Physics 222 UCSD/225b UCSB

Lecture 5

Mixing & CP Violation (2 of 3)

Today we walk through the formalism in more detail, and then focus on CP violation

Nomenclature

(These notational conventions are different from Jeff Richman's paper)

- We refer to the decays of a "pure" flavor state:

$$\langle f | B^0 \rangle = A \qquad \langle \bar{f} | B^0 \rangle = 0 \qquad \langle f_{CP} | B^0 \rangle = A$$

$$\langle \bar{f} | \overline{B^0} \rangle = \bar{A} \qquad \langle f | \overline{B^0} \rangle = 0 \qquad \langle f_{CP} | \overline{B^0} \rangle = \bar{A}$$

- The time evolution of a state that was a "pure" flavor state at t=0:

$$\langle f | H | B^0 \rangle = \langle f | B^0(t) \rangle \qquad \langle \bar{f} | H | B^0 \rangle = \langle \bar{f} | B^0(t) \rangle \qquad \langle f_{CP} | H | B^0 \rangle = \langle f_{CP} | B^0(t) \rangle$$

$$\langle \bar{f} | H | \overline{B^0} \rangle = \langle \bar{f} | \overline{B^0(t)} \rangle \qquad \langle f | H | \overline{B^0} \rangle = \langle f | \overline{B^0(t)} \rangle \qquad \langle f_{CP} | H | \overline{B^0} \rangle = \langle f_{CP} | \overline{B^0(t)} \rangle$$

Unmixed

Mixed

Can't tell because f is not flavor specific

Remember from last week

We have: mass eigenstates = B_H and B_L

flavor eigenstates = B^0 and $\overline{B^0}$

CP eigenstates = B_+ and B_-

Let's first set $|\Gamma_{12}/M_{12}| = 0$:

Define q, p via:

$$\begin{aligned} B_H &= p\,|B^0\rangle + q\,|\overline{B^0}\rangle \\ B_L &= p\,|B^0\rangle - q\,|\overline{B^0}\rangle \end{aligned} \quad \Rightarrow \quad \frac{q}{p} = +\frac{M_{12}^*}{|M_{12}|}$$

Define CP eigenstates:

$$\begin{aligned} CP|B_+\rangle &= +|B_+\rangle \\ CP|B_-\rangle &= -|B_-\rangle \end{aligned} \quad \Rightarrow \quad \begin{aligned} B_+ &= \tfrac{1}{\sqrt{2}}\,|B^0\rangle - \tfrac{1}{\sqrt{2}}\,|\overline{B^0}\rangle \\ B_- &= \tfrac{1}{\sqrt{2}}\,|B^0\rangle + \tfrac{1}{\sqrt{2}}\,|\overline{B^0}\rangle \end{aligned}$$

Where we have used that B^0 is a pseudoscalar meson.

Mixing

Probability for meson to keep its flavor:

$$
\begin{aligned}
|\langle f|H|B^0\rangle|^2 &= |\langle f|B^0(t)\rangle|^2 \\
&= \tfrac{1}{4|p|^2}|\langle f|B_L(t)\rangle + \langle f|B_H(t)\rangle|^2 \\
&= \frac{1}{4|p|^2}|pAe^{(-im_L-\Gamma_L/2)t} + pAe^{(-im_H-\Gamma_H/2)t}|^2 \\
&= \tfrac{1}{4}|A|^2(e^{-\Gamma_L t} + e^{-\Gamma_H t} + 2e^{-(\Gamma_H+\Gamma_L)t/2}\cos\Delta m t)
\end{aligned}
$$

Probability for meson to switch flavor:

$$
\begin{aligned}
|\langle \bar{f}|H|B^0\rangle|^2 &= |\langle \bar{f}|B^0(t)\rangle|^2 \\
&= \tfrac{1}{4|p|^2}|\langle \bar{f}|B_L(t)\rangle + \langle \bar{f}|B_H(t)\rangle|^2 \\
&= \frac{1}{4|p|^2}|q\bar{A}e^{(-im_L-\Gamma_L/2)t} - q\bar{A}e^{(-im_H-\Gamma_H/2)t}|^2 \\
&= \tfrac{1}{4}|\tfrac{q}{p}|^2|\bar{A}|^2(e^{-\Gamma_L t} + e^{-\Gamma_H t} - 2e^{-(\Gamma_H+\Gamma_L)t/2}\cos\Delta m t)
\end{aligned}
$$

