Physics 222 UCSD/225b UCSB

Lecture 12

Chapter 15: The Standard Model of EWK Interactions

A large part of today's lecture is review of what we have already done, just putting it all together in one place.

Summary so far

- In Chapter 13, we learned how $SU(2)_L$ x $U(1)$ unifies Weak and Electromagnetic interactions.
  - One triplet of currents, (W^+, W^-, W^3), plus one singlet, (B), mix in such a way as to produce the physical vector bosons W^+, W^-, Z, and photon.
- In Chapter 14, we learned
  - about Lagrangian formalism, and how imposing local phase invariance, also called Gauge Invariance, naturally leads to the Gauge bosons, and the interaction terms in the Lagrangian, and thus the "forces" in the theory.
  - How spontaneous symmetry breaking can give mass to the Gauge bosons, and predict the existence of a neutral scalar Higgs boson, as well as its interactions.
- In Chapter 15, we bring all the pieces together to arrive at the Standard Model of EWK interactions.

QED and Local U(1)

- Free Lagrangian: $L = i\overline{\psi}\gamma_\mu\partial^\mu\psi - m\overline{\psi}\psi$

- Gauge Transformation:

$$\psi(x) \to e^{i\alpha(x)Q}\psi(x) \qquad A^\mu \to A^\mu + \frac{1}{e}\partial^\mu\alpha$$

- Complete Lagrangian that is Gauge Invariant:

$$L = \underbrace{\overline{\psi}(i\gamma_\mu\partial^\mu - m)\psi}_{kinetic\ energy\ and\ mass\ of\ \psi} + \underbrace{e\overline{\psi}\gamma_\mu\psi A^\mu}_{Interaction} - \underbrace{\frac{1}{4}F^{\mu\nu}F_{\mu\nu}}_{kinetic\ energy\ of\ A_\mu}$$

Electroweak and SU(2)$_L$ x U(1)$_Y$

- The generators: T, Y
- Fermions and Gauge Transformations:

$$\chi_L \rightarrow e^{i\alpha(x)\cdot T + i\beta(x)\cdot Y}\chi_L$$

$$\psi_R \rightarrow e^{i\beta(x)\cdot Y}\psi_R$$

$$\chi_L = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix} \quad \text{With T = 1/2 ; Y = -1}$$

$$\psi_R = e_R^- \quad \text{With T = 0 ; Y = -2}$$

And for quarks:

$$\chi_L = \begin{pmatrix} u \\ d \end{pmatrix} \quad \text{With T = 1/2 ; Y = 1/3}$$

$$\psi_R = u_R \quad or \quad d_R \quad \text{With T = 0 ; Y = 4/3 and -2/3}$$

Electroweak Currents

- Currents of SU(2)$_L$:

$$\tau_+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}; \tau_- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

This triplet, J$^+$,J$^-$,J^3 of currents are typically written as a 3-component vector of currents.

$$J_\mu^\pm(x) = \overline{\chi}_L \gamma_\mu \tau_\pm \chi_L$$

$$J_\mu^3(x) = \overline{\chi}_L \gamma_\mu \frac{1}{2} \tau_3 \chi_L = \frac{1}{2} \overline{\nu}_L \gamma_\mu \nu_L - \frac{1}{2} \overline{e}_L \gamma_\mu e_L$$

- Current of U(1)$_Y$: $j_\mu^Y = \overline{\psi} \gamma_\mu Y \psi$

Interaction Terms for SU(2)$_L$ x U(1)$_Y$

- The general idea is to multiply:

  (coupling) x (current) x (gauge boson field)

$$-ig\overrightarrow{J_\mu}\overrightarrow{W^\mu} = -ig\overline{\chi}_L\gamma_\mu\vec{\tau}\overrightarrow{W^\mu}\chi_L$$

The arrows indicate the 3-vector nature of currents and Gauge fields.

$$-i\frac{g'}{2}j_\mu^Y B^\mu = -i\frac{g'}{2}\overline{\psi}\gamma_\mu Y\psi B^\mu$$

Next, we mix the neutral components of these two currents to get the physical photon and Z fields.

