8.2 Wave Nature of Matter

De Broglie Wavelength
Diffraction of electrons
Uncertainty Principle
Wave Function
Tunneling

Wave properties of matter

Material particles behave as waves with a wavelength given by the De Broglie wavelength (Planck's constant/momentum)

$$\lambda = \frac{h}{p}$$

The particles are diffracted by passing through an aperture in a similar manner as light waves.

The wave properties of particles mean that when you confine it in a small space its momentum (and kinetic energy) must increase. (uncertainty principle) This is responsible for the size of the atom.

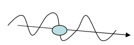
De Broglie Wavelength

Lois De Boglie

Momentum of a photon - inverse to wavelength.

$$p = \frac{E}{c}$$ Einstein's special relativity theory

since $E = \frac{hc}{\lambda}$



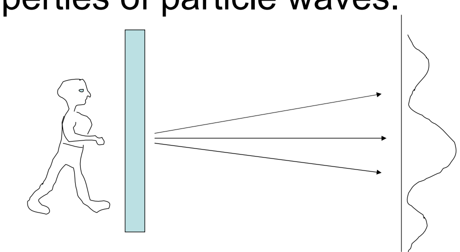
$$p = \frac{h}{\lambda}$$

Wavelength of a particle- inverse to momentum.

$$\lambda = \frac{h}{p}$$

De Broglie proposed that this wavelength applied to material particles as well as for photons. (1924)

properties of particle waves.



Suppose you walk into a room through a doorway.

In the wave picture you will be diffracted.

By a small amount since you are big.
But suppose you can shrink in size. Then the angle will increase.

Big particles- small wavelengths.

Find the De Broglie wavelength of a 100 kg man walking at 1 m/s.

p=mv

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.63\text{x}10^{-34}\text{Js}}{(100\text{kg})(1.0\text{m/s})} = 6.6\text{x}10^{-36}\text{m}$$

For macroscopic momenta the wavelengths are so small that diffraction effects are negligible.

Small particles-large wavelengths

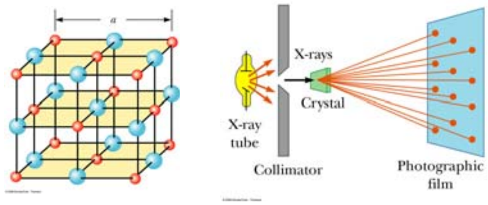
Find the wavelength of an electron traveling at 1.0 m/s (m_e =9.11x10^{-31} kg)

$$\lambda = \frac{h}{mv} = \frac{6.63\text{x}10^{-34}}{(9.11\text{x}10^{-31})(1)} = 7.3\text{x}10^{-4}\text{m} \quad = 0.73\text{ mm}$$

Diffraction effects should be observable for small particles.

The wavelength of the electron can be changed by varying it velocity.

Diffraction of light from crystals



Crystals act as a three-dimensional diffraction grating

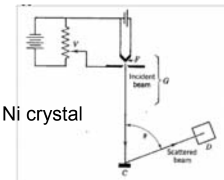
Light with wavelength close to the inter-atomic spacing (x-rays) is diffracted.

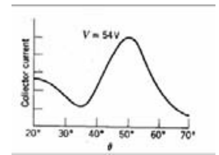
Diffraction of electrons from crystals

Davisson-Germer Experiment (1927)

An electron beam is scattered from a crystal

Ni crystal





The scattered beam shows a diffraction pattern expected for the crystal spacing.

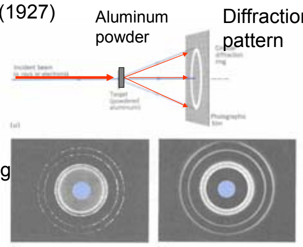
Comparison between electron diffraction and x-ray diffraction

George Thompson (1927)

Aluminum powder

Diffraction pattern

Either x-rays or electrons



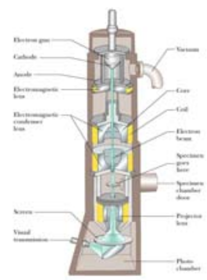
X-ray pattern electron pattern

The electron wavelength was adjusted to the same value as the x-ray by varying the voltage.

The diffraction pattern for x-rays and electrons are very similar.

Material particles have wave properties.

Electron microscopy



$$\lambda = \frac{h}{mv}$$

Shorter wavelength can be obtained by increasing v, the speed of the electron.

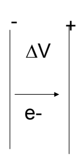
Wavelength in an electron microscope

Suppose an electron microscope accelerated electrons across a potential of 10^4 V. What would the wavelength of the electron be?

$$KE = \frac{1}{2}mv^2 = e\Delta V$$

$$v = \sqrt{\frac{2e\Delta V}{m_e}}$$

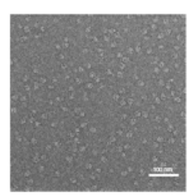
$$\lambda = \frac{h}{mv} = \frac{h}{m}\sqrt{\frac{m}{2e\Delta V}} = h\sqrt{\frac{1}{2me\Delta V}}$$



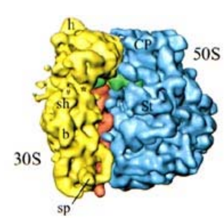
$$\lambda = 6.63x10^{-34}\sqrt{\frac{1}{2(9.11x10^{-31})(1.60x10^{-19})(10^4)}} = 1.22x10^{-11}m$$

0.0122 nm

Electron microscopy



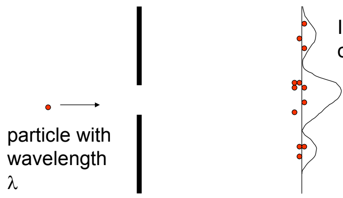
electron microscope pictures of ribosomes



model of ribosome

Electron microscopy can be used to image structures of molecules.