Anatomie of these Equations (1)

Unmixed:

$$|\langle f|H|B^0\rangle|^2 = \frac{1}{4}|A|^2(e^{-\Gamma_L t} + e^{-\Gamma_H t} + 2e^{-(\Gamma_H+\Gamma_L)t/2}\cos\Delta mt)$$

Mixed:

$$|\langle \bar{f}|H|B^0\rangle|^2 = \frac{1}{4}\left|\frac{q}{p}\right|^2|\bar{A}|^2(e^{-\Gamma_L t} + e^{-\Gamma_H t} - 2e^{-(\Gamma_H+\Gamma_L)t/2}\cos\Delta mt)$$

|q/p| =1 unless there is CP violation in mixing itself.

$|A| = |\bar{A}|$ unless there is CP violation in the decay.

We will discuss both of these in more detail later!

Anatomie of these Equations (2)

Unmixed:

$$|\langle f|H|B^0\rangle|^2 = \frac{1}{4}|A|^2(e^{-\Gamma_L t} + e^{-\Gamma_H t} \boxed{+} 2e^{-(\Gamma_H+\Gamma_L)t/2}\cos\Delta m t)$$

Mixed:

$$|\langle \bar{f}|H|B^0\rangle|^2 = \frac{1}{4}\left|\frac{q}{p}\right|^2|\bar{A}|^2(e^{-\Gamma_L t} + e^{-\Gamma_H t} \boxed{-} 2e^{-(\Gamma_H+\Gamma_L)t/2}\cos\Delta m t)$$

cosΔmt enters with different sign for mixed and unmixed!

$$\frac{\text{Unmixed - Mixed}}{\text{Unmixed + Mixed}} = \frac{2e^{-(\Gamma_H+\Gamma_L)t/2}}{e^{-\Gamma_L t}+e^{-\Gamma_H t}}\cos\Delta m t$$

Assuming no CP violation in mixing or decay.

Will explain when this is a reasonable assumption later.

Anatomie of these Equations (3)

Unmixed:

$$|\langle f|H|B^0\rangle|^2 = \frac{1}{4}|A|^2(e^{-\Gamma_L t} + e^{-\Gamma_H t} \boxed{+} 2e^{-(\Gamma_H+\Gamma_L)t/2}\cos\Delta m t)$$

Mixed:

$$|\langle \bar{f}|H|B^0\rangle|^2 = \frac{1}{4}\left|\frac{q}{p}\right|^2|\bar{A}|^2(e^{-\Gamma_L t} + e^{-\Gamma_H t} \boxed{-} 2e^{-(\Gamma_H+\Gamma_L)t/2}\cos\Delta m t)$$

cosΔmt enters with different sign for mixed and unmixed!

$$\frac{\text{Unmixed - Mixed}}{\text{Unmixed + Mixed}} \approx \cos\Delta m t$$

Assuming no CP violation in mixing or decay,
and $\frac{\Delta\Gamma}{\Gamma} << 1$

Anatomie of these Equations (4)

Unmixed:

$$|\langle f|H|B^0\rangle|^2 = \frac{1}{4}|A|^2(e^{-\Gamma_L t} + e^{-\Gamma_H t} + 2e^{-(\Gamma_H+\Gamma_L)t/2}\cos\Delta m t)$$

Mixed:

$$|\langle \bar{f}|H|B^0\rangle|^2 = \frac{1}{4}|\frac{q}{p}|^2|\bar{A}|^2(e^{-\Gamma_L t} + e^{-\Gamma_H t} - 2e^{-(\Gamma_H+\Gamma_L)t/2}\cos\Delta m t)$$

Now assume that you did not tag the flavor at production, and there is no CP violation in mixing or decay, i.e. $|q/p|=1$ and $|A| = |\bar{A}|$

$$|\langle f|H|B^0\rangle|^2 + |\langle \bar{f}|H|B^0\rangle|^2 = \frac{1}{2}|A|^2(e^{-\Gamma_L t} + e^{-\Gamma_H t})$$

All you see is the sum of two exponentials for the two lifetimes.

Summary so far

- We discussed the basic formalism for matter <-> antimatter oscillations.
- We showed how this is intricately related to:
  - Mass difference of the mass eigenstates
  - Lifetime difference of the mass eigenstates
  - CP violation in the decay amplitude
  - CP violation in the mixing amplitude
- We discussed how the formalism simplifies in the B-meson system due to natures choice of M_{12} and Γ_{12} .
- We showed how one can measure $\cos\Delta mt$.

