{2m_e} = \frac{h^2}{2m_e \lambda^2} = \frac{(hc)^2}{2m_e c^2 \lambda^2} =$$

$$= \frac{12,400^2}{2 \times 0.511 \times 10^6 \times 0.08^2} \, eV = \boxed{23,508 \, eV = E_a}$$

(b) The electron kinetic energy gives the maximum photon energy

$$E = E_a = hf = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{E_a} = \frac{12,400}{23,508} \, Å \Rightarrow$$

$$\boxed{\lambda = \lambda_m = 0.527 \, Å}$$ minimum wavelength of emitted X-rays

(c) Sharp peaks correspond to transitions to $n=1$ (K-series) with $Z-1$, $n$ to $n=2$ (L series) with $Z-7.4$, etc.

For $K_\alpha$ transition, wavelength of emitted photon is:

$$\frac{hc}{\lambda} = E_0 (Z-1)^2 \left(1 - \frac{1}{4}\right) = \frac{3}{4} E_0 (Z-1)^2 \Rightarrow$$

$$\lambda = \frac{hc}{\frac{3}{4} E_0 (Z-1)^2} = \frac{12,400}{0.75 \times 13.6 \times 78^2} \, Å = 0.20 \, Å < \lambda_m$$

$\Rightarrow$ it is not K series (other K lines have even smaller wavelength)

For $L_\alpha$ transition, $$\frac{hc}{\lambda} = E_0 (Z-7.4)^2 \left(\frac{1}{4} - \frac{1}{9}\right) = \frac{5}{9} E_0 (Z-7.4)^2$$

$$\Rightarrow \lambda = \frac{hc}{\frac{5}{9} E_0 (Z-7.4)^2} = 4.35 \, Å > \lambda_m$$ $\Rightarrow$ is in range emitted $\Rightarrow$

$\boxed{\text{The maximum wavelength in the group is } L_\alpha \text{ transition with } \lambda = 4.35 \, Å}$

Problem 3

$\Delta x \Delta k \sim 1 \Rightarrow \Delta x \sim \frac{1}{\Delta k}$ , $\Delta k = 4 \mathring{A}^{-1}$

$k_1 = 10 \mathring{A}^{-1}$
$k_2 = 14 \mathring{A}^{-1} \Rightarrow \Delta k = 4 \mathring{A}^{-1}$

$$\boxed{\Delta x \sim 0.25 \mathring{A}}$$

(b) $p = \hbar k$, $\Delta p = \hbar \Delta k = \hbar c \, \Delta k / c =$

$= 1973 \, eV \mathring{A} \cdot 4 \mathring{A}^{-1} / c \Rightarrow \boxed{\Delta p = 7892 \, eV/c}$

(c) $$\Psi(x) = \int_{k_1}^{k_2} dk \, a(k) e^{ikx} = a \frac{e^{ik_2x} - e^{ik_1x}}{ix} =$$

$$= a e^{i\frac{k_1+k_2}{2}x} \frac{e^{i\frac{k_2-k_1}{2}x} - e^{i\frac{k_1-k_2}{2}x}}{ix} =$$

$$= 2a e^{i\frac{k_1+k_2}{2}x} \frac{\sin \frac{k_2-k_1}{2} x}{x} =$$

$$= 2a e^{i 12 \mathring{A}^{-1} x} \frac{\sin 2\mathring{A}^{-1} x}{x}$$

At $x = 0$, $\frac{\sin 2\mathring{A}^{-1} x}{x} \to 2 \mathring{A}^{-1}$

At $x = 20 \mathring{A}$, $\frac{\sin 2\mathring{A}^{-1} x}{x} \sim \frac{1}{x} = \frac{1}{20} \mathring{A}^{-1}$

So ratio $$\boxed{\frac{\Psi(x = 20 \mathring{A})}{\Psi(x=0)} \sim \frac{1}{20 \times 2} = \frac{1}{40} = 0.025}$$