Combinations of lenses

- When two lenses are used in combination, the image of the first lens is the object for the second lens.
- The total magnification is the product of the magnifications of the first and second lens.

Find the image formed by two lenses in combination

Image formed by lens 1

$$q_1 = \frac{p_1 f_1}{p_1 - f_1} = \frac{(30)(10)}{30-10} = 15\text{cm}$$

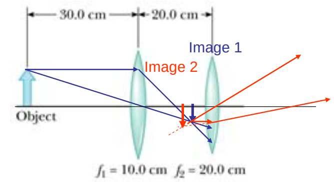
from lens 1

Image formed by lens 2

$p_2 = 20 - q_1 = 20 - 15 = 5$ cm

$$q_2 = \frac{p_2 f_2}{p_2 - f_2} = \frac{(5)(20)}{5-20} = -6.7\text{cm}$$

from lens 2

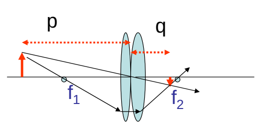


Magnification

$$M = M_1 M_2 = \left(-\frac{q_1}{p_1}\right)\left(-\frac{q_2}{p_2}\right) = \left(-\frac{15}{30}\right)\left(-\frac{-6.7}{5}\right) = -0.67$$

Inverted
Reduced

For two lenses in contact the total power is the sum of powers of the individual lenses



$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$P = P_1 + P_2$$

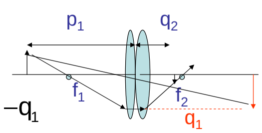
For two lenses in contact the total power is the sum of powers of the individual lenses

$$\frac{1}{p_1} + \frac{1}{q_1} = \frac{1}{f_1} \qquad \frac{1}{p_2} + \frac{1}{q_2} = \frac{1}{f_2}$$

$$-\frac{1}{q_1} + \frac{1}{q_2} = \frac{1}{f_2}$$

substitute

$$p_2 = -q_1$$



Eliminate q_1

$$\frac{1}{p_1} + \frac{1}{q_2} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{f}$$

The image of the first lens is the object for the second lens. Virtual object has a negative sign.

5.2 Optical Instruments

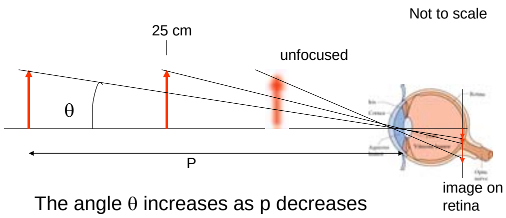
- Simple magnifier
- Compound microscope
- Telescope

Magnifiers

How do we image small objects?

- We can image a small object by bringing it close to our eye.
- But we cannot bring it closer than the near point. (we can't focus on it).
- A magnifier can produce a larger image of the object at the near point (or farther away) that can be focused on by the eye.
- Key concept - Angular magnification

Angular size

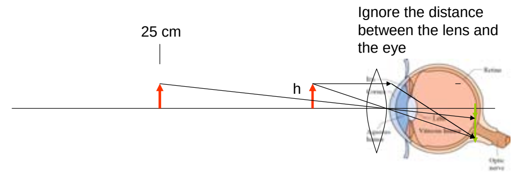


The angle θ increases as p decreases
The image size increases

Objects closer than the near point are not in focus.

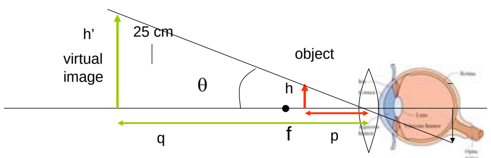
Simple Magnifier

A converging lens in combination with the lens of the eye forms an image on the retina from an object closer than the near point of the eye.



How does it work?

Produces an enlarged virtual image at a distance from the eye (from 25 cm to infinity) that the eye can focus on.



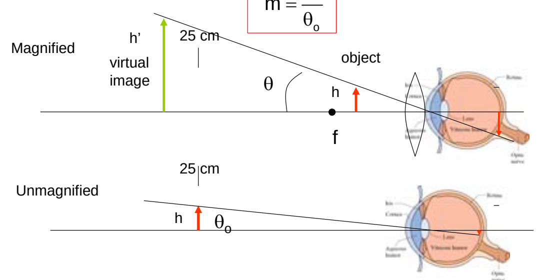
θ Increases

Simple Magnifier

Angular Magnification

The angular magnification is the ratio of θ for the magnified image compared to value of θ_o for the object at the near point of the eye. (25 cm)

$$m = \frac{\theta}{\theta_o}$$

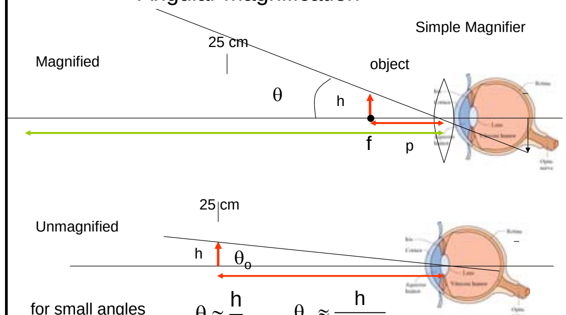


Angular magnification

The angular magnification for the simple magnifier can have a range of values because the focal length of the eye can vary due to accommodation.

