Homework 6: *Due March 4*

1. Consider a ferromagnetic system composed of N elementary magnets with nearest-neighbor interaction
$$\varepsilon_{ij} = -I\sigma_i\sigma_j \qquad (I > 0; \sigma_i, \sigma_j = 1 \text{ or } -1).$$
In the mean-field approximation, this expression may be replaced my $-I\langle\sigma\rangle^2$, i.e. $-Im^2$, where m is the "degree of magnetization" in the system.

   (a) Using this approximation, write down the total energy U_0 of the system <u>in the limit $H \to 0$</u> and show that this gives for the specific heat of the system
   $$C_0 = -NIqm_s\frac{\mathrm{d}m_s}{\mathrm{d}T},$$
   where q is the # of nearest neighbors that each elementary magnet has.

   (b) Next show that, as $T \to T_c$ from below, $C_0 \to \frac{3}{2}Nk$.

2. The equation of state of a ferromagnetic system, in the mean-field approximation, is given by
$$m = \tanh\left(\frac{\mu_B H + Iqm}{kT}\right),$$
where the various symbols have their usual meanings. Making no further approximations, show that the magnetic susceptibility of this system is given by the general expression
$$\chi = \frac{N\mu_B^2}{k}\frac{1-m^2}{T-(1-m^2)T_c}$$
and the specific heat difference by
$$C_H - C_M = Nk\frac{T(1-m^2)(\tanh^{-1}m)^2}{T-(1-m^2)T_c}.$$

   (a) What forms do these expressions take in the Curie regime?

   (b) What forms do they take in the limit $H \to 0$ and at temperature $T > T_c$ and at $T \lesssim T_c$?

3. (a) It was proved in the class that
   $$C_H - C_M = -T\left(\frac{\partial H}{\partial T}\right)_M\left(\frac{\partial M}{\partial T}\right)_H,$$
   where the various symbols have their usual meanings.
   Show that this result may be expressed in the form
   $$C_H - C_M = \frac{N^2\mu_B^2 T}{\chi}\left(\frac{\partial m}{\partial T}\right)_H^2,$$
   where χ is the magnetic susceptibility of the system and m the degree of magnetization.

   (b) Now show that, as $H \to 0$ and $T \to T_c$ from below, this quantity too approaches the value $\frac{3}{2}Nk$.