6.1 Diffraction

Diffraction grating
Single slit diffraction
Circular diffraction

Diffraction and Interference

- Diffraction and interference are similar phenomena.
- Interference is the effect of superposition of 2 coherent waves.
- Diffraction is the superposition of many coherent waves.

Diffraction grating

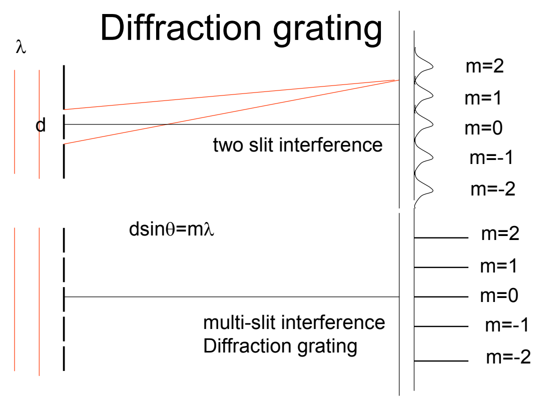
- Consists of a flat barrier which contains many parallel slits separated by a short distance d.
- A parallel monochromatic light beam passing through the grating is diffracted by an angle θ

$$d\sin\theta = m\lambda$$

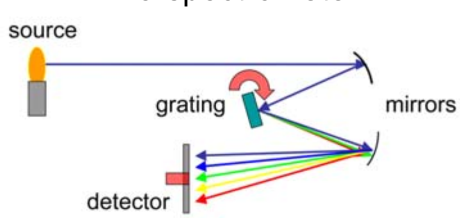
similar to two slit interference.

However, the intensity of the diffracted light is higher and the peaks are much narrower.

Diffraction grating



Use of a diffraction grating in a spectrometer



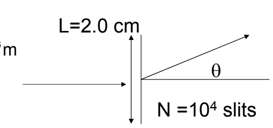
Dispersion of light of different wavelengths

Question

A grating in a spectrometer has a length of 2 cm and has contains 10^4 lines. Find the first order diffraction angle for light with a wavelength of 500 nm.

first order m=1

$$d = \frac{L}{N} = \frac{2\times10^{-2}\,m}{10^4} = 2\times10^{-6}\,m$$

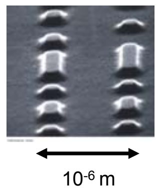


$d\sin\theta = m\lambda$

$$\sin\theta = \frac{m\lambda}{d} = \frac{500\times10^{-9}}{2\times10^{-6}} = 0.25$$

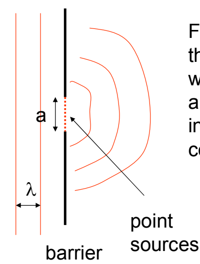
$\theta = 14.5^\circ$

Optical compact disc



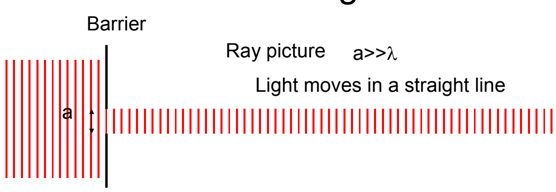
The closely spaced dots act like a diffraction grating.

Single slit diffraction



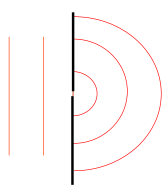
For single slit diffraction the complex behavior of the wave as a function of the aperture a is explained by interference between multiple coherent point sources in the slit.

Two Limiting Cases



Ray picture $a >> \lambda$

Light moves in a straight line

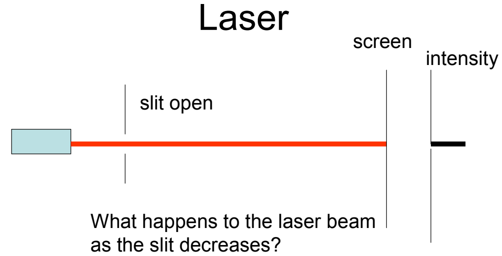


Wave picture $a << \lambda$

Light spreads out when passed through small aperture.

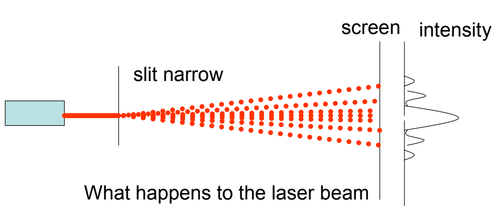
What happens to the light for the general case?

Laser



What happens to the laser beam as the slit decreases?

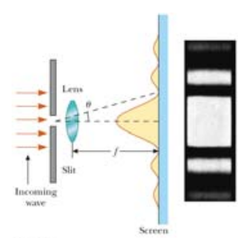
The pattern spreads out as the slit becomes narrower



What happens to the laser beam as the slit decreases?