CKM Convention

(same as Richman's paper)

- Down type quark -> up type quark = V_{ud}
- Anti-down -> anti-up = V_{ud}^*
- Up type quark -> down type quark = V_{ud}^*
- Anti-up -> anti-down = V_{ud}
- This means for mixing:

$$B^0 \overset{M_{12}^*}{\rightarrow} \overline{B^0}$$

$$\bar{b}d \overset{M_{12}^*}{\rightarrow} b\bar{d}$$

$$\Rightarrow \quad \boxed{\frac{M_{12}^*}{|M_{12}|} = \frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*}}$$

Another look at Unitarity of CKM

$UU^\dagger = U^\dagger U = 1$ $\Longrightarrow$ 9 constraints.

$$\begin{aligned} V_{1j}V_{1k}^* + V_{2j}V_{2k}^* + V_{3j}V_{3k}^* &= 0 \\ (j,k) &= (1,2),(1,3),(2,3) \end{aligned}$$

$$\begin{aligned} V_{j1}V_{k1}^* + V_{j2}V_{k2}^* + V_{j3}V_{k3}^* &= 0 \\ (j,k) &= (1,2),(1,3),(2,3) \end{aligned}$$

$$\begin{aligned} V_{j1}V_{j1}^* + V_{j2}V_{j2}^* + V_{j3}V_{j3}^* &= 1 \\ j &= 1,2,3 \end{aligned}$$

Top 6 constraints are triangles in complex plane.

Careful Look at CKM Triangles

Meson	columns to multiply		size of sides
B_d	$\vec{d}\,\vec{b^*}$	$=$	$O(\lambda^3)+O(\lambda^3)+O(\lambda^3)$
B_s	$\vec{s}\,\vec{b^*}$	$=$	$O(\lambda^4)+O(\lambda^2)+O(\lambda^2)$
K^0	$\vec{d}\,\vec{s^*}$	$=$	$O(\lambda)+O(\lambda)+O(\lambda^5)$

Meson	rows to multiply		size of sides
	$\vec{u}\,\vec{t^*}$	$=$	$O(\lambda^3)+O(\lambda^3)+O(\lambda^3)$
	$\vec{c}\,\vec{t^*}$	$=$	$O(\lambda^4)+O(\lambda^2)+O(\lambda^2)$
D^0	$\vec{u}\,\vec{c^*}$	$=$	$O(\lambda)+O(\lambda)+O(\lambda^5)$

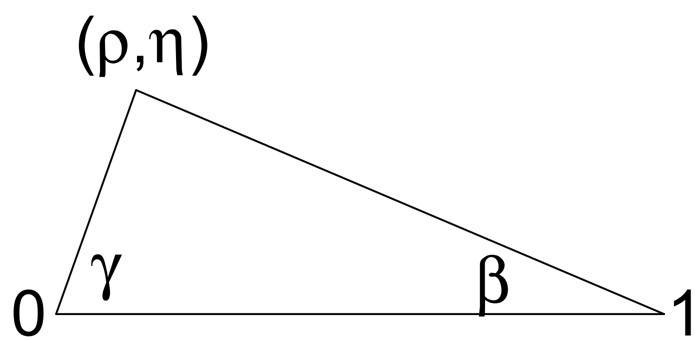
Top quark too heavy to produce bound states.
Most favorable aspect ratio is found in Bd triangle.

Standard CKM Conventions

(same as Richman's paper)

$$\begin{array}{ccccccc} V_{ub}^* V_{ud} & + & V_{cb}^* V_{cd} & + & V_{tb}^* V_{td} & = & 0 \\ (1-\frac{1}{2}\lambda^2)(\rho+i\eta) & & & + & (1-\rho-i\eta) & = & 1 \end{array}$$

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4)$$



$$V_{ub}^* = |V_{ub}| \times e^{i\gamma} \; ; \; V_{td}^* = |V_{td}| \times e^{i\beta}$$

Another Useful CKM

$$\begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & |V_{ub}|\times e^{-i\gamma} \\ -\lambda & 1-\frac{1}{2}\lambda^2 & |V_{cb}| \\ \lambda|V_{cb}|-|V_{ub}|\times e^{+i\gamma} & -|V_{cb}| & 1 \end{pmatrix}$$

As we will see on Tuesday, this is a useful way of writing the CKM matrix because it involves only parameters that can be measured via tree-level processes.

