Handout 3

(from "Heat & Thermodynamics" by Zemansky & Dittman)

3-11 WORK IN CHANGING THE MAGNETIZATION OF A MAGNETIC SOLID

Consider a sample of magnetic material in the form of a ring of cross-sectional area A and of mean circumference L. Suppose that an insulated wire is wound on top of the sample, forming a toroidal winding of N closely spaced turns, as shown in Fig. 3-7. A current may be maintained in the winding by a battery, and by moving the sliding contactor of a rheostat this current may be changed.

The effect of a current in the winding is to set up a magnetic field with magnetic induction $\mathcal{B}$. If the dimensions are as shown in Fig. 3-7, $\mathcal{B}$ will be nearly uniform over the cross-section of the toroid. Suppose that the current is changed and that in time $d\tau$ the magnetic induction changes by an amount $d\mathcal{B}$. Then, by Faraday's principle of electromagnetic induction, there is induced in the winding a back emf $\mathcal{E}$, where

$$\mathcal{E} = -NA\frac{d\mathcal{B}}{d\tau}.$$

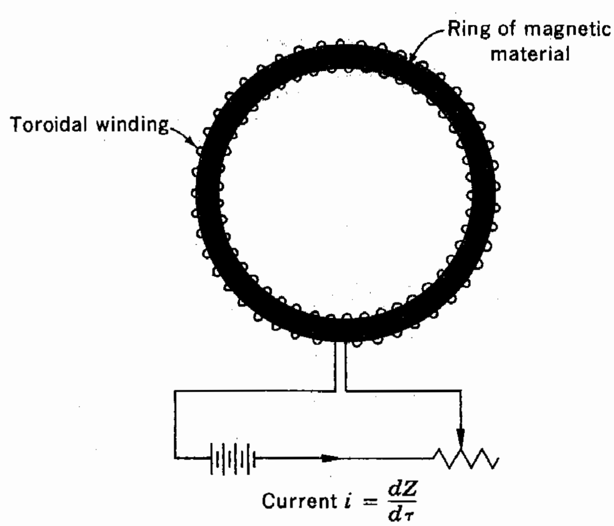

Figure 3-7 Changing the magnetization of a magnetic solid.

During the time interval $d\tau$, a quantity of charge dZ is transferred in the circuit, and the work done *by* the system to maintain the current is calculated as

$$
\begin{aligned}
đW &= -\mathcal{E}\, dZ \\
&= NA\frac{d\mathcal{B}}{d\tau}\, dZ \\
&= NA\frac{dZ}{d\tau}\, d\mathcal{B} \\
&= NAi\, d\mathcal{B},
\end{aligned}
$$

where i, equal to $dZ/d\tau$, is the momentary value of the current.

The magnetic intensity $\mathcal{H}$ due to a current i in a toroidal winding is given by

$$\mathcal{H} = \frac{Ni}{L} = \frac{NAi}{AL} = \frac{NAi}{V},$$

where V is the volume of magnetic material. Therefore,

$$NAi = V\mathcal{H}$$

and

$$đW = V\mathcal{H}\, d\mathcal{B}. \tag{3-9}$$

If M is the *total magnetic moment* of the material (assumed to be isotropic), or *total magnetization*, we have the relation

$$\mathcal{B} = \mu_0 \mathcal{H} + \mu_0 \frac{M}{V}. \tag{3-10}$$

Therefore,

$$dW = V\mu_0 \mathcal{H}\, d\mathcal{H} + \mu_0 \mathcal{H}\, dM.$$

If no material were present within the toroidal winding, M would be zero, $\mathcal{B}$ would equal $\mathcal{H}$, and

$$đW = V\mu_0 \mathcal{H}\, d\mathcal{H} \qquad \text{(vacuum only)}.$$

This is the work necessary to increase the magnetic field in a volume V *of empty space* by an amount $d\mathcal{H}$. The second term, $\mu_0 \mathcal{H}\, dM$, is the work done in increasing the magnetization of the material by an amount dM. We shall be concerned in this book with changes of temperature, energy, etc., of the material only, brought about by work done on or by the material. Consequently, for the purpose of this book,

$$\boxed{đW = \mu_0 \mathcal{H}\, dM.} \tag{3-11}$$

If $\mathcal{H}$ is measured in amperes per meter and M in ampere · square meters, then the work will be expressed in joules. If the magnetization is caused to change a finite amount from M_i to M_f, the work will be

$$W = \mu_0 \int_{M_i}^{M_f} \mathcal{H}\, dM.$$