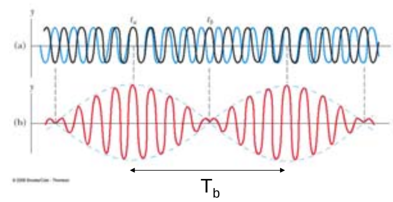
2.2 Doppler Effect

Beats
Doppler Effect.
Shock Waves

Beats

Superposition of two waves with different frequencies produce oscillation in amplitude.



Beat Frequency $f_b = \frac{1}{T_b} = |f_2 - f_1|$

Tuning musical instruments

The beat frequency for two musical instruments is zero when the two are in tune. (have the same frequency)

Doppler Effect

Doppler effect- the shift in frequency of a wave where the source and observer are moving relative to one another.

Doppler effect

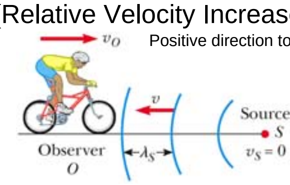
Two different cases:

Observer moving – Relative velocity changes
Source moving- Wavelength changes

$$f = \frac{v}{\lambda}$$

Observer moving toward a Stationary source (Relative Velocity Increases)

Positive direction toward source

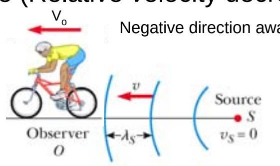


- Relative velocity of wave (v_o + v) increases.
- Frequency increases

$$f_o = \frac{v + v_o}{\lambda_s} \quad = \frac{v + v_o}{v} f_s = (1 + \frac{v_o}{v}) f_s$$

Observer moving away from a stationary source (Relative velocity decreases)

Negative direction away from source

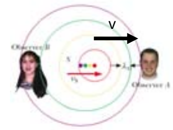


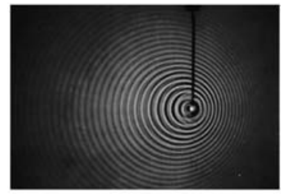
Relative velocity of wave ($v+v_o$) decreases.
Frequency decreases.

$$f_o = \frac{v + v_o}{v} f_s = (1 + \frac{v_o}{v}) f_s$$

v_o is negative

Source moving toward a stationary observer (wavelength in the medium decreases)

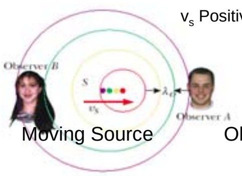




- When the source is moving the wavelength of the wave in the media is changed
  - source approaches observer A
    - Wavelength decreases and frequency heard by observer A increases
  - Source moves away from observer B.
    - Wavelength increases and frequency heard by observer B decreases.

Source Moving Toward observer A

v_s Positive direction toward observer



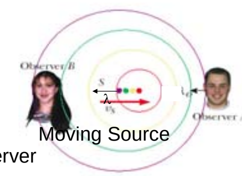
Distance traveled by source in one period

- Wavelength decreases $\lambda = \lambda_s - v_s T$
- Frequency increases $f_o = \frac{v}{\lambda_s - v_s T_s} = \frac{v}{vT_s - v_s T_s}$

$$f_o = \frac{v}{v - v_s} f_s$$

Source Moving Away from observer B

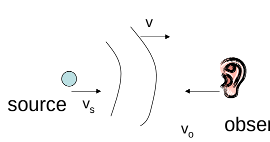
v_s negative direction for observer B



- Wavelength increases $\lambda = \lambda_s - v_s T$
- Frequency decreases $f_o = \frac{v}{v - v_s} f_s$

v_s is negative

Observer and source moving

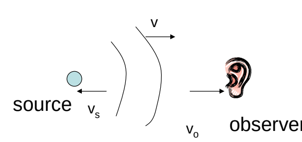


$$f_o = f_s \left(\frac{v + v_o}{v - v_s} \right)$$

v_s positive
v_o positive

- The frequency increases when the source and observer are moving toward each other.

Observer and source moving



$$f_o = f_s \left(\frac{v + v_o}{v - v_s} \right)$$

v_o negative
v_s negative

- The frequency decreases when the source and observer are moving away each other.