To the extend that new physics may show up primarily in loops, this way of looking at CKM is thus “new physics free”.

Reminder of CP Asymmetry Basics

- To have a CP asymmetry you need three incredients:
  - Two paths to reach the same fnal state.
  - The two paths differ in CP violating phase.
  - The two paths differ in CP conserving phase.
- Simplest Example: $A + Be^{i\delta}e^{i\phi} \xrightarrow{CP} A + Be^{i\delta}e^{-i\phi}$

$$\frac{\left|A + Be^{i\delta}e^{i\phi}\right|^2 - \left|A + Be^{i\delta}e^{-i\phi}\right|^2}{\left|A + Be^{i\delta}e^{i\phi}\right|^2 + \left|A + Be^{i\delta}e^{-i\phi}\right|^2} = \frac{2AB\sin\delta\sin\phi}{A^2 + B^2 + 2AB\cos\delta\cos\phi}$$

Three Types of CP Violation

- Direct = CP violation in the decay:

$$\frac{\left|\left\langle f \middle| B^0 \right\rangle\right|^2 - \left|\left\langle \bar{f} \middle| \overline{B^0} \right\rangle\right|^2}{\left|\left\langle f \middle| B^0 \right\rangle\right|^2 + \left|\left\langle \bar{f} \middle| \overline{B^0} \right\rangle\right|^2} = \frac{|A|^2 - |\overline{A}|^2}{|A|^2 + |\overline{A}|^2} \neq 0 \leftrightarrow \left|\frac{\overline{A}}{A}\right| \neq 1$$

- CP violation in mixing:

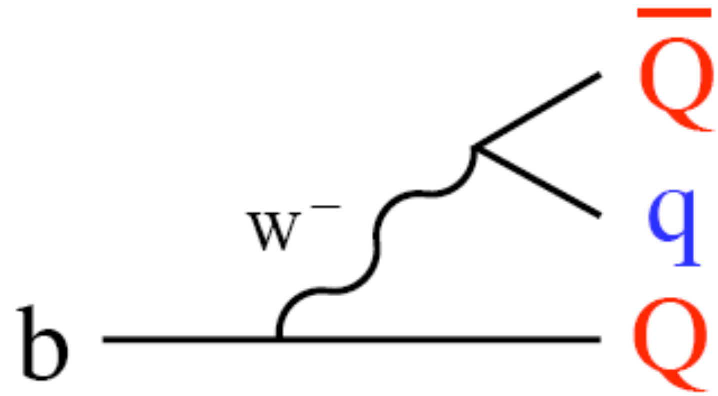
$$\left|\frac{q}{p}\right| \neq 1$$

- CP violation in interference of mixing and decay.

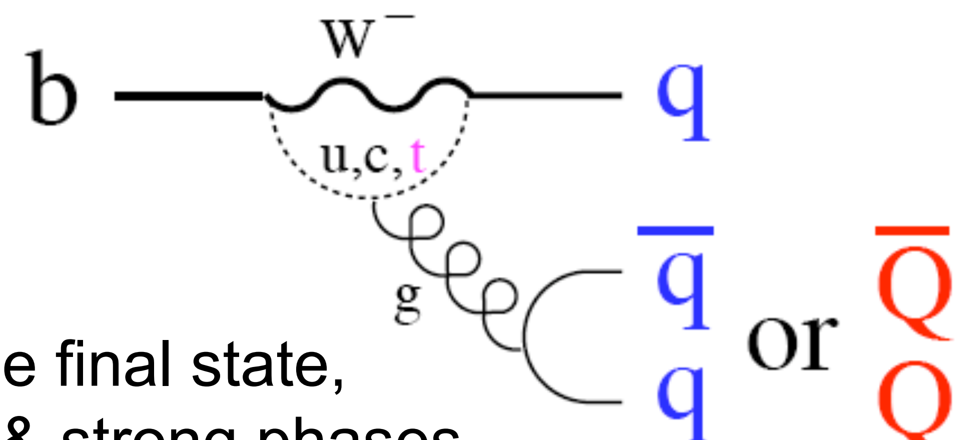
$$\frac{|\langle f_{cp}|H|\overline{B^0}\rangle|^2 - |\langle f_{cp}|H|B^0\rangle|^2}{|\langle f_{cp}|H|\overline{B^0}\rangle|^2 + |\langle f_{cp}|H|B^0\rangle|^2} \propto \mathrm{Im}\left(\frac{q}{p}\frac{\overline{A}}{A}\right) \neq 0$$