Single slit diffraction pattern

Single slit diffraction pattern



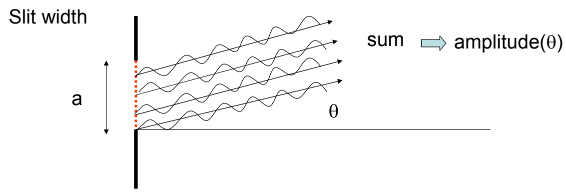
How do we account for the minima and maxima?

Analyze assuming Fraunhofer diffraction conditions.
a) Screen far from the slit or
b) Converging lens with screen at the focal length

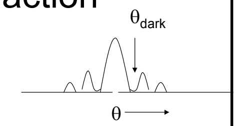
Single slit diffraction

Huygens principle – Each point in the wave in the slit acts as a source of spherical waves.

sum the waves with different phases



Single slit diffraction

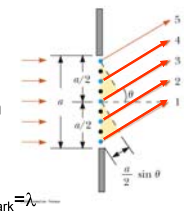


Find the angle at the first minimum

amplitude = 0

Divide the slit into 2 halves

Light from every point in the top half interferes destructively with a light from a point in the bottom half when

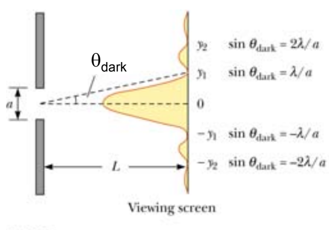


$$\frac{a}{2}\sin\theta_{dark} = \frac{\lambda}{2} \quad \Rightarrow \quad a\sin\theta_{dark} = \lambda$$

Condition for a minimum

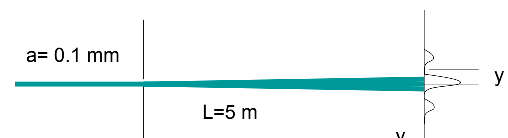
$a\sin\theta_{dark} = m\lambda$

m=± 1, ± 2, ...



Question

Laser light with a wavelength of 532 nm is passed through a slit with a width of 0.1 mm and illuminates a screen 5 m away. Find the distance from the central maxima to the first minima.

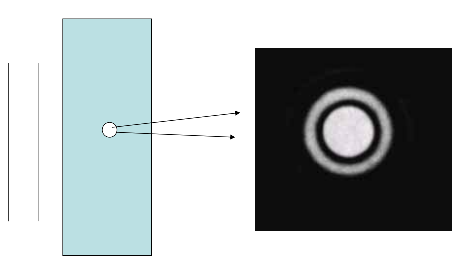


$a\sin\theta = \lambda$ for small angles $\sin\theta = \theta = \frac{y}{L}$

$$a\frac{y}{L} = \lambda$$

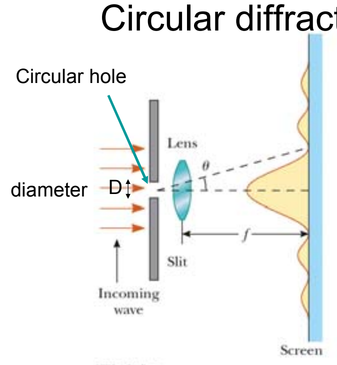
$$y = \frac{\lambda L}{a} = \frac{(532\times10^{-9}\,m)(5m)}{0.1\times10^{-3}\,m} = 2.7\times10^{-2}\,m = 2.7\,cm$$

Circular diffraction



Waves passing through a circular hole forms a a circular diffraction pattern.

Circular diffraction.



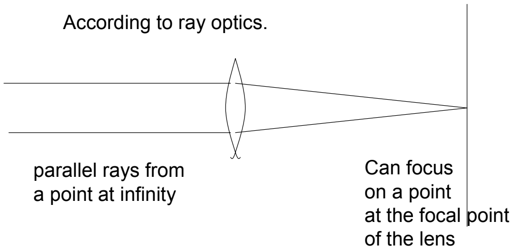
The first minimum occurs at

$$\theta_{min} = 1.22\frac{\lambda}{D}$$

Circular diffraction pattern

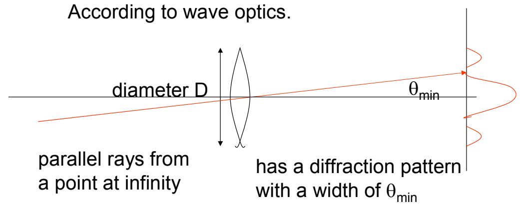
Circular diffraction limits the minimum size do the spot that can be formed by a lens.

According to ray optics.