Example

A fire truck is approaching an observer with a speed of 30 m/s. The siren has a frequency of 700 Hz. What frequency does the observer hear as the truck approaches? What frequency is heard after the truck passes. speed of sound 340 m/s

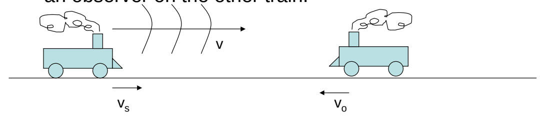
Approaching source v_s positive $v_o = 0$

$$f_o = f_s\left(\frac{v+v_o}{v-v_s}\right) = f_s\left(\frac{v}{v-v_s}\right) = 700\left(\frac{340}{340-30}\right) = 700(1.09) = 763\text{Hz}$$

Departing source v_s is negative

$$f_o = f_s\left(\frac{v+v_o}{v-v_s}\right) = f_s\left(\frac{v}{v-v_s}\right) = 700\left(\frac{340}{340+30}\right) = 700(0.92) = 643\text{Hz}$$

Two trains are approaching each other each moving at 34 m/s. One train sounds a whistle at a frequency of 1000 Hz. Find the frequency of sound heard by an observer on the other train.



$$f_o = \frac{v+v_o}{v+v_s} f_s = \frac{340+34}{340-34} f_s = 1.22 f_s$$

$$f_o = 1.22\text{x}10^3\text{Hz}$$

Note each motion increases the frequency by about v_{train}/v =10%. The net increase is about 20% This is true when $v_{source} << v$.

Approximate solution at low speeds.

Source moving toward observer.

$$f_o = \frac{v}{v-v_s} f_s = \frac{v}{v(1-\frac{v_s}{v})} f_s = \frac{1}{1-\frac{v_s}{v}} f_s$$

Using the relation

$$\frac{1}{1-x} \approx 1+x \qquad \text{When } x<<1$$

At low speed $v_s << v$

$$f_o \approx (1+\frac{v_s}{v}) f_s$$

Approximate solution for two trains approaching

$$f_o = \frac{v+v_o}{v-v_s} f_s = \left(\frac{v+v_o}{v}\right)\left(\frac{v}{v-v_s}\right) f_s \cong \left(1+\frac{v_o}{v}\right)\left(1+\frac{v_s}{v}\right) f_s$$

negligible

$$f_o = \left(1+\frac{v_s}{v}+\frac{v_o}{v}+\frac{v_s v_o}{v^2}\right) f_s \cong \left(1+\frac{v_s}{v}+\frac{v_o}{v}\right) f_s$$

$$f_o - f_s \cong \left(\frac{v_s}{v}+\frac{v_o}{v}\right) f_s$$

- The shift in frequency is approximately proportional to the ratio of the train velocities to speedof sound as we found in the previous example.
- This is a good approximation when the train velocities are slow compared to the speed of sound.
- This is a good approximation for the Doppler shift of electromagnetic waves.

Doppler shift of Electromagnetic waves

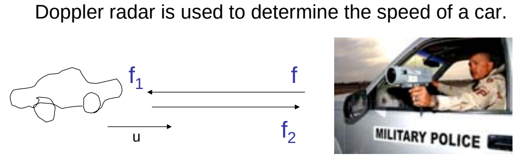
- Electromagnetic waves are also shifted by the Doppler effect.
- Since EM waves travel in a vacuum the equations governing the shift are different.
- The same shift is observed for moving source or moving observer.
- For motion with speeds less than the speed of light the relation is the same as for the approximate shift for sound waves when u<<v.

$$f = f_s(1 \pm \frac{u}{c})$$

u = relative velocity of source and observer.
c = speed of light =3.00x10[8] m/s
Positive sign when approaching
Negative sign when moving away.

Doppler Radar

Doppler radar is used to determine the speed of a car.



The beat frequency between the Doppler shifted frequency and the initial frequency is measured to determine the speed of the car.

negligible

$f_1 = f_s\,(1+u/c)$

$f_2 = f_1(1+u/c) = f_s\,(1+u/c)^2 = f_s\,(1+2u/c + (u/c)^2)$

beat frequency = $f_2 - f_s = 2\frac{u}{c} f_s$

Applications Doppler ultrasound

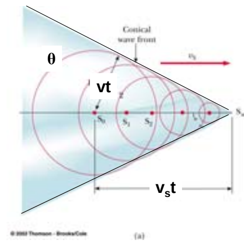
Ultrasonic waver 2-18 MHz.

Doppler shift used to measure blood flow speed.

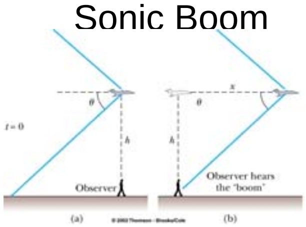
Shock Waves

When v_s > speed of sound, shock waves are formed.

$$\mathrm{Sin}\theta = \frac{v}{v_s}$$



Sonic Boom



When a sonic boom is heard the observer sees the plane at an angle of $\theta = 45^0$ What is the speed of the plane relative to the speed of sound?

$$\sin\theta = \frac{v}{v_s} = \sin 45 = 0.707$$

$$v_s = \frac{v}{\sin\theta} = \frac{v}{0.707} = 1.41v$$

The plane travels at 1.41 time the speed of sound.