Physics 222 UCSD/225b UCSB

Lecture 11

Spontaneous Symmetry Breaking

Discussion of what we've done.

- We studied a simple scalar field with a potential that has mirror symmetry.
- We found that this mirror symmetry can be broken by the ground state if we choose the traditional mass term in the Lagrangian ($\mu^2 \phi^2$) to have a non-traditional sign ($-|\mu|^2 \phi^2$) .
- To be able to do perturbation theory around the ground state, we introduce a new field, η, that is zero in the ground state.
- We find that the field η acquires a traditional mass term with the correct sign, and the new Lagragian is no longer exhibiting mirror symmetry in $+-\eta$.

Reminder of Lagrangians

- Original Lagrangian:

$$L = \frac{1}{2}\left(\partial_\mu \phi\right)^2 + \frac{1}{2}|\mu|^2 \phi^2 - \frac{1}{4}\lambda\phi^4 + O(\phi^6)$$

  – We made the minus sign of μ explicit here.

- Lagrangian around ground state:

$$L = \frac{1}{2}\left(\partial_\mu \eta\right)^2 \boxed{- \lambda v^2 \eta^2} - \lambda v \eta^3 - \frac{1}{4}\lambda\eta^4 + const$$

Repeat the Procedure for U(1) global phase symmetry.

$$L = \left(\partial_\mu \phi\right)^* \left(\partial^\mu \phi\right) - \mu^2 \phi^* \phi - \lambda(\phi^* \phi)^2$$

Complex scalar field for the global phase symmetry: $\phi = \dfrac{\phi_1 + i\phi_2}{\sqrt{2}}$

$$L = \frac{1}{2}\left(\partial_\mu \phi_1\right)^2 + \frac{1}{2}\left(\partial_\mu \phi_2\right)^2 - \frac{1}{2}\mu^2(\phi_1^2 + \phi_2^2) - \frac{1}{4}\lambda(\phi_1^2 + \phi_2^2)^2$$

The ground state now describes a circle in the ϕ_1 - ϕ_2 plane.

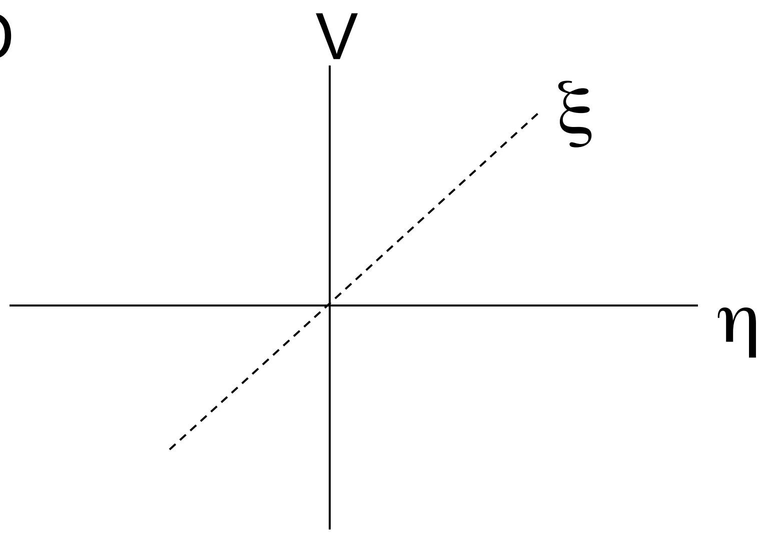
$$\phi_1^2 + \phi_2^2 = v^2 = \frac{-\mu^2}{\lambda}$$

Next we will again translate the field to its minimum energy, and rewrite the Lagrangian accordingly.

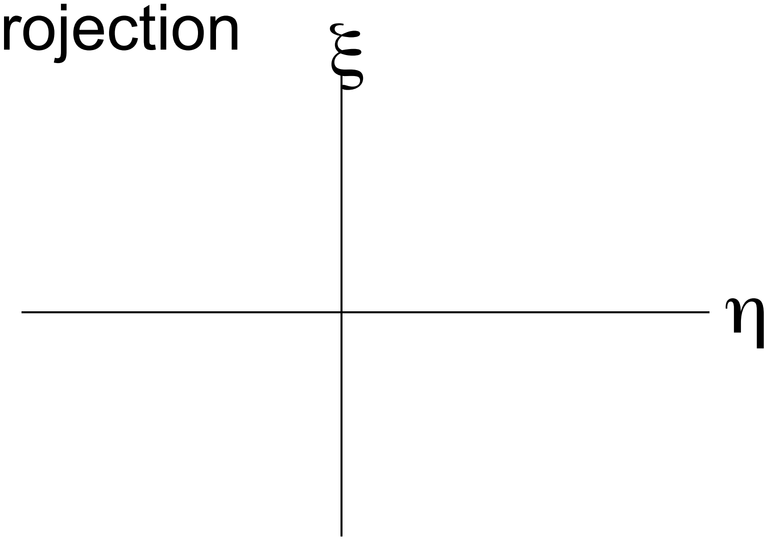
Aside on minimum

- In this 2d case, the minimum is a circle in the complex plane of the fields η and ξ.
- We need to pick a concrete point around which to write down L in terms of the new fields η and ξ.
  - For convenience, we pick the point $(\eta , \xi) = (\eta , 0)$
- We expect the circular symmetry to still be present!