Mixing of Neutral Currents

$$A_\mu = W_\mu^3 \sin\theta_W + B_\mu \cos\theta_W$$

$$Z_\mu = W_\mu^3 \cos\theta_W - B_\mu \sin\theta_W$$

$$-ig\left(J^3\right)^\mu W_\mu^3 - i\frac{g'}{2}\left(J^Y\right)^\mu B_\mu =$$

$$= -i\left(g\sin\theta_W J_\mu^3 + g'\cos\theta_W \frac{J_\mu^Y}{2}\right)A^\mu$$

$$-i\left(g\cos\theta_W J_\mu^3 - g'\sin\theta_W \frac{J_\mu^Y}{2}\right)Z^\mu$$

The physical interactions are then given by:

$$\boxed{= -iej_\mu^{em}A^\mu - \frac{ie}{\sin\theta_W \cos\theta_W}\left(J_\mu^3 - \sin^2\theta_W j_\mu^{em}\right)Z^\mu}$$

Lagrangian

- Now we are in a position to write down the Gauge invariant EWK Lagrangian for massless fields.
- Let's do this for the concrete case of just electron and its neutrino:

$$L_1 = \bar{\chi}_L \gamma^\mu \left[i\partial_\mu - g \frac{1}{2} \vec{\tau}\vec{W}_\mu - g'\left(\frac{-1}{2}\right) B_\mu \right] \chi_L$$

$$+ \bar{e}_R \gamma^\mu \left[i\partial_\mu - g'(-1) B_\mu \right] e_R$$

$$- \frac{1}{4} \left[W_{\mu\nu} W^{\mu\nu} + B_{\mu\nu} B^{\mu\nu} \right]$$

Where we have used the hypercharge quantum numbers.

Aside on Ekin of Gauge fields

- The field tensors depend on the symmetry group.
- This means we get:

this comes from
$ig[W_\mu, W_\upsilon]$

$$W_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu \boxed{- g W_\mu \times W_\nu}$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

Next note that the commutator part in: $W_{\mu\nu} W^{\mu\nu}$
Leads to triple and quartic gauge couplings.

Aside on triple & quartic gauge couplings

- The cross product guarantees that no ZZZ, ZZγ, or Z$\gamma\gamma$ couplings exist in SM.

- However, the corresponding quartic couplings: WWZZ, WWZγ, and WW$\gamma\gamma$ do exist following the same arguments.

The Gauge Group Structure determines the allowed couplings among the gauge bosons.
The coupling values are determined furthermore by the gauge mixing between W$_3$ and B.

Higgs Mechanism

- Finally, we are ready to add mass to the Gauge fields via the higgs mechanism.

$$L_2 = \left\| \left(i\partial_\mu - g\vec{\tau}\overrightarrow{W_\mu} - g'\left(\frac{Y}{2}\right)B_\mu \right)\phi \right\|^2 - V(\phi)$$

All we need to do is choose a higgs field and an appropriate minimum of the higgs potential.

Higgs Field

- There are in principle many choices one could make.
- The only constraint is that the higgs fields belong to some multiplet of SU(2) x U(1).
- The simplest choice is a complex doublet with Y = 1.
  - Complex for U(1)
  - Doublet for SU(2)

$$\begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix} \frac{1}{\sqrt{2}}$$

The superscript indicate the charge according to:

$Q = T^3 + Y/2$

Minimum of Higgs Potential

- This particular choice of multiplets is exactly what we need because it allows us to break both SU(2) and U(1)$_Y$, while at the same time allowing us to choose a ground state that leaves U(1)$_{em}$ unbroken.

- The latter is accomplished by choosing a ground state that leaves ϕ^+ = 0.

$$\begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \begin{pmatrix} 0 \\ v \end{pmatrix} \frac{1}{\sqrt{2}}$$

Aside

- Given that we clearly want to conserve charge symmetry, we really didn't have much choice in the ground state for the higgs potential.

- It had to be something where only the neutral scalar receives a vacuum expectation value.

