Physics 222 UCSD/225b UCSB
Winter 2010

Final Exam

This take home final is due on Tuesday, March 16th, 2pm, in Bernie Camberos' office.

This is completely open book. You may use anything except the solution sets to lectures taught by Claudio Campagnari years ago.

However, you must reference anything you do use to solve these problems!

Point Assignment

Problem 1 = 10 points

Problem 2 = 10 points

Problem 3 = 20 points

Problem 4 = 20 points

Problem 5 = 20 points

Problem 6 = 20 points

Problem 1

(a) Calculate numerical values for the ratios

$$\Gamma(\mathrm{B}^+ \to \tau^+\nu_\tau) \quad : \quad \Gamma(\mathrm{B}^+ \to \mu^+\nu_\mu) \quad : \quad \Gamma(\mathrm{B}^+ \to \mathrm{e}^+\nu_\mathrm{e})$$

(b) Estimate the ratio

$$\Gamma(\tau^- \to \mathrm{K}^-\nu_\tau) \quad : \quad \Gamma(\tau^- \to \pi^-\nu_\tau)$$

State your assumptions, if any. You may look up and use any useful formula and/or numerical value that you might need.

Problem 2

Horizontal vector bosons are neutral bosons that couple quarks and leptons across generations. For example, such bosons, if they existed, could couple, at the same vertex, e and μ, or s and d, etc. Use information from the PDG to do back-of-the-envelope estimates of lower limits on the masses of such bosons.
Do these by comparing limits for (unobserved) processes that would be mediated by these bosons with branching ratios for processes mediated by the W. For example: $K_L \to \mu e$ vs. $K^+ \to \mu\nu$

For the sake of argument, assume that the coupling to such a horizontal boson is the same as the weak coupling. Make sure to draw all Feynman diagrams and to explain your assumptions.

Horizontal bosons arise naturally in (extended) technicolor theories. These are classes of theories where the EWK symmetry is broken dinamically without resorting to the Higgs mechanism. These theories were quite popular some time ago, but they have now fallen out of fashion.

Problem 3

Show that if the Standard Model contained several Higgs multiplets with weak isospins T_i and hypercharges Y_i whose neutral states acquire vev's v_i, then the ρ parameter would be

$$\rho = \frac{M_W^2}{\cos^2\theta_W M_Z^2} = \frac{\sum v_i^2[T_i(T_i+1)-(Y_i/2)^2]}{2\sum v_i^2(Y_i/2)^2}$$

Note: this is the same as Problem 15.4 in Halzen and Martin. Do not use the result of equation 15.27 without proving it first.

Problem 4

Suppose we took Φ in equation 15.14 of Halzen and Martin to be a triplet representation of SU(2). In this representation the SU(2) generators are

$$\mathrm{T}_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad \mathrm{T}_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad \mathrm{T}_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

(a) Take

$$\Phi_0 = \begin{pmatrix} 0 \\ 0 \\ v \end{pmatrix}$$

Assign the hypercharge such that Φ_0 is neutral. Calculate $\mathrm{M_W}/\mathrm{M_Z}$ in this variant of the model.

(b) Now take

$$\Phi_0 = \begin{pmatrix} 0 \\ v \\ 0 \end{pmatrix}$$

Show that only the charged weak bosons acquire mass in this case.

Problem 5

- Consider the 2HDM type-I.

a) Specify the conditions within this model for which the low mass CP-even higgs does not couple to fermions at tree level without changing the coupling hWW nor hZZ from their standard model values. Pick for the remainder of this problem these parameters as your type-I model.

b) In class, we talked about higgs production mechanisms at a hadron collider. How are they modified in this model? Discuss the change in higgs production in this model versus the standard model for m_h = 110GeV to 200GeV.

c) Go through the higgs decay modes that we discussed in class, and discuss the changes in this model as compared to the standard model.

d) Consider for comparison a 2HDM type-II where only the coupling to down-type quarks is turned off but hWW, hZZ and couplings to up-type quarks is unchanged. What are the relevant values for the 2HDM parameters in this case? Compare the search strategies for h in these two models for m_h = 115 - 200GeV. Which of the two types of higgs is easier to find at a hadron collider? Why? Is either one of these easier to find than the standard model higgs? Why, or why not?

Problem 6:

Consider using polarized electrons in the process $e^+e^- \to Z \to f\bar{f}$, where f is a fermion assumed to be massless.

(a) Draw the allowed helicity configurations for the process. For each configuration, give the dependence of the amplitude on the fermion scattering angle θ with respect to the initial momentum of the electron. Include the relevant **relative** weak coupling parameters.

(b) Compute the relative total cross-sections for each initial e^+e^- helicity configuration, assuming that the polarization of the final state fermions is not measured.

(c) Compute the left-right asymmetry

$$A_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L - \sigma_R}$$

where σ_L and σ_R are the total cross-sections for left and right handed electrons respectively. **Do not calculate the Feynman diagrams explicitely. Just use helicity arguments and the fermion couplings to the Z.**

(d) The SLD collaboration at SLAC has published the result $A_{LR} = 0.1510 \pm 0.0025$. From this measurement, extract $\sin^2\theta_W$, including errors. This is the most precise single measurement of $\sin^2\theta_W$.