3D



2D projection



Lagrangian around minimum

$$\phi(x) = \sqrt{\frac{1}{2}}\left[v + \eta(x) + i\xi(x)\right]$$

$$L = \frac{1}{2}\left(\partial_\mu \xi\right)^2 + \frac{1}{2}\left(\partial_\mu \eta\right)^2 + \mu^2\eta^2 + O(\xi^3) + O(\eta^3) + const$$

Note the presence of an η mass term and the absence one for ξ .

The origin of this "Goldstone boson" is clearly the flatness of the potential in the ξ direction, or generally tangential to the circle that defines the minimum.
I.e. **V is independent of ξ near its minimum at (η , ξ) = (η , 0).**
Note: this does not depend on our choice of (η , ξ).
Any choice will have some direction for which V is flat!

Local U(1) Phase Symmetry -- The Higgs Mechanism --

$$L = \left(\partial^{\mu} + ieA^{\mu}\right)\phi^{*}\left(\partial_{\mu} - ieA_{\mu}\right)\phi - \mu^{2}\phi^{*}\phi - \lambda(\phi^{*}\phi)^{2} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

Now do the same as before: $\phi(x) = \sqrt{\frac{1}{2}}\left[v + \eta(x) + i\xi(x)\right]$

And you get:

$$L = \frac{1}{2}\left(\partial_{\mu}\xi\right)^{2} + \frac{1}{2}\left(\partial_{\mu}\eta\right)^{2} - \lambda v^{2}\eta^{2} + \frac{1}{2}e^{2}v^{2}A_{\mu}A^{\mu} - evA_{\mu}\partial^{\mu}\xi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \ldots$$

Doing it this way would be a bit tedious.
Let's be a little smarter than this.

Local U(1) Phase Symmetry -- The Higgs Mechanism --

$$L = \left(\partial^{\mu} + ieA^{\mu}\right)\phi^{*}\left(\partial_{\mu} - ieA_{\mu}\right)\phi - \mu^{2}\phi^{*}\phi - \lambda(\phi^{*}\phi)^{2} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

Now don't do the same as before, but rather be a little bit more clever:

$$\phi(x) = \sqrt{\frac{1}{2}}\left[v + h(x)\right]e^{i\theta(x)/v}$$

$v, h(x)$ are real and positive.

This is more clever because we know already that the Lagrangian with A added obeys local Gauge symmetry, if A is transformed accordingly:

$$A^{\mu} \rightarrow A^{\mu} + \frac{1}{ev}\partial^{\mu}\theta$$

We thus know that L is independent of θ !!!

What terms do we expect?

Higgs self-coupling:

mass

$$\left.\begin{array}{l} -\mu^2\phi^*\phi \rightarrow -\dfrac{1}{2}\mu^2(v+h)^2 \\ -\lambda(\phi^*\phi)^2 \rightarrow -\lambda\dfrac{1}{4}(v+h)^4 \end{array}\right\} \rightarrow -\lambda v^2 h^2 - \lambda v h^3 - \frac{1}{4}\lambda h^4$$

Triple & quartic higgs coupling

Kinetic Energy:

$$\partial^\mu \phi^* \partial_\mu \phi \rightarrow \frac{1}{2}\partial^\mu h \partial_\mu h = \frac{1}{2}(\partial_\mu h)^2$$

Note: We dropped the
Gauge terms.

What terms do we expect?

Massive Gauge Boson

$$e^2 A^\mu A_\mu \phi^* \phi \rightarrow \frac{1}{2} e^2 A^\mu A_\mu (v+h)^2 \rightarrow \frac{1}{2} e^2 \left[v^2 A^\mu A_\mu + h^2 A^\mu A_\mu \right] + e^2 v h A^\mu A_\mu$$

Quartic coupling of hh to VV.

Associative production
And Higgs->WW,ZZ decay.