Circular diffraction limits the minimum size do the spot that can be formed by a lens.

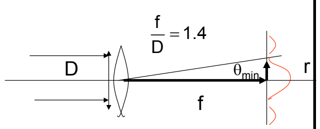
According to wave optics.



Example

A camera lens with an f- number (f/D) equal to 1.4 is used to focus light from a distant source. What is the diffraction limited diameter of the spot that can be formed for 500 nm light?

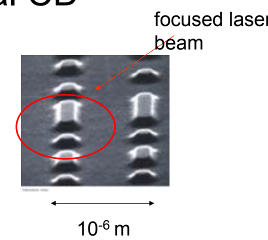
$$\theta_{min} = 1.22\frac{\lambda}{D} = \frac{r}{f}$$



$$d = 2r = 2(1.22\frac{\lambda f}{D}) = 2(1.22)(500x10^{-9})(1.4) = 1.7x10^{-6}m$$

about 3 x the wavelength of the light

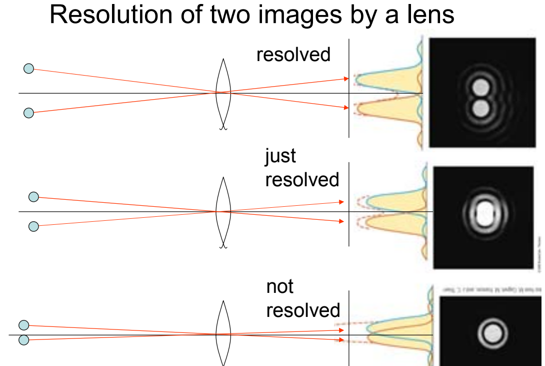
Optical CD



The amount of information that can be encoded is limited by the diameter of the diffraction-limited spot.

Diffraction limits the resolution of light microscopes (200 nm)

Resolution of two images by a lens

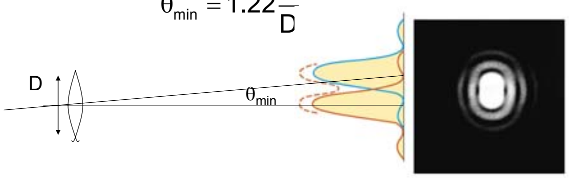


The resolution of images is limited by the diffraction pattern.

Rayleigh criterion

For resolution of two object by a circular lens of diameter D the diffraction limit of resolution occurs when the image of the second object is at position of the first minimum of the diffraction pattern of the first object.

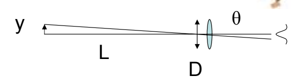
$$\theta_{min} = 1.22\frac{\lambda}{D}$$



GeoEye 1 Imaging Satellite

1.1 m diameter mirror
Resolution 0.41 m
Altitude of 684 km

Diffraction limit (λ=550nm)



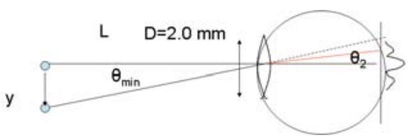
$$\theta = \frac{y}{L} = 1.22\frac{\lambda}{D}$$

$$y = \frac{1.22\lambda L}{D} = \frac{1.22(550x10^{-9}m)(684x10^{3}m)}{1.1m} = 0.41m$$

Satellite picture of Sea World

Diffraction Limit for the eye

The diffraction limit for the eye is similar to that for a camera. The wavelength of light in the eye is reduced to λ/n. However, this is compensated for because the light passing into the eye is deviated by Snell's law



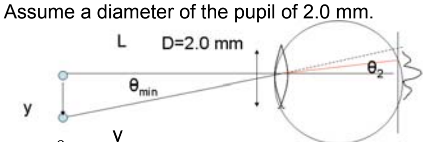
The net result is that the apparent angle to the first minimum

$$\theta_{min} = \frac{1.22\lambda}{D}$$

This differs from the book

Question

Two light sources (λ= 500nm) are separated vertically by 2.0 mm. How far away can these objects be resolved by the eye Assume a diameter of the pupil of 2.0 mm.



$$\theta_{min} = \frac{y}{L}$$ (for small angles)

$$\frac{y}{L} = 1.22\frac{\lambda}{D}$$

$$L = \frac{yD}{1.22\lambda} = \frac{(2x10^{-3})(2x10^{-3})}{1.22(500x10^{-9})} = 6.6m$$

Diffraction limit

- Diffraction limits the resolution of objects viewed through an optical system
- Resolution depends on the size of the aperture and the wavelegth of light.
- Consequences
  - atoms cannot be seen with a light microscope (shorter wavelengths are required)
  - Satellite cameras have a limited resolution.
- To attain higher resolution.
  - Larger diameter lenses
  - Shorter wavelengths.