Example Direct CP Violation

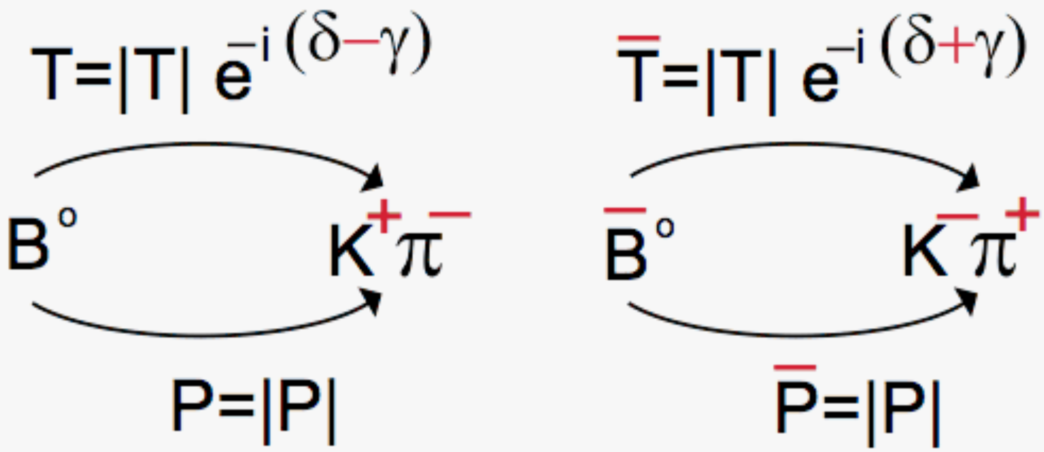
"Tree" Diagram



"Penguin" Diagram



Both can lead to the same final state,
And have different weak & strong phases.



Breaking CP is easy

⇒*Add complex coupling to Lagrangian.*

⇒*Allow 2 or more channels*

⇒*Add CP symm. Phase, e.g. via dynamics.*

$\delta =$ strong phase shift

$\gamma =$ difference in weak phase

CP $\gamma = -\gamma$ CP $\delta = +\delta$

T,P are real numbers.

$$A_{cp} = \frac{\mathcal{B}(B^0 \to K^+\pi^-) - \mathcal{B}(\bar{B}^0 \to K^-\pi^+)}{\mathcal{B}(B^0 \to K^+\pi^-) + \mathcal{B}(\bar{B}^0 \to K^-\pi^+)} = \frac{\left|P + Te^{-i(\delta-\gamma)}\right| - \left|P + Te^{-i(\delta+\gamma)}\right|}{\left|P + Te^{-i(\delta-\gamma)}\right| + \left|P + Te^{-i(\delta+\gamma)}\right|}$$

$$= \frac{-2|TP| \sin\gamma \sin\delta}{|T|^2 + |P|^2 + 2|TP| \cos\gamma \cos\delta}$$

The rest is simple algebra.

CP Violation in Mixing

- Pick decay for which there is only one diagram, e.g. semileptonic decay.

$$\frac{\Gamma(\overline{B^0}(t)\rightarrow l^+\nu X)-\Gamma(B^0(t)\rightarrow l^-\overline{\nu}X)}{\Gamma(\overline{B^0}(t)\rightarrow l^+\nu X)+\Gamma(B^0(t)\rightarrow l^-\overline{\nu}X)} =$$

$$= \frac{1-|q/p|^4}{1+|q/p|^4} = Im\frac{\Gamma_{12}}{M_{12}}$$

Verifying the algebra, incl. the sign, is part of HW.