What's left to show:

- Show how W,Z receive mass via the higgs mechanism.

- Show how the fermions receive mass via the higgs mechanism.

- Let's start with the massive gauge bosons.

Mass of W,Z

$$\left|\left(-ig\frac{\tau_a}{2}W_\mu^a - i\frac{g'}{2}B_\mu\right)\phi\right|^2 = \frac{1}{8}\left|\begin{pmatrix} gW_\mu^3 + g'B_\mu & g\left(W_\mu^1 - iW_\mu^2\right) \\ g\left(W_\mu^1 + iW_\mu^2\right) & -gW_\mu^3 + g'B_\mu \end{pmatrix}\begin{pmatrix} 0 \\ v \end{pmatrix}\right|^2$$

$$= \frac{g^2v^2}{8}\left(\left(W_\mu^1\right)^2 + \left(W_\mu^2\right)^2\right) + \frac{1}{8}v^2\left(W_\mu^3, B_\mu\right)\begin{pmatrix} gg & -gg' \\ -gg' & g'g' \end{pmatrix}\begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix}$$

Mass term for charged W

Off diagonal in neutral fields.
Eigenvalues are:
  0 for the photon
  $M_Z^2 = 1/4\ v^2\ (g^2 + g'^2)$

Mass of W,Z (2)

$M_Z^2 = 1/4\ v^2\ (g^2 + g'^2)$
$M_W^2 = 1/4\ v^2\ g^2$

In chapter 13, we learned:

$$g \sin\theta_W = g' \cos\theta_W = e$$

And thus:
$$\frac{M_W}{M_Z} = \frac{g}{\sqrt{g^2 + g'^2}} = \frac{1}{\sqrt{1 + \tan^2\theta_W}} = \cos\theta_W$$

Or, putting it differently:
$$\rho = \frac{M_W^2}{M_Z^2 \cos^2\theta_W} = 1$$

Summary on M_W, M_Z

- In chapter 13, we already showed that:

  g'/g = tanθ$_W$

- This was a direct consequence of getting back the EM current from the mixing of the two neutral Gauge bosons.

- In addition, we had a relationship between the strength of the neutral and charged weak current:

$$\rho = \frac{M_W^2}{M_Z^2 \cos^2\theta_W} \ \mathbf{= 1}$$

This value is predicted by the choice of higgs multiplets.

(see H&M problem 15.4)

Fermion Masses

- The Lagrangian that deals with Fermion masses is as follows:

$$L_3 = -G_e \left[\left(\overline{v}_e, \overline{e} \right)_L \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} e_R + \overline{e}_R \left(\phi^-, \overline{\phi^0} \right) \begin{pmatrix} v_e \\ e \end{pmatrix}_L \right]$$

- The higgs has exactly the right quantum numbers to couple to both right and left handed fermions, within a family.

- There's then one such term for each family of leptons. We'll get to quarks in a sec.

Lepton Masses

- After symmetry breaking, we find:

$$L_3 = -m_e \bar{e}e - \boxed{\frac{m_e}{v}\bar{e}eh}$$

Where we defined: $\quad m_e = \dfrac{G_e v}{\sqrt{2}}$

h -> e$^+$e$^-$ coupling

For every lepton, we have an arbitrary constant G. The lepton mass is thus not determined. However, the mass is related to the coupling to the higgs. This can thus be tested in principle by measuring the higgs partial width into each lepton species.

Quark Masses

- We proceed in two steps.
    - First, we show how up type quarks are treated as compared to down type quarks.
    - Second, we show how the CKM matrix relates to the higgs sector.
- The down type quarks receive identical treatment to the charged leptons if we ignore CKM for the moment.