Diffraction of particles Probabilistic Interpretation of the wave amplitude.



Intensity $\propto$ Amplitude^2 of the diffracted wave.

But each particle hits a certain point on the screen

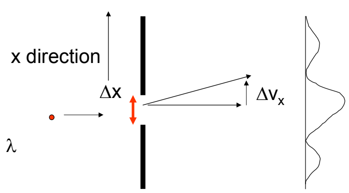
The amplitude^2 is interpreted as the probability of the particle hitting the screen at a certain position
This is true for electrons as well as photons.

Wavefunction

In quantum mechanics the result of an experiment is given in terms of a wavefunction Ψ. The square of the wavefunction Ψ^2 is the probability of the particle being at a certain position.

The wavefunction can be calculated using using the Schrödinger Equation. For instance for electrons in an atom.

Wave property of particles

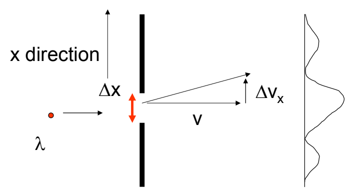


Decreasing the slit reduces Δx
But increases the width of the diffraction, Δv_x

When Δx decreases, Δp_x increases.

Uncertainty Principle

Particle passing through a slit --
The uncertainty in position is Δx
The uncertainty in the x component of momentum is $\Delta p_x = m\Delta v_x$



The particle is diffracted

$$\Delta x \sin\theta = \lambda = \frac{h}{mv}$$

$$\sin\theta = \theta = \frac{\Delta v_x}{v}$$

$$\Delta x \frac{\Delta v_x}{v} = \frac{h}{mv}$$

Therefore

$$\Delta x \Delta p_x = h$$

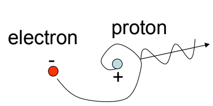
most often written as an inequality

$$\Delta x \Delta p_x \geq \frac{h}{4\pi}$$

The position and velocity cannot be know with unlimited certainty.

The size of an atom

What accounts for the size of the hydrogen atom?



$$PE = -\frac{k_o e^2}{r}$$

PE-> - infinity as r -> zero

Classical electrostatics predicts that the potential energy of the hydrogen atom should go to – infinity

The finite size of the atom is a quantum mechanical effect.

In the quantum limit – when the size of the atom r is comparable to the de Broglie wavelength. the kinetic energy increases with decreasing r due to the uncertainty principle.

Use linear momentum as a rough estimate.

$$\Delta x \Delta p_x \approx h$$

$$\Delta p_x \approx \frac{h}{\Delta x} = \frac{h}{r}$$

r =Δx

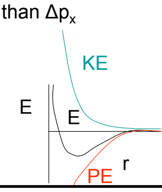
$$\Delta p_x = \frac{h}{2r} \approx p_x$$

p_x cannot be smaller than Δp_x

$$KE = \frac{1}{2} m v_x^2 = \frac{p_x^2}{2m} \approx \frac{h^2}{8r^2 m}$$

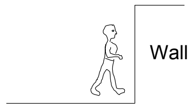
KE increases as $1/r^2$

E= KE+PE goes through a minimum as a function of r

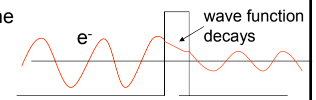


Tunneling across a barrier

a macroscopic object impinging on a barrier the object cannot penetrate within the barrier.

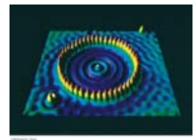


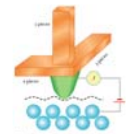
a wave particle impinging on a barrier can penetrate within the barrier for distance. and go through the barrier if it is thin enough.



The probability of tunneling decreases exponentially with the width of the barrier.

Electron Tunneling Microscope

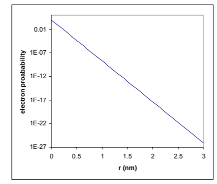




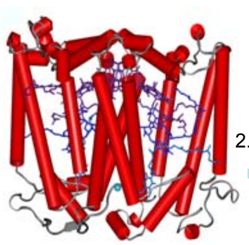
$$\text{Electron\_Probability} \propto |\Psi|^2 \approx \Psi_o^2 e^{-2\alpha r}$$

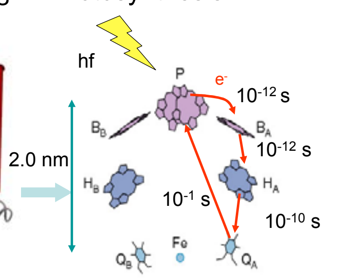
$$\alpha \sim 10\ \text{nm}^{-1}$$

The tunneling probability falls off exponentially with r and is a sensitive function of distance



Tunneling in Photosynthesis





Bacterial Reaction Center

Electron transfer time vs distance

The electron tunnels through the space between groups.
Electron tunneling times are a sensitive function of distance.
The back tunneling is 10^{11} times slower than the first step.