CP Asymmetry in mixing

$$\frac{\left|\left\langle \bar{f}\middle|H\middle|B^0\right\rangle\right|^2 - \left|\left\langle f\middle|H\middle|\overline{B^0}\right\rangle\right|^2}{\left|\left\langle \bar{f}\middle|H\middle|B^0\right\rangle\right|^2 + \left|\left\langle f\middle|H\middle|\overline{B^0}\right\rangle\right|^2} \propto Im\frac{\Gamma_{12}}{M_{12}}$$

Measuring cosΔmt in mixing

$$\frac{\left(\left|\left\langle f\middle|H\middle|B^0\right\rangle\right|^2 + \left|\left\langle \bar{f}\middle|H\middle|\overline{B^0}\right\rangle\right|^2\right) - \left(\left|\left\langle \bar{f}\middle|H\middle|B^0\right\rangle\right|^2 + \left|\left\langle f\middle|H\middle|\overline{B^0}\right\rangle\right|^2\right)}{\left(\left|\left\langle f\middle|H\middle|B^0\right\rangle\right|^2 + \left|\left\langle \bar{f}\middle|H\middle|\overline{B^0}\right\rangle\right|^2\right) + \left(\left|\left\langle \bar{f}\middle|H\middle|B^0\right\rangle\right|^2 + \left|\left\langle f\middle|H\middle|\overline{B^0}\right\rangle\right|^2\right)} \propto \cos\Delta mt$$

Summary Thus Far

(It's common for different people to use different definitions of $\Delta\Gamma$, and thus different sign!)

$$\Delta m = 2\left|M_{12}\right|$$

$$\Delta\Gamma = -2\left|\Gamma_{12}\right| \times \cos\left(Arg\left(\Gamma_{12}^{*} M_{12}\right)\right)$$

$$\frac{\left|\left\langle \bar{f}\middle|H\middle|B^0\right\rangle\right|^2 - \left|\left\langle f\middle|H\middle|\overline{B^0}\right\rangle\right|^2}{\left|\left\langle \bar{f}\middle|H\middle|B^0\right\rangle\right|^2 + \left|\left\langle f\middle|H\middle|\overline{B^0}\right\rangle\right|^2} \propto \left|\frac{\Gamma_{12}}{M_{12}}\right| \times \sin\left(Arg\left(\Gamma_{12}^{*} M_{12}\right)\right)$$

It's your homework assignment to sort out algebra and sign.
I was deliberately careless here!
Make sure you are completely clear how you define $\Delta\Gamma$!!!

Aside on rephasing Invariance

- Recall that we are allowed to multiply quark fields with arbitrary phases.
- This is referred to as “rephasing”, and directly affects the CKM matrix convention as follows:

$$\overline{(u,\ c,\ t)}_L \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L = \overline{(u,\ c,\ t)}_L \begin{pmatrix} V_{ud}\, e^{-i\phi} & V_{us} & V_{ub} \\ V_{cd}\, e^{-i\phi} & V_{cs} & V_{cb} \\ V_{td}\, e^{-i\phi} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d\, e^{i\phi} \\ s \\ b \end{pmatrix}_L$$

All physical observables must depend on combinations of CKM matrix elements where a quark subscript shows up as part of a V and a V^* .

Examples:

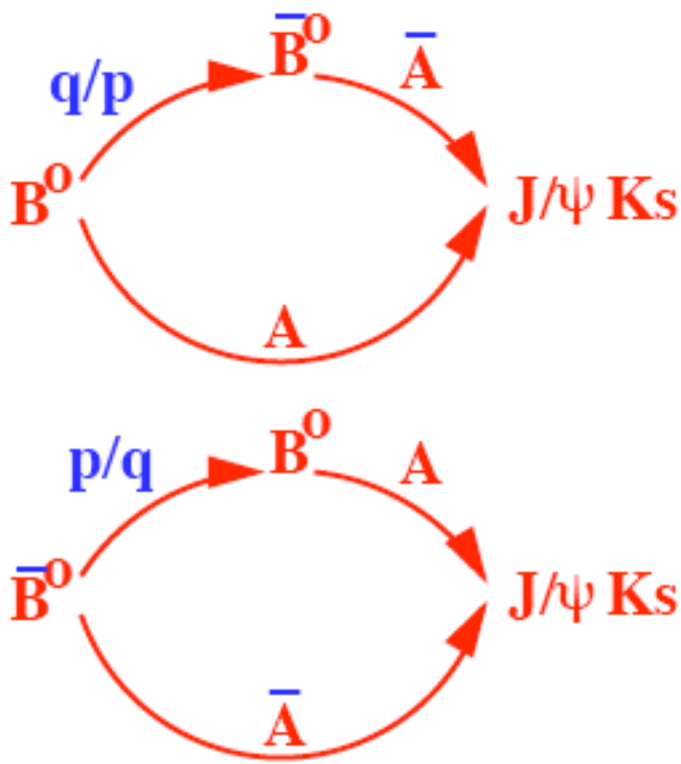
- Decay rate if the process is dominated by one diagram:
  - $|A|^2 \propto V_{cb} V_{ud}^* \ V_{cb}^* V_{ud}$
- Mixing: $\frac{M_{12}^*}{|M_{12}|} = \frac{V_{tb}^* V_{ts}}{V_{tb} V_{ts}^*}$; $\frac{\Gamma_{12}^*}{|\Gamma_{12}|} = \frac{V_{cb}^* V_{cs}}{V_{cb} V_{cs}^*}$