The simplest case is the magnification for the relaxed eye. (focused at infinity)

Angular magnification



for small angles
$\tan\theta \sim \theta$

$$\theta \approx \frac{h}{f} \qquad \theta_o \approx \frac{h}{25\text{cm}}$$

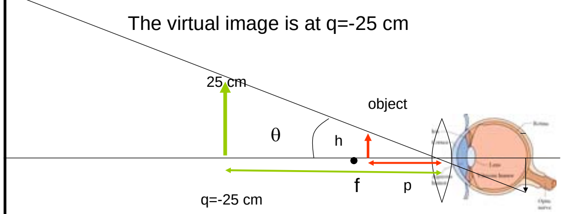
Angular magnification

$$m = \frac{\theta}{\theta_o} = \frac{25\text{cm}}{f}$$

f is the focal length of the lens in cm.

Simple magnifier

The maximum magnification of a simple magnifier.



$$m = 1 + \frac{25\text{cm}}{f}$$

Simple magnifier.

A simple magnifier with a focal length of 5.0 cm is used to view an insect. What is the angular magnification for a relaxed eye?

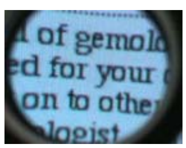
$$m = \frac{25\text{cm}}{f} = \frac{25\text{cm}}{5.0\text{cm}} = 5.0$$

What is the magnification for the accommodated eye?

$$m = 1 + \frac{25\text{cm}}{f} = 1 + \frac{25\text{cm}}{5\text{cm}} = 6$$

Simple magnifiers.

The angular magnification for a single lens is limited by aberration to about 4. Combination lenses can have magnification to about 20.



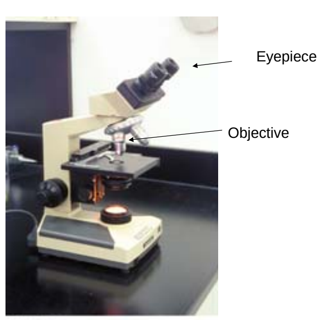
Compound Microscopes.

Magnification by 2 lenses.

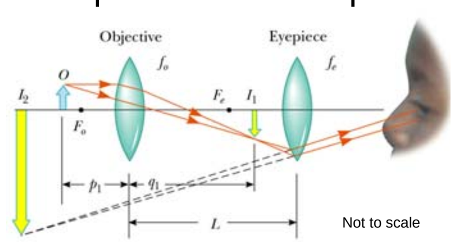
- Objective lens – Produces an enlarged real image of the object.
- Eyepiece – Used like a simple magnifier to view the image.

The net angular magnification of the product of the two magnifications.

Compound microscope



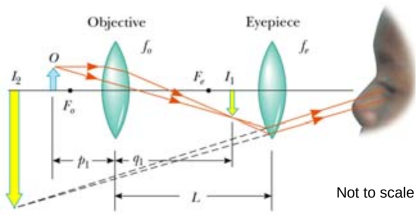
Compound microscope



The objective lens produces a magnified real image I_1
The image is viewed through the eyepiece.

M
a

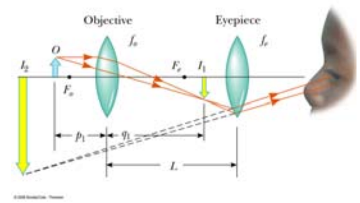
Two stages of magnification by
1) objective and 2) eyepiece.



The objective lens produces a magnified real image I_1
The image is viewed through the eyepiece.

$M_o = -\frac{q_1}{p_1} \approx -\frac{L}{f_o}$ $m_e = \frac{25cm}{f_e}$ For relaxed eye

Total Magnification is the product



$$m = M_1 m_e = -\frac{L}{f_o}\frac{(25cm)}{f_e}$$

Magnification increases when f_e and f_e get smaller.

Magnification

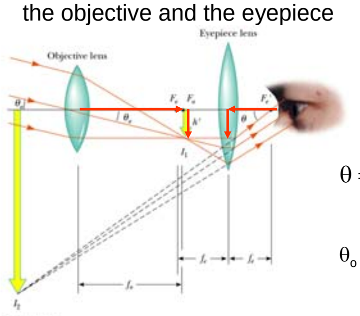
A compound microscope has an objective lens and eyepiece with a focal lengths of 1.5 cm and 2.0 cm respectively. The microscope is 20 cm long. Find the angular magnification

$$m = -\frac{L}{f_o}\frac{(25cm)}{f_e} = -\frac{20}{1.5}\frac{(25cm)}{2.0} = -167$$

Refracting Telescope

Two lenses
Objective lens – produces a reduced image of a distant object near the focal point.
Eyepiece – used to magnify the image.

Angular magnification - ratio of the focal length of the objective and the eyepiece



$$\theta = \frac{h'}{f_e}$$

$$\theta_o = \frac{h'}{f_o}$$

focus at infinity

Angular magnification $m = \frac{\theta}{\theta_o} = \frac{f_o}{f_e}$

Telescope

The Hubble space telescope has an objective mirror with a focal length of 57.8 m viewed with optics equivalent to an eyepiece with a focal length of $7.2x10^{-3}$m What is the angular magnification?

$$m = \frac{f_o}{f_e} = \frac{57.8}{7.2x10^{-3}} = 8.0x10^3$$

Hubble Telescope Image of M100 Spiral Galaxy (NASA)

Limits to magnification

Why can't we use light microscopes to see atoms?

- For refracting optics there are problems of chromatic and spherical aberration.
- Problems in precision in constructing the refracting and reflecting surfaces.
- Diffraction – A basic problems having to do with the wave nature of light (discussed next week)