Higgs doublet for up type quarks

- Recall that the ground state enforced that both neutrinos and photons remain massless.

$$\begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \begin{pmatrix} 0 \\ v \end{pmatrix} \frac{1}{\sqrt{2}}$$

- Use SU(2) properties to form a related higgs doublet:

$$\phi_c = -i\tau_2\phi^* = \begin{pmatrix} -\overline{\phi}^0 \\ \phi^- \end{pmatrix} \rightarrow \begin{pmatrix} v \\ 0 \end{pmatrix} \frac{1}{\sqrt{2}}$$

Higgs Lagrangian for Quarks

- Ignoring CKM we then get:

$$L_4 = -G_d \left(\bar{u}, \bar{d}\right)_L \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} d_R - G_u \left(\bar{u}, \bar{d}\right)_L \begin{pmatrix} -\bar{\phi}^0 \\ \phi^- \end{pmatrix} u_R + h.c.$$

$$L_4 = -m_d \bar{d}d - m_u \bar{u}u - \frac{m_d}{v}\bar{d}dh - \frac{m_u}{v}\bar{u}uh$$

Let's look at the structure of this for 3 generations.

$$\left(\left(\bar{u}, \bar{d}\right)_L, \left(\bar{c}, \bar{s}\right)_L, \left(\bar{t}, \bar{b}\right)_L\right) C_q \phi \begin{pmatrix} d \\ s \\ b \end{pmatrix}_R + \left(\left(\bar{u}, \bar{d}\right)_L, \left(\bar{c}, \bar{s}\right)_L, \left(\bar{t}, \bar{b}\right)_L\right) C_q' \phi_c \begin{pmatrix} u \\ c \\ t \end{pmatrix}_R$$

We now have the freedom to rotate the quark fields by some arbitrary 3x3 unitary matrix.

Freedom to rotate Quark fields

$$\left(\left(\bar{u}, \bar{d} \right)_L , \left(\bar{c}, \bar{s} \right)_L , \left(\bar{t}, \bar{b} \right)_L \right) U_L C_q \phi D_R \begin{pmatrix} d \\ s \\ b \end{pmatrix}_R + \left(\left(\bar{u}, \bar{d} \right)_L , \left(\bar{c}, \bar{s} \right)_L , \left(\bar{t}, \bar{b} \right)_L \right) U_L C'_q \phi_c U_R \begin{pmatrix} u \\ c \\ t \end{pmatrix}_R$$

***This freedom allow me to diagonalize
either C_q or C_q' , but not both!***

$$C_q \rightarrow U_L C_q D_R$$

$$C'_q \rightarrow U_L C'_q U_R$$

The fact that the left handed doublet is involved in giving
mass to both the up and down type quarks means that we
can not make both mass matrices diagonal at the same time.
We are left with a CKM mixing in either the up or down sector.

EWK Standard Model Lagrangian

$$-\frac{1}{4}\left[W_{\mu\nu}W^{\mu\nu} + B_{\mu\nu}B^{\mu\nu}\right]$$

Kinetic energy of W,Z,γ and interactions among gauge bosons.

$$\bar{\chi}_L\gamma^\mu\left[i\partial_\mu - g\frac{1}{2}\vec{\tau}\overrightarrow{W_\mu} - g'\left(\frac{-1}{2}\right)B_\mu\right]\chi_L$$

$$+ \bar{e}_R\gamma^\mu\left[i\partial_\mu - g'(-1)B_\mu\right]e_R$$

Lepton & quark kin. E. and their interactions with W,Z,γ

$$\left|\left(i\partial_\mu - \frac{1}{2}g\vec{\tau}\overrightarrow{W_\mu} - g'\left(\frac{Y}{2}\right)B_\mu\right)\phi\right|^2 - V(\phi)$$

Mass of W,Z,γ ,h and higgs interactions with vector bosons.

$$-\bar{L}(CKM)\phi R - \bar{L}\phi_C R$$

Lepton & quark masses, flavor mixing, fermion higgs couplings.

Summary

- The higgs mechanism in general allows us to give mass to W,Z while the photon remains massless.
- The minimal higgs doublet furthermore predicts:
  - Relationship between M_W ,M_Z , and the photon-Z mixing angle.
  - Relationships between all fermion masses and their couplings to the higgs.
  - hVV and hhVV couplings that are related to the respective M_V
  - Higgs self-couplings hhh and hhhh that are related to the higgs mass.
- Additional higgs doublets may lead to different predictions, in addition to adding additional physical higgs states.