Finally, terms like this cancel:

$$-ie\phi A_\mu \partial^\mu \phi^* + ie\phi A^\mu \partial_\mu \phi \rightarrow A^\mu \partial_\mu h \quad terms \quad cancel$$

Complete Lagrangian

$$L = \frac{1}{2}(\partial_\mu h)^2 - \lambda v^2 h^2 - \lambda v h^3 - \frac{1}{4}\lambda h^4$$

$$+ \frac{1}{2}e^2 v^2 A^\mu A_\mu + \frac{1}{2}e^2 h^2 A^\mu A_\mu + e^2 v h A^\mu A_\mu$$

$$- \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

Summary so far

- We have shown that we can break the U(1) local phase symmetry spontaneously such that its Gauge boson becomes massive.
- This introduces one new massive scalar particle, the higgs boson.
- It also introduces triple and quartic higgs self coupling, as well as h<->VV and hh<->VV couplings.

What's left to do?

- In nature, the photon is massless while the W+ W- Z are massive.
- We thus need to apply the higgs mechanism to $SU(2)_L$ instead of U(1).
- So let's repeat the exercise one more time!

Higgs & SU(2)

$$L = \left(\partial_\mu \phi\right)^{*T}\left(\partial^\mu \phi\right) - \mu^2 \phi^{*T}\phi - \lambda(\phi^{*T}\phi)^2$$

$$\phi = \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix} \qquad \phi \rightarrow e^{i\alpha_a(x)\tau_a/2}\phi$$

3 Pauli matrices for a=1,2,3

This is a local phase, coupled with a "rotation" in the space SU(2) acts upon. The local symmetry transformation is thus significantly more involved.

SU(2) Gauge Fields

$$\partial_\mu \rightarrow \partial_\mu + ig\frac{\tau_a}{2}W_\mu^a$$

$$W_\mu^a \rightarrow W_\mu^a - \frac{1}{g}\partial_\mu \alpha - \left(\alpha \times W_\mu^a\right)$$

SU(2) "rotation"

The W fields are the same fields W^+, W^-, W^3 that we encountered earlier in chapter 13.

The field tensor to build the kinetic energy term is given as:

$$W_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu - \boxed{g W_\mu \times W_\nu}$$

Needed because the generators of SU(2) don't commute, i.e. non-Abelian group.

Gauge invariant Lagrangian

$$L = \left(\partial_\mu \phi + ig\frac{\boldsymbol{\tau}_a}{2}W_\mu^a\right)^{*T}\left(\partial^\mu \phi + ig\frac{\boldsymbol{\tau}_a}{2}W_\mu^a\right)$$

$$-\mu^2\phi^{*T}\phi - \lambda(\phi^{*T}\phi)^2$$ ← Higgs potential

$$-\frac{1}{4}W_{\mu\nu}W^{\mu\nu}$$ ← Kinetic Energy of gauge fields

For $\mu^2 > 0$ this describes a set of 4 massive scalar fields, interacting with 3 massless Gauge fields.

Taking $\mu^2 < 0$

- Minima of the higgs potential on the SU(2) invariant manifold defined by:

$$\phi^{*T}\phi = \frac{1}{2}\left(\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2\right) = \frac{-\mu^2}{2\lambda}$$

As usual, we now pick a specific point on this manifold, and rewrite the Lagrangian in terms of fields around that minimum.

For simplicity, we pick a point where $\phi_3 = v \neq 0$.

$$\phi(x) = \sqrt{\frac{1}{2}}\begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$$

Gauge Transformation of Scalar

$$\phi(x) = \sqrt{\frac{1}{2}}\begin{pmatrix} 0 \\ v + h(x) \end{pmatrix} \rightarrow \sqrt{\frac{1}{2}}\begin{pmatrix} 0 \\ v + h(x) \end{pmatrix} e^{i\tau\theta(x)/v}$$

As before, we argue that by construction the Lagrangian is independent of local Gauge transformations.

Masses of Gauge Fields

- As before, the masses of the Gauge fields come from the product term of Gauge fields and scalar field:

$$\left| ig\frac{\tau_a}{2}W_\mu^a\phi \right|^2 = \frac{g^2}{8}\left| \begin{pmatrix} W_\mu^3 & W_\mu^1 - iW_\mu^2 \\ W_\mu^1 + iW_\mu^2 & W_\mu^3 \end{pmatrix}\begin{pmatrix} 0 \\ v \end{pmatrix} \right|^2$$

$$= \frac{g^2v^2}{8}\left(\left(W_\mu^1\right)^2 + \left(W_\mu^2\right)^2 + \left(W_\mu^3\right)^2 \right)$$

$$\Rightarrow M_W = \frac{1}{2}gv$$

Summary

- We have shown how spontaneous symmetry breaking gives masses to the Gauge bosons, while adding a new scalar particle, the Higgs boson to the theory.
    - As an aside, we get concrete predictions for the existence of couplings involving higgs and gauge bosons.
- We have not shown that this leads to a renormalizable theory in the end.
- The proof of this is beyond the scope of this course.
- Handwaving, we might find it plausible because at high energies, i.e. energies we ran into trouble in our loops before, the hidden symmetry ought to reveal itself, restoring the massless nature of the Gauge bosons, and thus making the theory renormalizable.