Physics 211B : Problem Set #1

[**1**] Recall that a quantity $F(\boldsymbol{k})$ that is conserved between collisions satisfies

$$\frac{d}{dt}\langle F(\boldsymbol{r})\rangle = \int\limits_{\hat{\Omega}} \frac{d^3k}{(2\pi)^3}\, F(\boldsymbol{k}) \left(\frac{\partial f}{\partial t}\right)_{\boldsymbol{k},\mathrm{coll}} .$$

In this context, we see how the relaxation time approximation,

$$\left(\frac{\partial f}{\partial t}\right)_{\boldsymbol{k},\mathrm{coll}} = -\frac{\delta f(\boldsymbol{r},\boldsymbol{k},t)}{\tau(\boldsymbol{k})} ,$$

where $\delta f = f - f^0$, violates number conservation. To remedy this situation, one can orthogonalize the relaxation approximation collision integral to this collisional invariant. This is the so-called BTK collision integral, first discussed in P. L. Bhatnagar, E. P. Gross, and M. Krook, *Phys. Rev.* **94**, 511 (1954):

$$\left(\frac{\partial f}{\partial t}\right)_{\boldsymbol{k},\mathrm{coll}} = -\frac{f}{\tau} + \frac{\mathrm{Tr}\,\big(f/\tau\big)}{\mathrm{Tr}\,\big(f^0/\tau\big)} \cdot \frac{f^0}{\tau} ,$$

where $\mathrm{Tr} \leftrightarrow \int\limits_{\hat{\Omega}} \frac{d^3k}{(2\pi)^3}$. How would you write down a linearized collision integral which would preserve both particle number and energy?

[**2**] In eqns. 1.91 – 1.97 of the notes, I write the linearized Boltzmann equation with $\boldsymbol{\mathcal{E}} = 0$ in terms of the amplitudes A_{LM} in the different angular momentum channels. Work out the equation corresponding to 1.95 when $\boldsymbol{\mathcal{E}}$ is included. Show that the coefficients of the spherical harmonics all decay to zero except for the case $L = 1$. How do you identify the transport lifetime?

[**3**] Thoroughly study problem #5 from the example problems for chapter 1. Compute the real part of $\sigma_{xx}(\boldsymbol{B},\omega)$ as a function of field amplitude B for the direction $\hat{B}$ and frequency given in the problem, both for Si and Ge. Assume different values for $\omega\tau$ and find the value of τ which best fits the data in each case.

[**4**] The spin-orbit Hamiltonian is

$$H_{\mathrm{SO}} = \frac{\hbar}{4m_{\mathrm{e}}^2c^2}\,\boldsymbol{\sigma} \cdot \boldsymbol{\nabla} V \times \boldsymbol{p} .$$

Write down the Boltzmann equation for scattering within a single band, treated within the effective mass approximation, for a collection of randomly distributed but otherwise identical spin-orbit scatterers. You should *only* consider spin-orbit scattering in this problem (a highly artificial situation) and neglect any potential scattering. You should derive coupled

Boltzmann equations for $\delta f_{\boldsymbol{k}\uparrow}$ and $\delta f_{\boldsymbol{k},\downarrow}$. It will make your life easier if, for each $\boldsymbol{k}$, you choose $\boldsymbol{k}$ as the quantization axis for the spin. Writing

$$\begin{aligned}\delta f_{\boldsymbol{k},\mathrm{s}} &= \tfrac{1}{2}\big(\delta f_{\boldsymbol{k},\uparrow} + \delta f_{\boldsymbol{k},\downarrow}\big)\\ \delta f_{\boldsymbol{k},\mathrm{a}} &= \tfrac{1}{2}\big(\delta f_{\boldsymbol{k},\uparrow} - \delta f_{\boldsymbol{k},\downarrow}\big)\ ,\end{aligned}$$

and then taking

$$\delta f_{\boldsymbol{k},\alpha} = \sum_{L,M} A_{LM\alpha}(k,t)\, Y_{LM}(\hat{\boldsymbol{k}})\ ,$$

where $\alpha =$ s or a, derive equations for the rate of change of the coefficients $A_{LM\alpha}$ in the absence of any external fields.