Neither of M_{12} nor Γ_{12} is rephasing invariant by themselves. However, the product $M_{12}\Gamma_{12}^*$ is rephasing invariant.

$V_{tb} V_{ts}^* \ V_{cb}^* V_{cs}$ = rephasing invariant

- In principle, these three measurements allow extraction of all the relevant parameters.
- In practice, Γ_{12} for both B_d and B_s is too small to be easily measurable.
- Extraction of the phase involved is thus not easily possible.
- Thankfully, there's another way of determining "the phase of mixing".

Interference of Mixing and Decay



J/psi Ks is a CP eigenstate.

Flavor tag B at production.
Measure rate as a function of proper time between production and decay.

This allows measurement of the relative phase of A and q/p.

$$A_{cp}(t) = \frac{|\langle f_{cp}|H|\overline{B^0}\rangle|^2 - |\langle f_{cp}|H|B^0\rangle|^2}{|\langle f_{cp}|H|\overline{B^0}\rangle|^2 + |\langle f_{cp}|H|B^0\rangle|^2}$$

Simplifying Assumptions and their Justification

- There is no direct CP violation
    - b->c cbar s tree diagram dominates
    - Even if there was a penguin contribution, it would have (close to) the same phase: $Arg(V_{tb}V_{ts}^*) \sim Arg(V_{cb}V_{cs}^*)$
- Lifetime difference in Bd system is vanishingly small -> effects due to Γ_{12} can be ignored.
- Top dominates the box diagram.
    – See HW.

$$
\begin{aligned}
A_{cp}(t) &= \frac{|\langle f_{cp}|H|\overline{B^0}\rangle|^2 - |\langle f_{cp}|H|B^0\rangle|^2}{|\langle f_{cp}|H|\overline{B^0}\rangle|^2 + |\langle f_{cp}|H|B^0\rangle|^2} \\
&= \boxed{\eta_{cp} Im\left(\frac{q}{p}\frac{\overline{A}}{A}\right)} \cdot sin\Delta mt
\end{aligned}
$$

Let's look at this in some detail!

$$
\begin{aligned}
J^{PC}(J/\psi) &= 1^{--} \Rightarrow CP \text{ even} \\
J^{PC}(K_s) &= 0^{--} \Rightarrow CP \text{ even}
\end{aligned}
$$

J/psi Ks must be P-wave => ***overall CP of the final state = -1***

$$
\begin{aligned}
A_{cp}(t) &= -Im\left(\frac{M_{12}^*}{|M_{12}|}\frac{\overline{A}}{A}\right)\cdot\sin\Delta m t \\
&= -Im\left(\frac{V_{tb}^*V_{td}}{V_{tb}V_{td}^*}\cdot\frac{V_{cb}V_{cs}^*}{V_{cb}^*V_{cs}}\cdot\boxed{\frac{V_{cs}V_{cd}^*}{V_{cs}^*V_{cd}}}\right)\sin\Delta m t \\
&= -Im\left(\boxed{\frac{V_{tb}^*V_{td}}{V_{tb}V_{td}^*}\cdot\frac{V_{cb}V_{cd}^*}{V_{cb}^*V_{cd}}}\right)\sin\Delta m t
\end{aligned}
$$

Some comments are in order here:

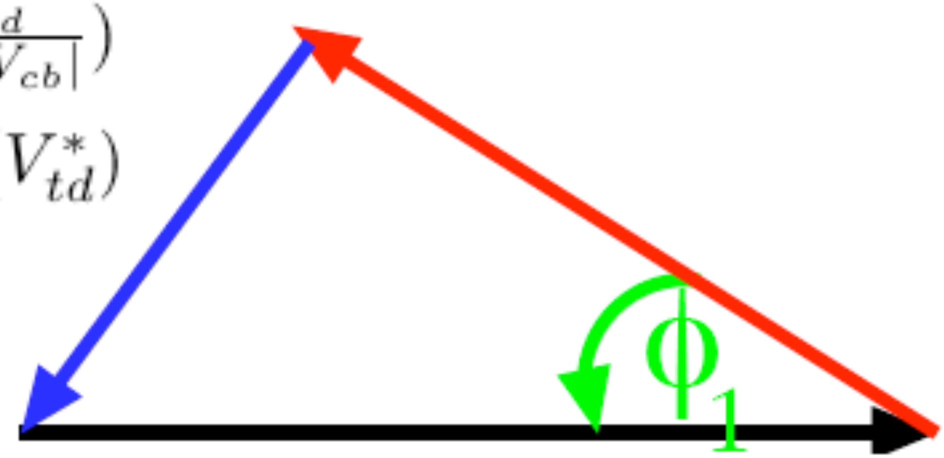
The extra CKM matrix elements enter because of Kaon mixing.
We produce s dbar or sbar d and observe K_s *.*

They are crucial to guarantee rephasing invariant observable: $V_{tb}^* V_{td} V_{cb} V_{cd}^*$

Connection To Triangle

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$
$$\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} + 1 + \frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*} = 0$$

$$Arg\left(\frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*}\right) = Arg\left(\frac{V_{td}}{-\lambda|V_{cb}|}\right) = \pi - Arg(V_{td}^*)$$

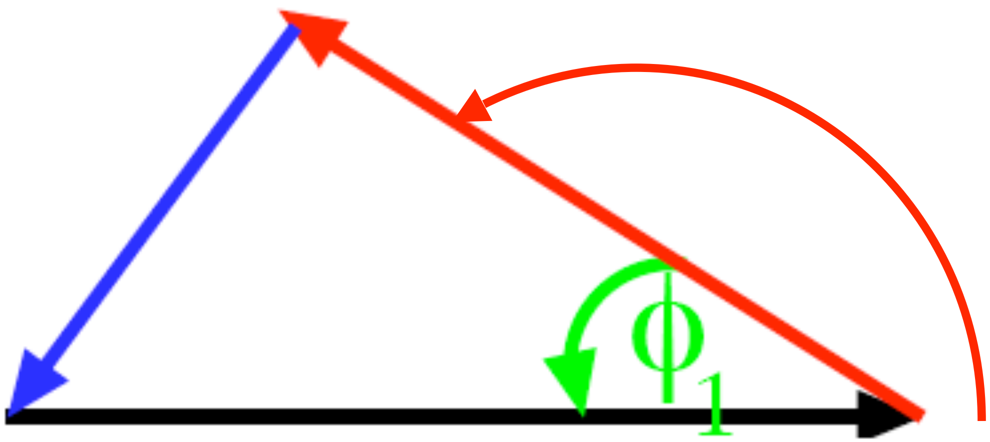


$$\frac{V_{td}}{-\lambda|V_{cb}|} = e^{i(\pi - Arg(V_{td}^*))} \times \left|\frac{V_{td}}{\lambda V_{cb}}\right|$$

Connection To Triangle

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$
$$\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} + 1 + \frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*} = 0$$

$$Arg(\frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*}) = Arg(\frac{V_{td}}{-\lambda|V_{cb}|})$$
$$= \pi - Arg(V_{td}^*)$$



$$\phi_1 = \pi - Arg(\frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*})$$
$$= \pi - (\pi - Arg(V_{td}^*))$$
$$= Arg(V_{td}^*)$$
$$= \beta$$

Putting the pieces together

$$A_{CP}(t) = \eta_{CP}\,\mathrm{Im}\left(\frac{q}{p}\frac{\bar{A}}{A}\right)\sin\Delta mt$$

$$= -\mathrm{Im}\left(\frac{V_{td}V_{tb}^*}{V_{td}^*V_{tb}}\frac{V_{cb}V_{cd}^*}{V_{cb}^*V_{cd}}\right)\sin\Delta mt$$

$$= -\sin\left(2Arg\left(V_{td}\right)\right)\sin\Delta mt$$

$$\boxed{A_{CP}(t) = \sin 2\beta \sin\Delta mt}$$

For B->J/psi Ks

Note: I do not use the same sign conventions as Jeff Richman !!!

Accordingly, I get the opposite sign for A_{CP} .

In HW, you are asked to do this yourself. Make sure you state clearly how you define A_{CP} !!!