Physics 4E   Quiz 4 Solutions   Spring 2010

Problem 1

$$E_{n_1,n_2} = \frac{\hbar^2\pi^2}{2m_e}\left(\frac{n_1^2}{L_1^2} + \frac{n_2^2}{L_2^2}\right) = 37.6\,eV\left(\frac{n_1^2}{4} + \frac{n_2^2}{9}\right)$$

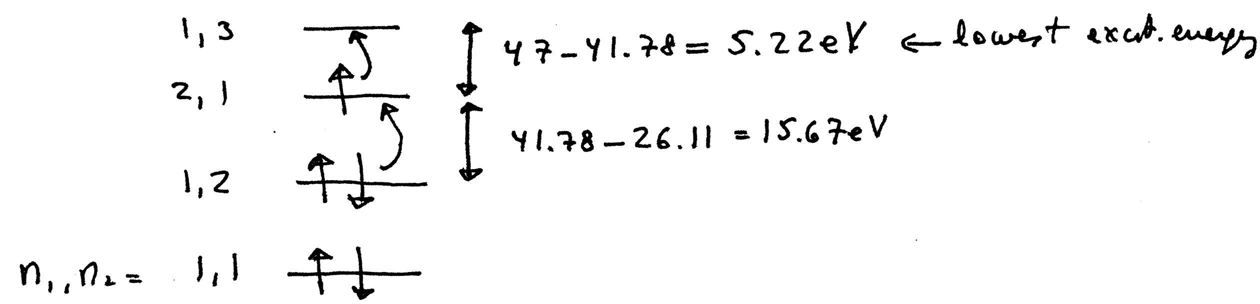
$$E_{11} = 37.6\,eV\left(\frac{1}{4} + \frac{1}{9}\right) = 13.6\,eV$$

$$E_{12} = 37.6\,eV\left(\frac{1}{4} + \frac{4}{9}\right) = 26.11\,eV$$

$$E_{21} = 37.6\,eV\left(1 + \frac{1}{9}\right) = 41.78\,eV$$

$$E_{13} = 37.6\,eV\left(\frac{1}{4} + 1\right) = 47.00\,eV$$

$$E_{22} = 37.6\,eV\left(1 + \frac{4}{9}\right) = 54.32\,eV$$



Ground state energy:

$$E = 2E_{11} + 2E_{12} + E_{21} = \boxed{121.2\,eV}$$

(c) Lowest energy photon: excites electron from 2,1 to 1,3

$$\Delta E = 47\,eV - 41.78\,eV = 5.22\,eV = \frac{hc}{\lambda} \Rightarrow \boxed{\lambda = 2375\,\text{Å}}$$

Problem 2

$$\psi(r,\theta,\phi) = C r^3 e^{-r/2a_0} \sin^3\theta \, e^{-i\phi}$$

(a) $e^{-i\phi} \Rightarrow \boxed{m=-1}$; $\sin^3\theta \Rightarrow \boxed{l=3}$

the fact that the radial part $\propto r^3 \Rightarrow \boxed{n=4}$

Then the exponent is $e^{-Zr/na_0} \Rightarrow \frac{Z}{n} = \frac{1}{2} \Rightarrow Z = \frac{n}{2} \Rightarrow \boxed{Z=2}$

(b) $P(r) = r^2 R^2(r) = C^2 r^8 e^{-r/a_0}$

$$\langle \tfrac{1}{r} \rangle = \frac{\int_0^\infty dr \, \frac{1}{r} P(r)}{\int_0^\infty dr \, P(r)} = \frac{\int_0^\infty dr \, r^7 e^{-r/a_0}}{\int_0^\infty dr \, r^8 e^{-r/a_0}} = \frac{7! \, a_0^{-9}}{a_0^{-8} \cdot 8!} = \frac{1}{8a_0}$$

$$\boxed{\langle \tfrac{1}{r} \rangle = \frac{1}{8a_0}}$$

(c) In the Bohr atom, $r_n = \frac{a_0 n^2}{Z}$, and $\langle \frac{1}{r} \rangle = \frac{1}{r_n}$ since it's a circular orbit.

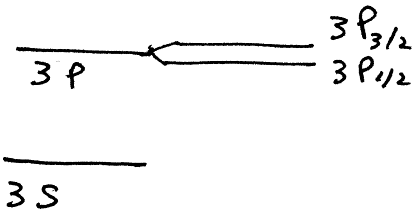
$$\frac{1}{r_n} = \frac{Z}{a_0 n^2} = \frac{2}{16 a_0} = \boxed{\frac{1}{8a_0}}$$

$\Rightarrow$ agrees with (b).

Problem 3

(a) $1s^2 2s^2 2p^6 3s^1$

(b) In Na the 3p state is higher than the 3s state



The energy to remove electron from 3s state is 5.14 eV

The energy difference between 3s and 3p states is related to the wavelength

$5890\,\text{Å}$ (or $5896\,\text{Å}$), $\Delta E = \dfrac{hc}{\lambda} = 2.11\,\text{eV}$

$\Rightarrow$ energy to remove electron from p-state is difference

$$5.14\,\text{eV} - 2.11\,\text{eV} = \boxed{3.03\,\text{eV}}$$

(c) Orbital angular momentum is $l = 2$, spin is $s = 1/2 \Rightarrow$

$j = \frac{5}{2}$ or $j = \frac{3}{2}$ $\Rightarrow$ $|J| = \sqrt{\frac{3}{2}\left(\frac{3}{2}+1\right)}\,\hbar = \sqrt{\frac{15}{4}}\,\hbar$

or $|J| = \sqrt{\frac{5}{2}\left(\frac{5}{2}+1\right)}\,\hbar = \sqrt{\frac{35}{4}}\,\hbar$

Because the d-electron has higher angular momentum than the p electron it is farther away from the nucleus $\Rightarrow$ screening is better $\Rightarrow$ its energy is higher than that of the 3p electron $\Rightarrow$ to remove this electron from the sodium atom takes less energy than 3.03 eV.

$j = 5/2$ is higher energy than $j = 3/2$ because the electron magnetic moment is antiparallel to the internal magnetic field for $j = 5/2$.